# 4A13 - Combustion & Engines
## 2023 - Crib



(Q1)

(a) Reactive heat flux = Conductive flux

$$\dot{\omega}\delta_f \Delta H_c = \lambda \frac{(T_b - T_u)}{\delta_f}$$

$\Delta H_c$ - heat of combustion

$$\dot{\omega}\delta_f c_p (T_b - T_u) = \lambda \frac{(T_b - T_u)}{\delta_f}$$

$$\Rightarrow \boxed{\delta_f^2 = \frac{\lambda}{c_p \dot{\omega}}}$$

Convective mass flux = Reactive flux

$$\rho_u S_l = \dot{\omega}\delta_f$$

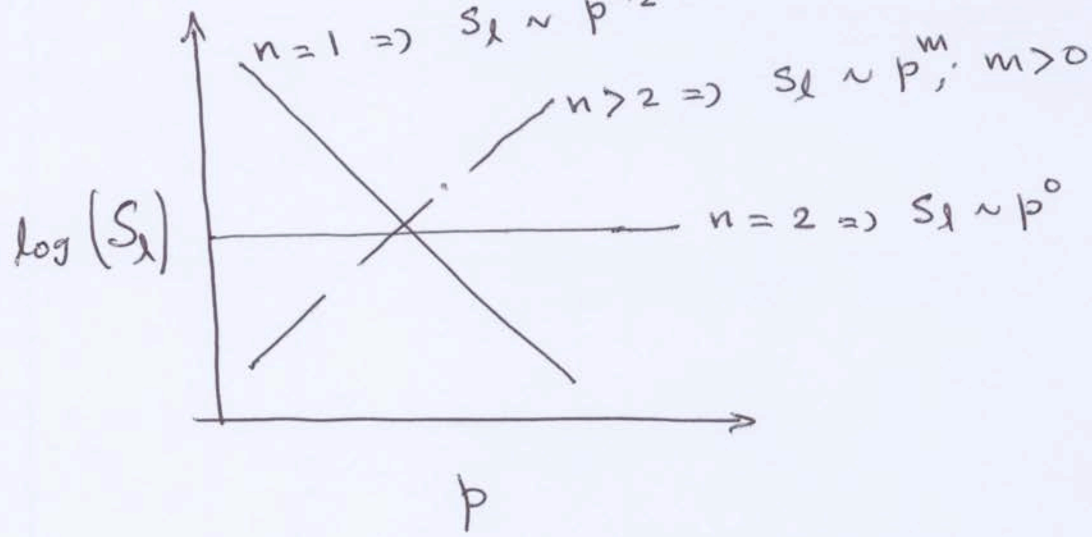
$$\Rightarrow \boxed{S_l = \frac{1}{\rho_u}\sqrt{\frac{\lambda \dot{\omega}}{c_p}}}$$

(b) $\dot{\omega} = p^n Y_f A \exp\left(-\frac{T_a}{T_b}\right)$

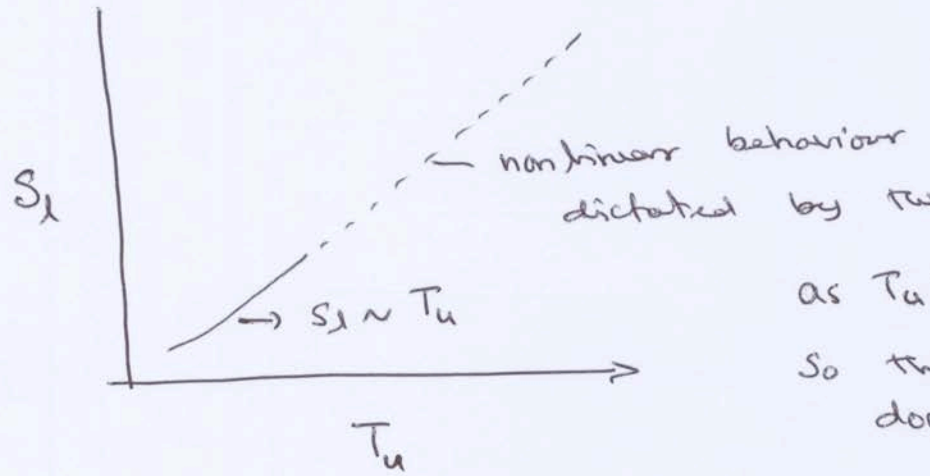
$$\Rightarrow S_l = \frac{1}{\rho_u}\sqrt{\frac{\lambda}{c_p} A Y_f}\; p^{n/2} \exp\left(-\frac{T_a}{2T_b}\right)$$

but $\rho_u = \frac{p}{RT_u}$

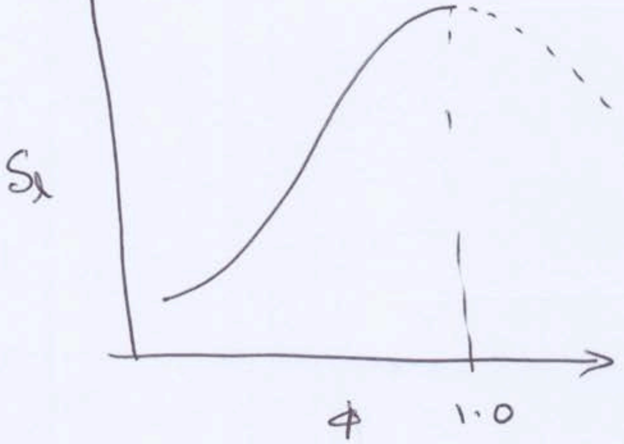
$$\Rightarrow S_l = \sqrt{\frac{\lambda}{c_p} A Y_f R^2}\; T_u\, p^{\frac{n-2}{2}} \exp\left(-\frac{T_a}{2T_b}\right)$$




$S_l$ Vs $p$ variation depends on the overall order of the reaction. The available experimental data suggests that $S_l \sim p^{-1/2}$, a drop in $S_l$ with $p$ over a typical range of interest.




dictated by the exponential term as $T_u \uparrow$ $T_b \uparrow$ so the exponential term dominates.




As $\phi \uparrow$ $Y_f \uparrow$ but $T_b$ will also increase so, the final behavior is dictated by the compounded effect of $Y_f$ & $T_b$. Also $A$ can change with $\phi$.

for the rich side ($\phi > 1$), the expression doesn't fully represent the chemistry as it is good for lean mixtures.

(c) mass balance across the flame

$$\rho_u S_l = \rho_b u_b$$

if $P$ = const & molecular wt of the mixture is also constant then

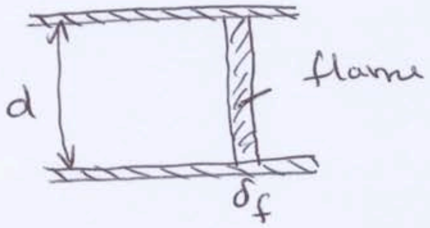
$$\frac{\rho_u}{\rho_b} = \frac{T_b}{T_u} \quad \text{through}$$

$$\Rightarrow u_b = \frac{T_b}{T_u} S_l$$

$$u_b - S_l = S_l \left( \frac{T_b - T_u}{T_u} \right) = \tau S_l$$

$$\therefore \boxed{u_b = (1+\tau) S_l} \quad \text{as required.}$$

(d)




heat generated in the flame } ≈ conducted away through the wall

$$\dot{\omega} c_p (T_b - T_u) \frac{\pi d^2}{4} \delta_f$$

$$\simeq \lambda \pi d \delta_f \frac{(T_b - T_w) 2}{d}$$

using $\dot{\omega} = \frac{\rho_u S_l}{\delta_f}$ and rearranging

$$\Rightarrow d \simeq \left\{ \frac{8 \lambda (T_b - T_w) \delta_f}{\rho_u c_p S_l (T_b - T_u)} \right\}^{1/2}$$

for the flame to be quenched (or when it is quenched)

$$T_w = T_u$$

$$\Rightarrow d \leq 2\sqrt{2} \left\{ \frac{\lambda}{\rho_u c_p} \frac{\delta_f}{S_l} \right\}^{1/2}$$

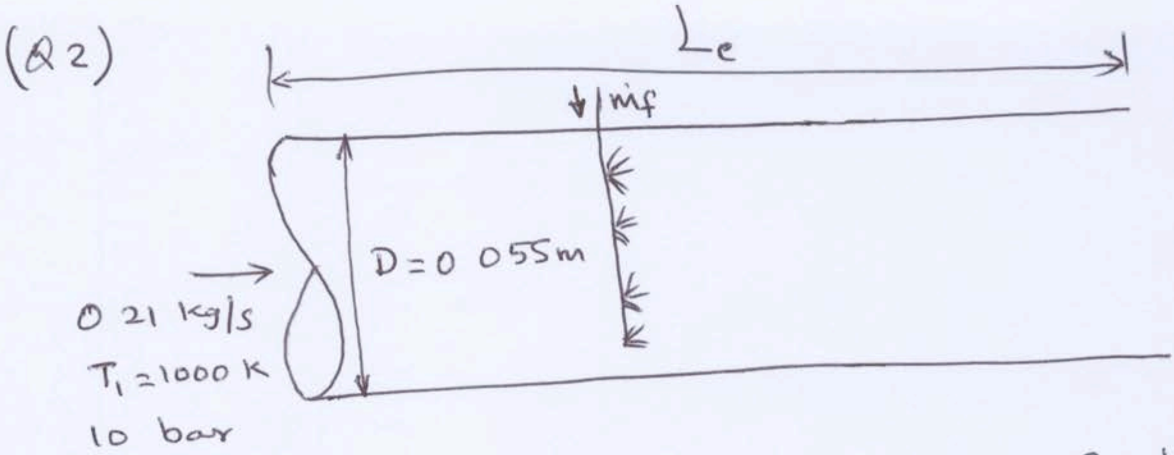
but $\frac{\lambda}{\rho_u c_p} = \alpha$ thermal diffusivity

& $\alpha \sim \delta_f S_L$

$$\Rightarrow \boxed{d \leq 2\sqrt{2} \, \delta_f}$$

in the limit $\underline{d \simeq 2\sqrt{2} \, \delta_f}$ for the flame to be quenched

(Q2)




$T_a = 15000$ K, $\quad \dot{m}_f = 0.01$ kg/s, $\quad C_{20}H_{38}O_4$

$d_d = 30 \times 10^{-6}$ m $\quad T_{a_{bio}} = 1.05\, T_a$

(a) $$\varphi = \frac{(\dot{m}_f/\dot{m}_a)_{actual}}{(\dot{m}_f/\dot{m}_a)_{st.}} = \frac{(0.01/0.21)}{(\quad)_{st.}}$$

$$C_{20}H_{38}O_4 + \frac{a}{\varphi}\left(O_2 + \frac{0.79}{0.21}N_2\right) \rightarrow 20\,CO_2 + 19\,H_2O + \frac{a}{\varphi}\frac{0.79}{0.21}N_2$$

($\varphi \to 1$)

O atom balance:

$$4 + 2a = 40 + 19 \Rightarrow \boxed{a = \frac{55}{2}}$$

$$\therefore \left(\frac{\dot{m}_f}{\dot{m}_a}\right)_{st} = \frac{342}{\frac{55}{2}\left(32 + \frac{0.79}{0.21}28\right)} = 0.0906$$

$$\therefore \varphi = \frac{(0.01/0.21)}{0.0906} = 0.526$$

$$\boxed{\varphi = 0.526}$$

(b)


fuel droplet $\qquad d = 30 \times 10^{-6}$ m

$$m = \rho_f \, \frac{4}{3}\pi \left(\frac{d_d}{2}\right)^3$$

$$\frac{dm}{dt} = -\dot{m}'' A = -\rho_f \frac{B}{4 d_d} \, 4\pi \frac{d_d^2}{4}$$

$$\rho_f \, \frac{4}{3}\pi \left(\frac{d_d}{2}\right)^2 \frac{3}{2} \, \frac{d d_d}{dt} = -\rho_f \frac{B}{4 d_d} \, 4\pi \frac{d_d^2}{4}$$

$$\Rightarrow \quad \frac{d d_d}{dt} = -\frac{B}{2 d_d}$$

$$\int_0^t \Rightarrow \quad d_d^2(t) = d_d^2(t=0) - Bt$$

$$d_d(t) = 0 \;\; @ \;\; t_{evap} \Rightarrow \quad t_{evap} = \frac{d_d^2(t=0)}{B}$$

$$t_{evap} = \frac{(30 \times 10^{-6})^2}{1.5 \times 10^{-7}} = 6 \times 10^{-3} \text{ s}$$

$$\boxed{t_{evap} = 6 \text{ ms}}$$

$$L_v = U \, t_{evap} \qquad \dot{m} = \rho U A$$

$$A = \frac{\pi}{4} \times (0.055)^2 = 2.3758 \times 10^{-3} \text{ m}^2$$

The mixture is

$$C_{20}H_{38}O_4 + \frac{55}{2 \times 0.526}\left(O_2 + \frac{0.79}{0.21} N_2\right)$$

$$\Rightarrow x_f = \frac{1}{\left(1 + 52.281 + 52.281 \times \frac{0.79}{0.21}\right)} = 0.004$$

$$x_{O_2} = 0.2092; \qquad x_{N_2} = 0.7868$$

$$\Rightarrow \boxed{MW_{mix} = 30.092}$$

$$\rho = \frac{P\,MW_{mix}}{R T_1} = \frac{10^6 \times 30.092}{8314 \times 1000} = 3.619 \text{ kg/m}^3$$

$$U = \frac{(0.21 + 0.01)}{3.619 \times 2.3758 \times 10^{-3}} = 25.58 \text{ m/s}$$

$$\therefore L_v = 25.58 \times 6 \times 10^{-3} = 15.35 \text{ cm}.$$

$$\boxed{L_v = 15.35 \text{ cm}}$$ also $\underline{L_v < L_c}$.

(c) $L_{ign} = U \tau_{ign}$ $\qquad \tau_{ign} \sim e^{-T_a/T_1}$

$$\frac{L_{ign,\,bio}}{L_{ign,\,fossil}} = \frac{\tau_{ign,\,bio}}{\tau_{ign,\,foss}} = \frac{e^{-1.05T_a/T_1}}{e^{-T_a/T_1}}$$

$$= e^{(0.05T_a/T_1)} = 2.117$$

$\Rightarrow L_{ign,\,bio} = 2.117\, L_{ign,\,foss}$.

$L_{ign,\,foss} \approx 0.5\, L_c$

$$\Rightarrow \boxed{L_{ign,\,bio} \approx 1.059\, L_c.}$$

$\Rightarrow$ Auto ignition occurs outside the Combustor, which is unacceptable. So Combustor Length for the biofuel use can be increased, which is impractical. $T_1$ can be increased.

(d) $$\frac{L_{ign, bio}}{L_{ign, foss}} = \frac{e^{-1.05 Ta / T_{1,bio}}}{e^{-Ta / T_{1, foss}}} = 1$$

$$\Rightarrow \boxed{T_{1, bio} = 1.05 \, T_{1, foss}}$$

This implies that $T_1$ has to be increased to 1050 K to allow safe operation using the biofuel

This increase of 50 K in the mixture temperature can be achieved either by

(i) Pre heating the fuel or

(ii) increasing the compression ratio.

Q3 (a) $NH_3$ / air flames have much lower flame speed than hydrocarbon-fuelled flames. Hence both engines & gas turbines must be adapted, or the fuel must be strengthened with a "promoter", such as a little $CH_4$ or $H_2$. The $H_2$ could come from the $NH_3$ itself, so some transformation of $NH_3$ into $N_2$ & $H_2$ (ie. chemical cracking) is needed. This adds complexity & cost.

$NH_3$ flames are weak, hence the possibility of "$NH_3$-slip" is higher. $NH_3$ is toxic.

Also, there may be strong emission of nitrogen oxides ($NO$, $NO_2$, $N_2O$), especially $N_2O$ may be large. $N_2O$ is a potent greenhouse gas, which means some of the key advantage of $NH_3$ (i.e. the absence of C, hence no $CO_2$) is negated.

$NH_3$ gas turbines are under development at present. They have the potential to reduce $CO_2$ in our energy system.

(b) The stability of a combustor depends, approximately, on the Damköhler number. This can be defined as

$$Da = \frac{\text{physical time}}{\text{chemical time}} = \frac{L/u}{\nu/S_L^2}$$

$L$: lengthscale of combustor
$u$: inlet velocity
$\nu$: kinematic viscosity
$S_L$: laminar flame speed

If Da is to stay the same as $P_0$ increases to $2P_0$, and since L is fixed, and since residence time ($=L/u$) must remain the same, this means $\nu/S_L^2$ stays constant, from condition 0 $(P_0, T_0)$ to condition 1 $(2P_0, T_1)$.

$$\nu = \frac{\mu}{\rho} \qquad \mu \sim \mu_0 \left(\frac{T_0}{T}\right)^{-\alpha} \quad (\alpha \sim 0.7)$$

$$P = \rho R T \Rightarrow P_0 = \rho_0 R T$$

$$P_1 = \rho_1 R T_1 \Leftrightarrow 2P_0 = \rho_1 R T_1$$

$$\Leftrightarrow \rho_1 = \frac{2P_0}{RT_1} = 2\rho_0 \frac{T_0}{T_1} \Rightarrow \nu_1 = \nu_0 \cdot 2 \left(\frac{T_0}{T_1}\right)^{-\alpha} \left(\frac{T_0}{T}\right)^{-1}$$

$$\Rightarrow S_L = S_{L,0} \left(\frac{2P_0}{P_0}\right)^m \left(\frac{T_1}{T_0}\right)^n \Rightarrow S_L^2 = S_{L,0}^2 \, (2)^{2m} \left(\frac{T_1}{T_0}\right)^{2n}$$

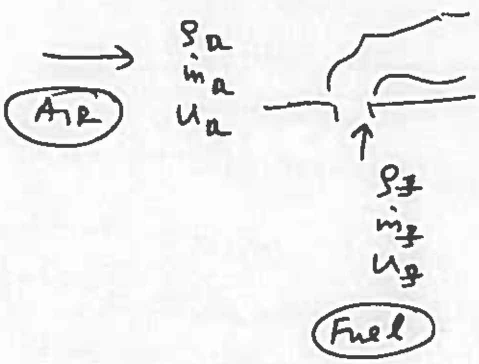
$\Rightarrow$ If $\frac{\nu}{S_L^2}$ stays constant, we must have:

$$2 \cdot (2)^{-2m} \cdot \left(\frac{T_0}{T_1}\right)^{-1-\alpha+2n} = 1 \quad (\text{or}) \quad 2^{(1-2m)} \left(\frac{T_1}{T_0}\right)^{1+\alpha-2n} = 1$$

Depending on $\alpha, m, n$, we get $T_1/T_0$.

This problem demonstrates the complexity of scaling-up combustion problems.

(c) The jet in cross flow is determined by the mass & momentum ratios between the two streams




$$\dot{m}_f = N \cdot \left(\frac{\pi d^2}{4}\right) \rho_f u_f$$

$$J_f = N \left(\frac{\pi d^2}{4}\right) \rho_f u_f^2$$

$$\Leftrightarrow J_f = \dot{m}_f u_f$$

We are told that when we switch from fuel 1 to fuel 2, power must stay the same, i.e. $\dot{m}_{f,1} Q_1 = \dot{m}_{f,2} Q_2$ ($Q$: calorific value)

$Q_{CH_4} = 50$ MJ/kg ($Q_1$)

$Q_{H_2} = 120$ MJ/kg ($Q_2$)

$\dot{m}_{a,1} = \dot{m}_{a,2}$ & $J_{a,1} = J_{a,2}$ ($u_{a,1} = u_{a,2}$)

Since momentum flow rate ratio must also stay the same, then $(\dot{m}_{f,1} u_1) = (\dot{m}_{f,2} u_2)$

$$\Rightarrow u_{f2} = u_{f1} \frac{Q_2}{Q_1}$$

Since $J_f$ stays constant,

$$N_1 \rho_1 \frac{\pi d_1^2}{4} u_1^2 = N_2 \rho_2 \frac{\pi d_2^2}{4} u_2^2$$

$$d_1^2 N_1 = \left(\frac{Q_2}{Q_1}\right)^2 d_2^2 N_2 \Rightarrow \boxed{\frac{d_2}{d_1} = \sqrt{\frac{N_1 \rho_1 Q_2}{N_2 \rho_2 Q_1}}}$$

$\Rightarrow N$ & $d$ are related in the new combustor

Q4 (a)

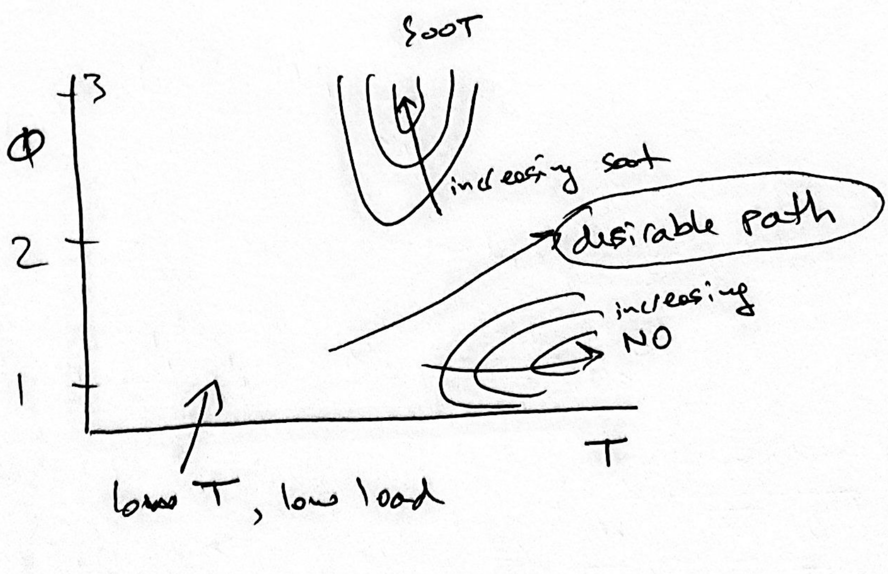
| | $\eta$ | NOx | PM | weight |
|---|---|---|---|---|
| Diesel engine | ~45-50% | low with aftertreatment | high aftertreatment needed | high |
| Gasturb | ~40% (single cycle) ~60-64% (combined cycle) | Can be very low, no aftertreatment needed | Can be low | low |

Both can be fuel-flexible (i.e. can be made to burn most fuels).

Maintenance is much higher in diesel engine.

In single cycle, exhaust T of gt gases is high, therefore a combined cycle can be implemented. A lot of the fuel energy is lost in cooling water in the diesel engine & this is low-grade heat that cannot be used easily.

(b)



Soot is produced at high T & rich pockets.
NOx is produced at high T & leaner regions & long residence times.

To reduce NOx, we must operate lean & at short residence times, or include EGR so that flame T reduces.

More premixing (fine droplets, high-P injection) leads to reduced soot.

"Low-T combustion", multiple injection strategies lead to low NOx & low soot.

If we overdo EGR, we will start having a lot of CO & unburnt hydrocarbon emissions.