①

(a). Upstream of the stator flow uniform ⇒ P_0, T_0 uniform
downstream of the stator T_0 uniform, assume $\frac{\partial P_0}{\partial r}$ small along span. for $drV_\theta/dr = 0$, $\partial V_x/\partial r = 0$ ∴ no streamline curvature for cylindrical annulus. SRE is valid.

from simple radial equilibrium, $dP/dr = \rho \frac{V_\theta^2}{r}$

$$\int dp = \int \rho \frac{(rV_\theta)^2}{r^3} dr = -\rho (r_c V_{\theta,c})^2 \cdot \frac{1}{2r^2}\Big|_{r_h}^{r_c} \quad ; \quad P(r) = P_{hub} + \frac{\rho r_c^2 V_{\theta c}^2}{2}\left[\frac{1}{r_h^2} - \frac{1}{r_c^2}\right]$$

$$\frac{\Delta P}{\rho V_{\theta,c}^2} = \frac{P_c - P_h}{\rho V_{\theta,c}^2} = \frac{r_c^2}{2}\left[\frac{1}{r_h^2} - \frac{1}{r_c^2}\right] = \frac{1}{2}\left[\frac{1}{\left(\frac{r_h}{r_c}\right)^2} - 1\right]$$

[30%]

(b). for low r_h/r_c and large α_c, $\Delta P = \frac{1}{2}\rho V_{\theta c}^2 \cdot \left[\frac{1}{\left(\frac{r_h}{r_c}\right)^2} - 1\right]$ increases rapidly, causing low P at the hub thus driving down the reaction.

[10%]

(c) By leaning the blade ~~towards~~ with the pressure surface downwards, a force field is created which provide extra force downward required to maintain radial equilibrium. thus the large radial pressure gradient can be reduced.

[10%]

(d). A radial component of the blade force should be added to the radial equilibrium equation. For a mean blade force in tangential direction $\overline{\Delta p}$, its component in r direction is $\overline{\Delta p} \cdot h/s \cdot \tan\gamma$, h: blade height, s mean pitch, and γ the lean angle. the radial equilibrium is: $\rho \frac{V_\theta^2}{r} = \frac{dP}{dr} + \frac{\overline{\Delta P}}{s} \cdot h \cdot \tan\gamma$

[20%]

(e). momentum of moment: $\overline{\Delta p} \cdot C_x \cdot r \cdot dh = \rho V_x dh \cdot S \cdot r V_\theta$, $V_x = V_\theta (\tan\alpha)^{-1}$

$\overline{\Delta p} \cdot C_x = \rho \cdot S \cdot V_{\theta,m} \cdot (\tan\alpha_c)^{-1} V_{\theta,c} = \rho V_{\theta,c}^2 \cdot S \cdot \frac{r_c}{r_m} \cdot (\tan\alpha_c)^{-1}$; $\overline{\Delta p} = \rho V_{\theta,c}^2 \frac{S}{C_x} \cdot \frac{r_c}{r_m} \cdot (\tan\alpha_c)$

$\overline{\Delta p}$ | in r direction component $= \overline{\Delta p} \cdot \tan\gamma \frac{h}{s}$; $r_m = \frac{1}{2}(r_c + r_h)$; $\frac{r_c}{r_m} = \frac{2}{1 + r_h/r_c}$.

$$\frac{\overline{\Delta p}}{\rho V_{\theta,c}^2}\Big|_r = \frac{2}{1 + r_h/r_c} \frac{h\,S}{S\,C_x} \frac{\tan\gamma}{\tan\alpha_c} = \frac{2}{1 + \frac{r_h}{r_c}} \cdot \frac{h}{C_x} \frac{\tan\gamma}{\tan\alpha_c}$$

[30%]

2. (a) (i) $\eta_c = \frac{\Delta h_{0is}}{\Delta h_0} = \frac{\Delta h_0 - T_2 \Delta S}{\Delta h_0} = 1 - \frac{T_2 \Delta S}{\Delta h_0}$

$$\eta_T = \frac{\Delta h_0}{\Delta h_{0is}} = \frac{\Delta h_0}{\Delta h_0 + T_2\Delta S} = \frac{\Delta h_0}{\Delta h_0 + T_2 \Delta S}$$

assume $T\Delta S \ll \Delta h_0$, and at the exit $T_2 \simeq T_{02}$. [10%]
~~section having constant radius~~.

(ii) for small $\Delta P_0/P_0$ and constant T_0 in a frame of ref. to the blade, $\frac{\Delta S}{R} \doteq \frac{\Delta P_0}{P_0}$ $\Delta S = R\left(\frac{P_{01}-P_{02}}{P_{01}-P_1}\right)\frac{(P_{01}-P_1)}{P_{01}} = R \cdot Y_p \cdot \left(1 - \frac{P_1}{P_{01}}\right)$

$$= R Y_p \left[1 - \left(\frac{T}{T_{01}}\right)^{\frac{\gamma}{\gamma-1}}\right] = R \cdot Y_p \left[1 - \left(\frac{T_{01}}{T_1}\right)^{\frac{-\gamma}{\gamma-1}}\right] = R Y_p \left\{1 - \left(1 + \frac{\gamma-1}{2} M_{in}^2\right)^{\frac{-\gamma}{\gamma-1}}\right.$$

with constant radius, [20%]

(iii). shock loss across normal shock with $M_{in} = 1.4$.

$\frac{P_{02}}{P_{01}} = 0.9582 \Rightarrow \frac{\Delta S}{R} = 1 - 0.9582$, $\Delta S = 287 \cdot 0.0418 = 12\ \mathrm{J/kg}$

$\frac{T_{02is}}{T_{01}} = 1.6^{\frac{\gamma-1}{\gamma}} = 1.6^{0.2857} = 1.1437 \Rightarrow T_{02is} = 329.39\ \mathrm{K}$, $T_{02} = \frac{T_{02is} - T_{01}}{\eta_c} + T_{01} = 335.03\ \mathrm{K}$

$T_s / T_{02ref} = 1.2547 \cdot 0.7184 = 0.9014$. $T_s = T_2 = 302.0\ \mathrm{K}$.

$$\Delta\eta_s = \frac{T_2 \cdot \Delta S}{\Delta h_0} = \frac{302.0 \cdot 12}{1005 \cdot 47} = \underline{\underline{0.0767}}$$

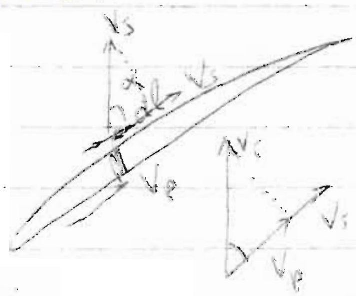
$\Delta S_{s+v} = \frac{0.12 \cdot \Delta h_0}{T_2} = 18.77$. $\Delta S_v = 18.77 - 12 = 6.77$

$Y_{p_v} = \frac{\Delta S_v}{R} \cdot \left[1 - \left(1 + \frac{\gamma-1}{2} M_{in}^2\right)^{\frac{-\gamma}{\gamma-1}}\right]$ $[\quad] = 0.663$

$Y_{p.v} = \frac{6.77}{287} \cdot 0.663 = 0.0156 \Rightarrow$ small.

~~compo~~ relatively, the shock loss is still dominating loss in a transonic compressor. if it is a normal shock. [30%]

(b). (i). For a small tip section dl,



mixing loss $dh = \frac{\dot m_j}{\dot m_m} V_m^2 \left(1 - \frac{V_j}{V_m}\cos\alpha\right)$

here $\dot m_j = d\dot m$, $\dot m_m = \rho V_x S h$ at inlet.

$V_j = V_s$, $V_m = V_s$, $\cos\alpha = \frac{V_p}{V_s}$ Substitute

$\frac{V_p}{V_s} = \cos\alpha$.

$dh = \frac{d\dot m}{\dot m_m} V_s^2 \left(1 - \frac{V_s}{V_s}\frac{V_p}{V_s}\right) = \frac{d\dot m}{\dot m} V_s^2\left(1 - \frac{V_p}{V_s}\right)$

entropy increase due ~~to~~ mixing $ds = \frac{dh}{T} = \frac{1}{T}\frac{d\dot{m}}{\dot{m}} V_s^2 (1 - \frac{V_p}{V_s})$

energy loss due to mixing $= T\Delta s = T ds \cdot \dot{m}_m = d\dot{m}\, V_s^2 (1 - \frac{V_p}{V_s})$

$\therefore T\Delta S = V_s^2 (1 - \frac{V_p}{V_s}) \cdot d\dot{m}$ [25%]

(iii) The leakage flow in a <u>shrouded</u> ~~tip~~ turbine blade is driven by the pressure difference between the blade leading edge and trailing edge across the blade row. This leakage flow has much smaller flow angle compared with the main passage flow thus a mixing loss which is proportional ~~to~~ the leakage mass flow rate. it also enters the next blade row with very different flow angle thus turns very little. and reduces the work done ~~p~~ capacity of the flow as well as to create extra mixing loss. The leakage flow in a <u>unshroude</u> turbine blade is driven by the blade loading at the tip. The pressure difference between the pressure surface and the suction surface. the mixing loss is ~~due to~~ proportional to the leakage mass flow rate. For low reaction blade the cross blade row pressure difference is small but blade loading is high. thus if unshroude the blade-to-blade leakage flow will be high. thus the mixing loss. but due to very low cross blade row pressure difference the leakage through the shrouded tip gap is very low thus for low reaction stage the shrouded tip is preferable whilst for a reaction stage the reaction level at tip section usually is high thus the static pressure drop across the blade which ~~[illegible]~~ is a large driving force for the leakage flow through the shrouded gap but as the relatively the ~~pressure~~ pressure difference across the pressure surface to the suction surface is smaller than that of ~~the~~ an impulse blade, the unshrouded blade tip is preferred. [15%]

a) i) Must area average mass flux across blade pitch.

$$\overline{\rho_2 V_{x2}} = \frac{1}{S}\int_0^S \rho_2(y)\, v_{x2}(y)\, dy$$

This gives the average value consistent with mass conservation at the mid span location.

$\boxed{10\%}$

(ii) <u>Compressor:</u> Relatively small secondary flow (compared to turbine) and high diffusion across the blade row causes the endwall boundary layers to grow, thus increasing hub & casing endwall blockage. Thus in a compressor:

$$AVDR = \frac{\overline{\rho_2 V_{x2}}}{\rho_1 V_{x1}} > \frac{A_1}{A_2}$$

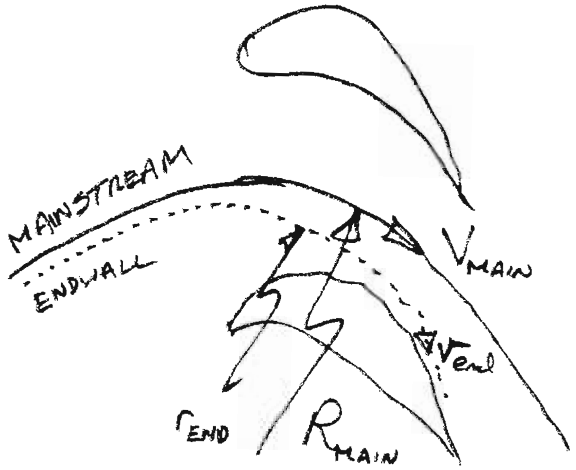
<u>Turbine:</u> There is a large secondary flow which sweeps the incoming endwall boundary layer onto the blade surface. The high acceleration across the blade row causes the new end-wall boundary layer to be relatively thin compared to the incoming one. Hence, in a turbine

$$AVDR = \frac{\overline{\rho_2}\,\overline{V_{x2}}}{\rho_1 V_{x1}} < \frac{A_1}{A_2}$$

$\boxed{20\%}$

b) i) PRESSURE & CURVATURE:

Blade pressure difference is essentially determined by the mainstream (mid-span) flow. Thus



$$\frac{\Delta P}{\text{pitch}} \sim \rho \frac{V_{MAIN}^2}{R_{MAIN}}$$

Endwall boundary layer has a much smaller velocity $v_{end} << V_{MAIN}$ but undergoes the same pressure field. Thus

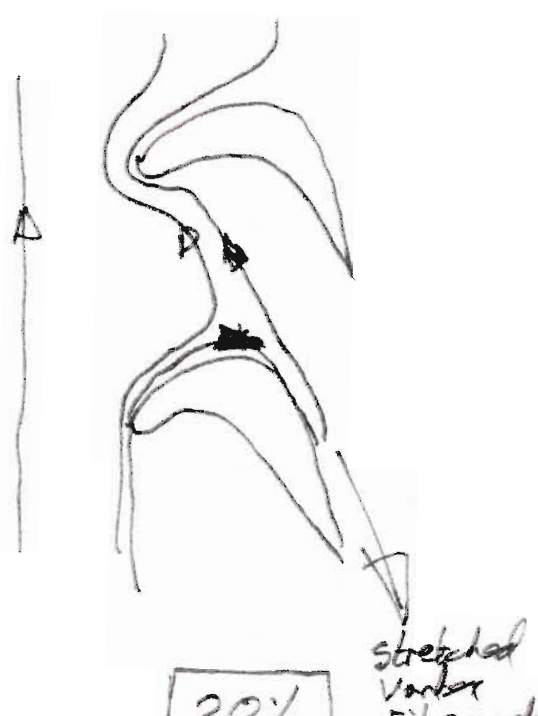
$$\frac{\Delta P}{\text{pitch}} \sim \rho \frac{V_{MAIN}^2}{R_{MAN}} \approx \rho \frac{v_{end}^2}{r_{end}}$$

$\Rightarrow$ since $v_{end} << V_{MAIN}$, $r_{end} << R_{MAIN}$.

Thus endwall boundary layer overturned towards the suction surface.

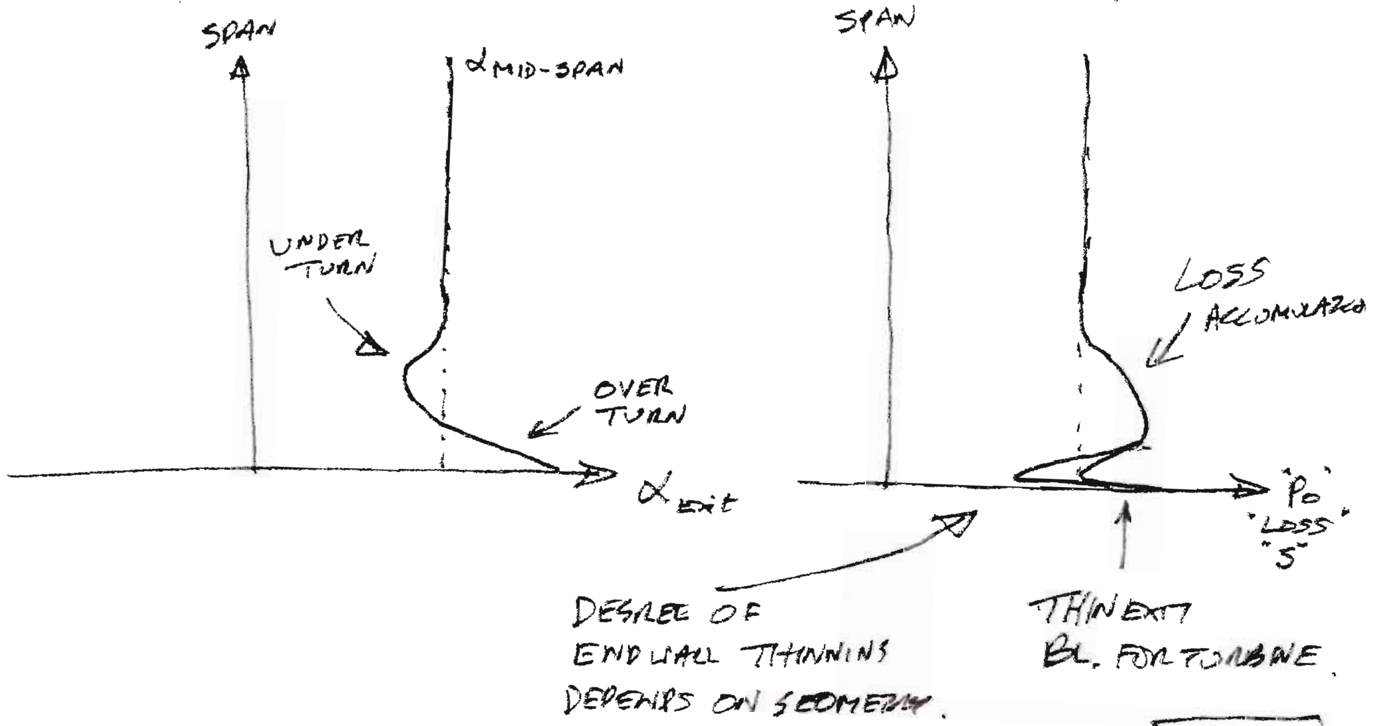
VORTICITY:

Incoming boundary layer has tangential vorticity – these vortex lines (filaments) cannot be broken, so must stretch and accumulate along endwall-suction surface corner.



$\boxed{20\%}$

b) ii) Movement of endwall fluid towards suction surface (& movement up/along the span) creates the classic "over-turning" - "underturning"



20%

c) (i) Annulus flare causes axial velocity to decrease so the leaving exit KE is reduced so overall total-to-static efficiency is higher. 10%

(ii) Lower axial velocity means lower tangential velocity (at fixed blade angle) thus expect less work unless blade angles are increased. 10%

(iii) Annulus flare increases the endwall & blade surface diffusion so can thicken boundary layers and may cause separation.

10%