Version LX/6

EGT3
ENGINEERING TRIPOS PART IIB

Tuesday 29 April 2014 2 to 3.30

Module 4A11

TURBOMACHINERY II

Answer not more than ***two*** *questions.*

All questions carry the same number of marks.

The ***approximate*** *percentage of marks allocated to each part of a question is indicated in the right margin.*

Write your candidate number ***not*** *your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
Attachment: Extract from Compressible Flow Tables (1 page)
Engineering Data Book

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

1 In this question, all symbols not specifically defined have their usual meanings.

A low speed model axial turbine stage has cylindrical casing and hub lines, with radially stacked blades. The flow is uniform and axial at the turbine inlet and axial at the stage exit. A free vortex design for the rV_θ distribution is used in the stator/rotor gap. The flow in the stage can be assumed as incompressible, steady and axisymmetric.

(a) Justify the use of the Simple Radial Equilibrium equation downstream of the stator to determine the pressure distribution along the span and use the Simple Radial Equilibrium equation to find the static pressure distribution downstream of the stator. Find the non-dimensional pressure difference coefficient defined as $\frac{p_c - p_h}{\rho V_{\theta,c}^2}$ in terms of the hub-to-tip radial ratio r_h / r_c, where the subscripts h and c refer to the hub and the casing respectively. [30%]

(b) It is found that for low r_h / r_c and high stator exit flow angle, the hub section of the stage has low reaction. Name the primary cause of the low reaction at the hub. [10%]

(c) It is suggested that the situation may be relieved by leaning the stator circumferentially. Comment why and how leaning the stator can change the reaction near the hub. [10%]

(d) Explain how the Simple Radial Equilibrium equation must be modified to take account of the radial blade force produced by leaning the blade. [20%]

(e) Estimate the change of the non-dimensional pressure difference coefficient due to the blade lean in terms of the lean angle γ, the stator exit tangential flow angle at the casing α_c, the stator aspect ratio $(r_c - r_h)/C_x$, and r_h / r_c. [30%]

2. In this question, all symbols not specifically defined have their usual meanings.

(a) (i) For a compressor, derive the expression for isentropic efficiency in terms of the entropy increase across the compressor. State carefully the approximations involved. What is the equivalent expression for a turbine? [10%]

(ii) Show that the non-dimensional entropy increase defined as $\Delta s / R$ in an air compressor blade section operating at an inlet relative Mach number M_{in} can be related approximately to the stagnation pressure loss coefficient Y_p by

$$\frac{\Delta s}{R} = Y_p \left(1 - (1 + \frac{\gamma - 1}{2} M_{in}^2)^{\frac{-\gamma}{\gamma - 1}} \right) .$$

State any assumptions you make in deriving this expression. [20%]

(iii) A transonic air compressor rotor section has a stagnation pressure ratio of 1.6 and an isentropic efficiency of 0.88 when operating at a relative inlet Mach number $M_{in} = 1.35$ and with a relative inlet stagnation temperature of 366 K. The absolute stagnation temperature at the inlet is 288 K. It is found that at this condition the section has a single passage shock wave normal to the flow and a shock upstream Mach number $M_{shock} = 1.4$. Calculate the loss of efficiency due to the entropy generation across the shock wave and that due to viscous dissipations outside the shock wave. What is the value of Y_p attributable to the viscous effects? Comment on your results. Take $\gamma = 1.4$ and $R = 287$ J kg^{-1}K^{-1} . [30%]

(b) (i) Using the jet mixing kinetic energy loss equation $\Delta h = \frac{\dot{m}_j}{\dot{m}_m} V_m^2 (1 - \frac{V_j}{V_m} \cos\alpha)$, show that the mixing loss due to the leakage flow over an infinitesimally small blade chord length dl of an unshrouded compressor blade tip can be expressed as

$$T\Delta\dot{S} = \dot{m}_m T ds = V_s^2 (1 - \frac{V_p}{V_s}) d\dot{m} \ ;$$

where subscript j denotes jet and m the main stream, $\dot{m}_j << \dot{m}_m$, $d\dot{m}$ the leakage mass flow rate through a small chord length dl , V_s and V_p the local isentropic velocities on the suction and the pressure surfaces respectively, and α the angle between the leakage jet velocity and the main stream velocity. [25%]

(ii) Describe the mechanisms for entropy generation in shrouded and unshrouded turbine blade tips. Discuss why the stage reaction is an important parameter when determining whether to use a shrouded or an unshrouded blade. [15%]

3 (a) The Axial Velocity Density Ratio across the mid-span of a blade row in an axial flow turbomachine is defined as:

$$\text{AVDR} = \frac{\overline{\rho_2 v_{x2}}}{\overline{\rho_1 v_{x1}}}$$

where ρ is the density, v_x is the axial velocity, subscripts 1 and 2 refer to just upstream and just downstream of the blade row respectively and the over bar means pitchwise average of the quantity beneath.

(i) Explain how the quantity $\overline{\rho_2 v_{x2}}$ must be evaluated to account for the presence of blade wakes. [10%]

(ii) Explain the significance of the AVDR and how its value is likely to differ between a compressor and turbine blade row. [20%]

(b) The flow through a high aspect ratio turbine stator blade row can exhibit very different characteristics in the endwall flow regions compared with those at mid-span.

(i) Give two, physically based, explanations for this phenomenon. [20%]

(ii) Describe the basic characteristics of the flow turning, boundary layer and entropy production in the endwall region. [20%]

(c) The last stage turbine blade rows in many industrial gas turbines have an increasing annular cross-sectional area. Comment on:

(i) why this can be beneficial to the overall performance; [10%]

(ii) the likely effect on the work extraction from the stage; [10%]

(iii) the likely effect on the blade surface and end-wall boundary layers. [10%]

END OF PAPER

Attachment: Extract from Compressible Flow tables

M	$\frac{T}{T_0}$	$\frac{p}{p_0}$	$\frac{\rho}{\rho_0}$	$\frac{V}{\sqrt{c_p T_0}}$	$\frac{\dot{m}\sqrt{c_p T_0}}{A p_0}$	$\frac{\dot{m}\sqrt{c_p T_0}}{A p}$	$\frac{F}{\dot{m}\sqrt{c_p T_0}}$	$\frac{4 c_f L_{max}}{D}$	$\frac{\frac{1}{2}\rho V^2}{p_0}$	M_s	$\frac{P_{0s}}{P_0}$	$\frac{P_s}{P}$	$\frac{P_{0s}}{P}$	$\frac{T_s}{T}$	ν	M
1.260	0.7590	0.3809	0.5019	0.6943	1.2195	3.2015	1.0066	0.0517	0.4233	0.8071	0.9857	1.6855	2.5875	1.1657	5.09	1.260
1.270	0.7561	0.3759	0.4971	0.6984	1.2152	3.2331	1.0077	0.0549	0.4244	0.8016	0.9842	1.7151	2.6186	1.1720	5.36	1.270
1.280	0.7532	0.3708	0.4923	0.7026	1.2107	3.2648	1.0089	0.0582	0.4253	0.7963	0.9827	1.7448	2.6500	1.1783	5.63	1.280
1.290	0.7503	0.3658	0.4876	0.7067	1.2061	3.2967	1.0100	0.0615	0.4262	0.7911	0.9811	1.7748	2.6816	1.1846	5.90	1.290
1.300	0.7474	0.3609	0.4829	0.7108	1.2014	3.3287	1.0112	0.0648	0.4270	0.7860	0.9794	1.8050	2.7136	1.1909	6.17	1.300
1.310	0.7445	0.3560	0.4782	0.7149	1.1965	3.3608	1.0124	0.0682	0.4277	0.7809	0.9776	1.8355	2.7459	1.1972	6.44	1.310
1.320	0.7416	0.3512	0.4736	0.7189	1.1916	3.3931	1.0136	0.0716	0.4283	0.7760	0.9758	1.8661	2.7784	1.2035	6.72	1.320
1.330	0.7387	0.3464	0.4690	0.7229	1.1866	3.4255	1.0149	0.0750	0.4289	0.7712	0.9738	1.8971	2.8112	1.2099	7.00	1.330
1.340	0.7358	0.3417	0.4644	0.7270	1.1815	3.4581	1.0161	0.0785	0.4294	0.7664	0.9718	1.9282	2.8444	1.2162	7.28	1.340
1.350	0.7329	0.3370	0.4598	0.7309	1.1763	3.4907	1.0174	0.0820	0.4299	0.7618	0.9697	1.9596	2.8778	1.2226	7.56	1.350
1.360	0.7300	0.3323	0.4553	0.7349	1.1710	3.5236	1.0187	0.0855	0.4303	0.7572	0.9676	1.9912	2.9115	1.2290	7.84	1.360
1.370	0.7271	0.3277	0.4508	0.7388	1.1656	3.5566	1.0200	0.0890	0.4306	0.7527	0.9653	2.0231	2.9455	1.2354	8.13	1.370
1.380	0.7242	0.3232	0.4463	0.7427	1.1601	3.5897	1.0213	0.0926	0.4308	0.7483	0.9630	2.0551	2.9798	1.2418	8.41	1.380
1.390	0.7213	0.3187	0.4418	0.7466	1.1546	3.6229	1.0226	0.0962	0.4310	0.7440	0.9607	2.0875	3.0144	1.2482	8.70	1.390
1.400	0.7184	0.3142	0.4374	0.7505	1.1490	3.6563	1.0240	0.0997	0.4311	0.7397	0.9582	2.1200	3.0492	1.2547	8.99	1.400
1.410	0.7155	0.3098	0.4330	0.7543	1.1433	3.6899	1.0253	0.1033	0.4312	0.7355	0.9557	2.1528	3.0844	1.2612	9.28	1.410
1.420	0.7126	0.3055	0.4287	0.7581	1.1375	3.7236	1.0267	0.1069	0.4312	0.7314	0.9531	2.1858	3.1198	1.2676	9.57	1.420
1.430	0.7097	0.3012	0.4244	0.7619	1.1317	3.7574	1.0281	0.1106	0.4311	0.7274	0.9504	2.2191	3.1555	1.2741	9.86	1.430
1.440	0.7069	0.2969	0.4201	0.7657	1.1258	3.7914	1.0295	0.1142	0.4310	0.7235	0.9476	2.2525	3.1915	1.2807	10.15	1.440
1.450	0.7040	0.2927	0.4158	0.7694	1.1198	3.8255	1.0308	0.1178	0.4308	0.7196	0.9448	2.2863	3.2278	1.2872	10.44	1.450
1.460	0.7011	0.2886	0.4116	0.7732	1.1138	3.8598	1.0323	0.1215	0.4306	0.7157	0.9420	2.3202	3.2643	1.2938	10.73	1.460
1.470	0.6982	0.2845	0.4074	0.7769	1.1077	3.8942	1.0337	0.1251	0.4303	0.7120	0.9390	2.3544	3.3011	1.3003	11.02	1.470
1.480	0.6954	0.2804	0.4032	0.7805	1.1016	3.9287	1.0351	0.1288	0.4299	0.7083	0.9360	2.3888	3.3382	1.3069	11.32	1.480
1.490	0.6925	0.2764	0.3991	0.7842	1.0954	3.9634	1.0365	0.1324	0.4295	0.7047	0.9329	2.4235	3.3756	1.3136	11.61	1.490
1.500	0.6897	0.2724	0.3950	0.7878	1.0891	3.9983	1.0379	0.1361	0.4290	0.7011	0.9298	2.4583	3.4133	1.3202	11.91	1.500