EGT3
ENGINEERING TRIPOS PART IIB

Tuesday 22 April 2014 9.30 to 11

Module 4A15

AEROACOUSTICS

*Answer not more than **three** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

*Write your candidate number **not** your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
Attachment: 4A15 Aeroacoustics data sheet (6 pages).

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

1 Consider a sphere of radius a undergoing a small-amplitude pulsation with an angular frequency ω. The radial velocity on the surface of the sphere is given by $u_a \cos(\omega t)$, where u_a is a constant.

(a) Show that the acoustic pressure radiated by the sphere, at a distance r from the center of the sphere, is given by

$$p(r,t) = \Re\left\{\frac{a}{r}\frac{ika}{1+ika}\rho_0 c_0 u_a e^{i\omega t} e^{-ik(r-a)}\right\},$$

where $\Re\{\dots\}$ denotes the real part of the argument, $k = \omega/c_0$, c_0 is the speed of sound and ρ_0 is the ambient density. [60%]

(b) Use this result to explain why we hear only high frequency sounds when we hold an in-ear headphone away from the ear. [20%]

(c) When we use the headphones in our ears, we can hear the low frequencies. How is this possible? [20%]

2 The sound field radiated by a wavepacket can be described by the following inhomogeneous wave equation:

$$\left(\frac{\partial^2}{\partial t^2} - c_0^2\nabla^2\right) p' = Ae^{-\sigma x_1^2}\delta(x_2)\delta(x_3)e^{i(\omega t-\alpha x_1)},$$

where c_0 is the speed of sound, x_1, x_2 and x_3 represent the three-dimensional cartesian coordinates, t represents time, A is a dimensional amplitude and σ and α are positive constants.

(a) Show that the farfield acoustic pressure radiated by the wavepacket is given by

$$\frac{A}{4c_0^2 x\sqrt{\pi\sigma}}e^{i(\omega t-kx)}e^{-\frac{(kcos\theta-\alpha)^2}{4\sigma}},$$

where $k = \omega/c_0$, $x = |\boldsymbol{x}|$ and θ is the polar angle between the observer and the x_1 axis. Hint 1: For $x \gg y$, $|\boldsymbol{x}-\boldsymbol{y}| = x(1-\frac{\boldsymbol{x}\cdot\boldsymbol{y}}{x^2}+\ldots)$.

Hint 2: $\int_{-\infty}^{\infty} e^{-\sigma y^2}e^{i\gamma y}dy = \sqrt{\frac{\pi}{\sigma}}e^{-\frac{\gamma^2}{4\sigma}}$. [70%]

(b) What is the phase speed of the wave packet? [10%]

(c) If $\sigma \ll 1$, show that the phase speed of the wavepacket must be supersonic in order for it to radiate sound. [20%]

3 The container shown in Fig. 1 has a neck of cross-sectional area A and effective length of l and a bulb of volume V.

(a) State clearly the conditions under which $m'(t)$, the rate of mass flow out of the bulb, is related to $p_2'(t)$ the pressure perturbation in the bulb by both

$$\dot{p}_2 = -\frac{c_0^2}{V} m'(t)$$

and

$$p_2' - p_1' = \frac{l}{A}\dot{m}$$

where $p_1'(t)$ is the pressure perturbation at the neck opening, c_0 is the speed of sound and the dot denotes a time derivative. [50%]

(b) When these conditions hold, determine the relationship between $p_1'(t)$ and $m'(t)$ for fluctuations of frequency ω. [15%]

(c) By considering the container as a monopole point source, determine the ratio of the sound pressure at radial distance r from the neck to pressure fluctuations $p_1'(t)$ at frequency ω.

Hint: For a monopole point source the sound pressure at radial distance r is related to $m'(t)$, the rate of mass outflow from the source by

$$p'(r,t) = \frac{\dot{m}(t - r/c_0)}{4\pi r}$$

[25%]

(d) In practice, what physical mechanism would control the amplitude of the far-field sound at the resonant frequency? [10%]

(cont.

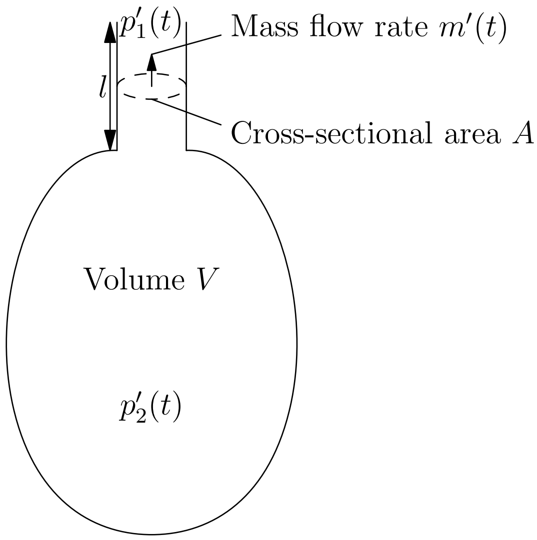

Fig. 1

4 (a) Near a fjord where cold water enters the sea, the speed of sound has a maximum at a depth of 100 m. Sketch the ray pattern produced by a source on the surface. [20%]

(b) The speed of sound increases linearly with depth from 1450 ms^{-1} at the sea surface to 1500 ms^{-1} at a depth of 100 m. At depths greater than 100 m, the speed of sound decreases with increasing depth. Consider the ray that forms the boundary of the shadow zone produced by a source on the sea surface.

(i) At what angle does this ray leave the source? [20%]

(ii) At what horizontal distance from the source does this ray again meet the surface? [60%]

END OF PAPER