Version AA/7

EGT3
ENGINEERING TRIPOS PART IIB

Tuesday 3 May 2022 9.30 to 11.10

Module 4A15

AEROACOUSTICS

*Answer not more than **three** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

*Write your candidate number **not** your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
Attachment: 4A15 Aeroacoustics data sheet (5 pages)
Engineering Data Book

10 minutes reading time is allowed for this paper at the start of the exam.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

You may not remove any stationery from the Examination Room.

1 Power lines sometimes ‘sing’ in windy conditions because, when the wind flows transversely to the wire, it can induce a periodic stream of vortices downstream of the wire. The periodic vortex stream generates a periodic force that produces sound. Assume that the force per unit length exerted by the wire on the fluid has a magnitude of $f\cos(\omega t)$ in a direction perpedicular to the flow, where ω is the angular frequency, and the wire can be treated as spatially compact in all directions, and thus a point source. Let L be the length of the wire (into the page).

(a) Show that the farfield sound generated by the wire is given by

$$p'(\mathbf{x},t) = -\frac{1}{4\pi}\frac{L\cos\theta}{r}\frac{\omega}{c_0} f\sin(\omega(t - r/c_0)),$$

where c_0 is the speed of sound, $r = |\boldsymbol{x}|$ and θ is the angle between $\boldsymbol{x}$ and $\boldsymbol{f}$ (see Fig. 1). [70%]

(b) Find the time-averaged power of the sound radiated by the wire. [20%]

(c) Would this result be valid for any value of L? Explain your answer. [10%]

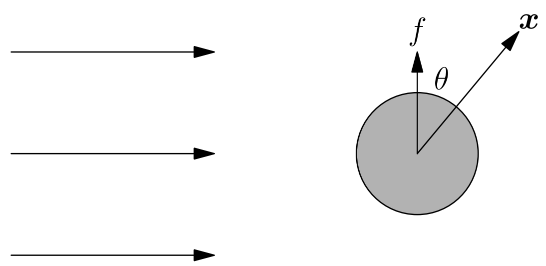

Fig. 1

2 A conical horn is attached to a cylindrical duct to increase the sound transmission out of the duct, as shown in Fig. 2. Assume that the waves propagating in the duct are plane with angular frequency ω, and in the horn are segments of spherical waves that originate from a virtual apex, A. The distance of A from the connection between the duct and the horn is r_0. The terminating impedance of the duct can be assumed to be the impedance presented by the waves in the horn.

(a) Show that the terminating impedance of the duct is given by

$$Z = \rho_0 c_0 \frac{i\,kr_0}{1 + i\,kr_0},$$

where $k = \omega/c_0$ and ρ_0 and c_0 are mean density and speed of sound, respectively. [40%]

(b) The reflection coefficient, R is defined as the ratio of the amplitude of the reflected wave to the incident wave. Show that, for waves travelling down the tube toward the horn, $R = -1/(1 + i\,2kr_0)$ [40%]

(c) Using the result in part (b), comment on the effectiveness of the horn in aiding the transmission of the waves out of the cylindrical duct. [20%]

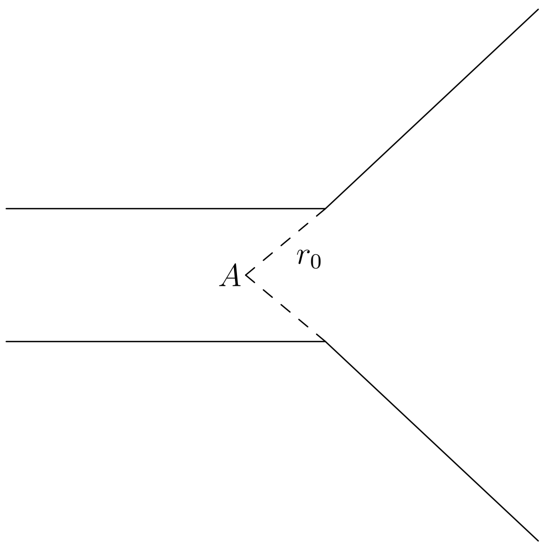

Fig. 2

3 (a) Explain the meaning of the term "cut-off" in connection with acoustic modes in a duct. [20%]

(b) A 3-bladed fan of diameter 300 mm is to be operated in a cylindrical duct of circular cross-section of the same diameter. Table 1 shows the values of z_{mn}, the m^{th} zero of $dJ_n(z)/dz$, where J_n is the n^{th} order Bessel function of the first kind. For $|n| > 6$, use $z_{1n} \approx |n| + 0.80861|n|^{1/3}$. Use the data in Table 1 to determine R_{max}, the maximum number of revolutions per minute if all rotor-alone modes are to be cut-off at atmospheric conditions. Formulae on the data sheet may be used without proof. [40%]

(c) The fan rotor in (b) is operated at $10,000$ rpm. Choose a suitable number of blades for a downstream stator row, explaining clearly the reasons for your choice. With your choice of stator blade number which, if any, of the rotor-stator interaction modes at the blade-passing frequency (bpf) and at 2 bpf propagate? [40%]

	$n=0$	$n=\pm1$	$n=\pm2$	$n=\pm3$	$n=\pm4$	$n=\pm5$	$n=\pm6$
$m=1$	0.00000	1.84118	3.05424	4.20119	5.31755	6.41562	7.50127
$m=2$	3.83170	5.33144	6.70613	8.01524	9.28240	10.51986	11.73494
$m=3$	7.01558	8.53632	9.96947	11.34592	12.68191	13.98719	15.26818
$m=4$	10.17346	11.70600	13.17037	14.58585	15.96411	17.31284	18.63744
$m=5$	13.32369	14.86359	16.34752	17.78875	19.19603	20.57551	21.93172

Table 1

4 To attenuate sound of angular frequency ω travelling as plane waves in a duct of cross-sectional area S, a Helmholtz resonator with volume V is connected to the side wall, as shown in Fig. 3. The neck of the resonator has a length l and cross-sectional area A. Show that the transmission loss, L_T, is given by

$$L_T = 10\log_{10}\left(\frac{|I|^2}{|T|^2}\right) = 10\log_{10}\left(1 + \frac{1}{4S^2}\left(\frac{c_0}{\omega V} - \frac{\omega l}{c_0 A}\right)^{-2}\right)$$

where I and T are the strengths of the incident and transmitted waves and c_0 is the speed of sound. [100%]

Hint: apply conditions of 1) continuity of mass flow rate into and out of the control volume across the neck of the Helmholtz resonator and the duct upstream and downstream of it, and 2) matching of pressure at $x = 0$.

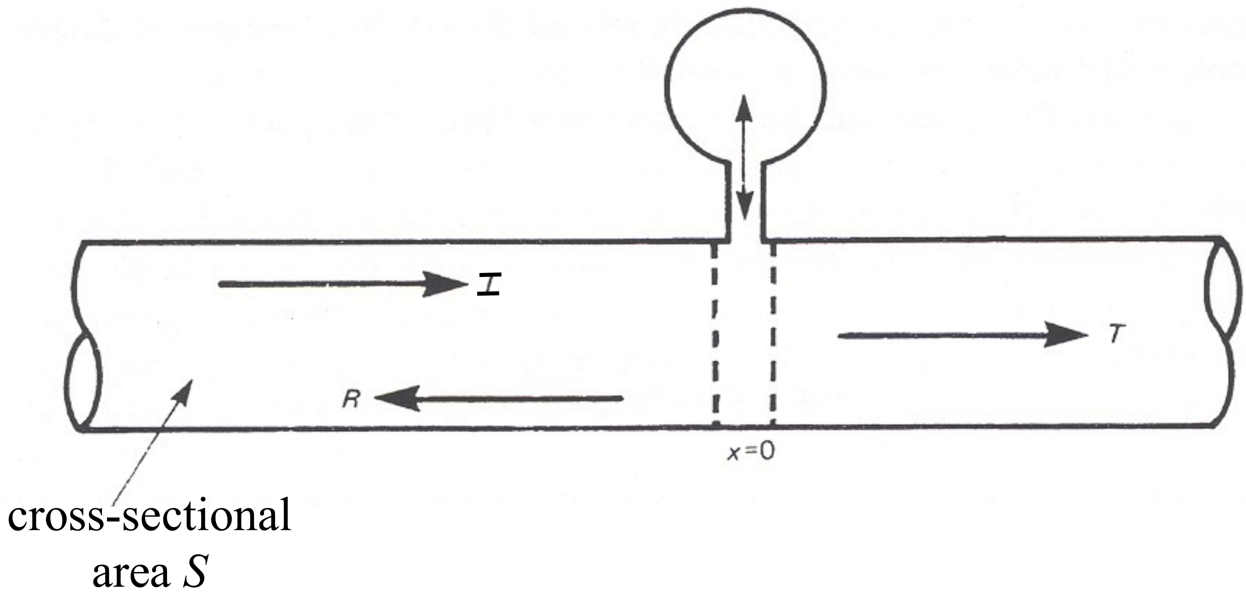

Fig. 3

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Module 4A15 Aeroacoustics Data Sheet

USEFUL DATA AND DEFINITIONS

Physical Properties

Speed of sound in an ideal gas $\sqrt{\gamma RT}$, where T is temperature in Kelvins

Units of sound measurement

$$\begin{aligned}
\text{SPL (sound pressure level)} &= 20\log_{10}\left(\frac{p'_{rms}}{2\times 10^{-5}\text{Nm}^{-2}}\right)\text{dB} \\
\text{IL (intensity level)} &= 10\log_{10}\left(\frac{\text{intensity}}{10^{-12}\text{watts m}^{-2}}\right)\text{dB} \\
\text{PWL (power level)} &= 10\log_{10}\left(\frac{\text{sound power output}}{10^{-12}\text{watts}}\right)\text{dB}
\end{aligned}$$

Definitions

Surface impedance Z_s, relates the pressure perturbation applied to a surface, p', to its normal velocity v'; $p' = Z_s v'$

Characteristic impedance of a fluid $\rho_0 c_0$

Specific impedance of a surface $Z_s/(\rho_0 c_0)$

Wavenumber $k = \omega/c_0 = 2\pi/\lambda$, where λ is the wavelength

Helmholtz number (or compactness ratio) = kD, where D is a typical dimension of the source.

Strouhal number $= \omega D/(2\pi U)$ for sound of frequency ω (in rad/s), produced in a flow with speed U, length scale D.

Basic equations for linear acoustics

Conservation of mass

$$\frac{\partial \rho'}{\partial t} + \rho_0 \nabla \cdot \mathbf{v}' = 0$$

Conservation of momentum

$$\rho_0 \frac{\partial \mathbf{v}'}{\partial t} + \nabla p' = 0$$

Isentropic

$$c_0^2 = \left. \frac{dp}{d\rho} \right|_S$$

Wave equation

$$\frac{1}{c_0^2} \frac{\partial^2 p'}{\partial t^2} - \nabla^2 p' = 0$$

Energy density

$$e = \frac{1}{2} \rho_0 v'^2 + \frac{1}{2\rho_0 c_0^2} p'^2$$

Intensity $\mathbf{I} = p'\mathbf{v}'$

Velocity potential ϕ' satisfies the wave equation and $p' = -\rho_0 \frac{\partial \phi'}{\partial t}$, $\mathbf{v}' = \nabla \phi'$.

Autocorrelation $F(\xi)$, the autocorrelation of $f(y)$ is given by

$$F(\xi) = \overline{f(y) f(y+\xi)}$$

$$F(0) = \overline{f^2}$$

Integral length scale, l

$$l\overline{f^2} = \int_{-\infty}^{\infty} F(\xi) d\xi$$

Sound power

Sound power from a source is defined as

$$P = \int_S \bar{\mathbf{I}} \cdot \mathbf{dS} = \int_{S_\infty} \frac{\overline{p'^2}}{\rho_0 c_0} \mathbf{dS}$$

for a statistically stationary source. For an outward propagating spherically symmetrical sound field $P = \frac{\overline{p'^2}}{\rho_0 c_0} 4\pi r^2$, where p' is the acosutic pressure at radius r.

For a sound field, which is a function of spherical polar coordinates r, θ only, and is independent of the azimuthal angle,

$$P = 2\pi r^2 \int_0^{\pi} \frac{\overline{p'^2}}{\rho_0 c_0} \sin\theta d\theta$$

Simple wave fields

1D or plane wave

The general solution of the 1D wave equation is $p'(x,t) = f(t - x/c_0) + g(t + x/c_0)$, where f and g are arbitrary functions. In a plane wave propagating to the right $p' = \rho_0 c_0 u'$; in a plane wave propagating to the left $p' = -\rho_0 c_0 u'$, u' being the particle velocity.

Spherically symmetric sound fields

The general spherically symmetric solution of the 3D wave equation is

$$\phi'(r,t) = \frac{f(t - r/c_0)}{r} + \frac{g(t + r/c_0)}{r},$$

where r is the distance from the source; f and g are arbitrary functions.

$\cos\theta$ dependence

The general solution of the 3D wave equation with $\cos\theta$ dependence is

$$p'(\mathbf{x},t) = \frac{\partial}{\partial x}\left[\frac{f(t - r/c_0)}{r} + \frac{g(t + r/c_0)}{r}\right] = \cos\theta \frac{\partial}{\partial r}\left[\frac{f(t - r/c_0)}{r} + \frac{g(t + r/c_0)}{r}\right]$$

Useful mathematical formulae

Spherical polar coordinates (r, θ, ψ)

Gradient

$$\nabla p' = \left(\frac{\partial p'}{\partial r}, \frac{1}{r}\frac{\partial p'}{\partial \theta}, \frac{1}{r\sin\theta}\frac{\partial p'}{\partial \psi}\right)$$

Divergence

$$\nabla \cdot \mathbf{v}' = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 v'_r\right) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta v'_\theta\right) + \frac{1}{r\sin\theta}\frac{\partial v'_\phi}{\partial \psi}$$

Laplacian

$$\nabla^2 p' = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial p'}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial p'}{\partial \theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 p'}{\partial \psi^2}$$

Delta functions

Kronecker Delta

$$\delta_{ij} = \begin{cases} 1 & i = j \\ 0 & i \neq j \end{cases}$$

1D δ-function $\delta(x) = 0$ for $x \neq 0$ and $\int_{-\infty}^{\infty} \delta(ax-b) f(x) dx = f(b/a)/|a|$

3D δ-function $\delta(\mathbf{x}) = \delta(x_1)\delta(x_2)\delta(x_3)$

Convolution algebra

Convolution of $f(\mathbf{x})$ and $g(\mathbf{x})$

$$(f \star g)(\mathbf{x}) = \int_{-\infty}^{\infty} f(\mathbf{y}) g(\mathbf{x}-\mathbf{y}) d\mathbf{y}$$

Commutative properties

$$f \star g = g \star f$$

$$\frac{\partial}{\partial x_i}(f \star g)(\mathbf{x}) = f \star \frac{\partial g}{\partial x_i} = \frac{\partial f}{\partial x_i} \star g$$

Green's function

3D Green's function for wave equation

$$\left(\frac{\partial^2}{\partial t^2} - c_0^2 \nabla^2\right) g(\mathbf{x}, t|\mathbf{y}, \tau) = \delta(t-\tau)\delta(\mathbf{x}-\mathbf{y})$$

$$g(\mathbf{x}, t|\mathbf{y}, \tau) = \frac{\delta\left\{|\mathbf{x}-\mathbf{y}| - c_0(t-\tau)\right\}}{4\pi c_0 |\mathbf{x}-\mathbf{y}|}$$

Lighthill's Acoustic Analogy

Lighthill's equation

$$\left(\frac{\partial^2}{\partial t^2} - c_0^2 \nabla^2\right) \rho' = \frac{\partial^2 T_{ij}}{\partial x_i \partial x_j}.$$

For cold, isentropic, low Mach-number jets, T_{ij} can be approximated as:

$$T_{ij} = \rho_0 u_i u_j$$

Lighthill eight power law Acoustic power,

$$P_a \sim \frac{\rho_o d_j^2}{c_0^5} u_j^8,$$

where d_j and u_j are the jet exit diameter and velocity, respectively.

In a cylindrical duct of radius a

The pressure field is given by

$$p'(\mathbf{x},t) = e^{i(\omega t + n\theta)} J_n(z_{mn} r/a)(Ae^{-ikx_3} + Be^{ikx_3}),$$

where z_{mn} is the mth zero of $\dot{J}_n(z)$ and $k = (k_0^2 - z_{mn}^2/a^2)^{1/2}$.

For large azimuthal wavenumber, n

$$z_{1n} \approx n + 1.85 n^{1/3}$$

In a duct of varying area $A(x)$

Webster horn equation

$$\frac{1}{c_0^2}\frac{\partial^2 p'}{\partial t^2} - \frac{1}{A}\frac{\partial}{\partial x}\left(A\frac{\partial p'}{\partial x}\right) = 0$$

Modified Webster horn equation $\psi(x) = \hat{p}(x) A^{1/2}$, $A = \pi a^2$

$$\frac{d^2\psi}{dx^2} + \left(k^2 - \frac{1}{a}\frac{d^2 a}{dx^2}\right)\psi = 0$$