Module 4A3 - TURBOMACHINERY 1
2022

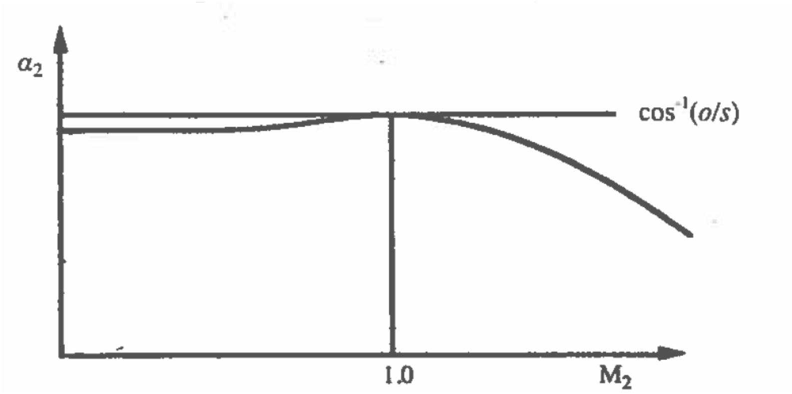
1(a) When the blade throat is choked the average Mach number across the throat is choked.

$$\frac{\dot{m}\sqrt{c_p T_{01}}}{hop_0{}^*} = f(1) = 1.281$$

For air. Relating to the exit condition

$$\cos\alpha_2 = \frac{o}{s}\frac{f(1)}{f(M_2)}\frac{p_0{}^*}{p_{02}}$$

The first term is geometric and sets the Mach 1 exit flow angle. The second term is due to supersonic deviation. The third term is due to loss between the throat and downstream. In the supersonic regime the second and third terms rise. In the subsonic regime the deviation is small due to the boundary layers being thin.



(b) $\frac{p_{01}}{p_2} = 2.2$ and $M_2 = 1.1$

$$Y_p = \frac{p_{01} - p_{02}}{p_{02} - p_2} = \frac{\frac{p_{01}}{p_2} - \frac{p_{02}}{p_2}}{\frac{p_{02}}{p_2} - 1}$$

$$\frac{p_{02}}{p_2} = \frac{1}{0.4684} = 2.13$$

$$Y_p = \frac{\frac{p_{01}}{p_2} - \frac{p_{02}}{p_2}}{\frac{p_{02}}{p_2} - 1} = \frac{2.2 - 2.13}{2.13 - 1} = 0.062$$

A loss coefficient of 6.2%.

(c) $\alpha_1 = 30°$ and $M_1 = 0.3°$

$$f(M_1) = \frac{\dot{m}\sqrt{c_p T_{01}}}{hscos\alpha_1 p_{01}} = 0.630$$

$$f(M_2) = \frac{\dot{m}\sqrt{c_p T_{01}}}{hscos\alpha_2 p_{02}} = 1.271$$

$$\cos\alpha_2 = \frac{s\cos\alpha_1}{s}\frac{f(M_1)}{f(M_2)}\frac{p_{01}}{p_{02}}$$

$$\frac{p_{01}}{p_{02}} = \frac{\frac{p_{01}}{p_2}}{\frac{p_{02}}{p_2}} = \frac{2.2}{2.13} = 1.033$$

$$\cos\alpha_2 = \cos 45° \frac{0.630}{1.271} 1.033 = 0.3621$$

$$\alpha_2 = 68.8°$$

(d) Two thirds of the loss occurs down stream of the throat and so

$$Y_{pT} = \frac{p_0{}^* - p_{02}}{p_{02} - p_2} = \frac{2}{3}Y_p = \frac{2}{3} \times 0.062 = 0.041$$

$$Y_{pT} = \frac{\frac{p_0{}^*}{p_{02}} - 1}{1 - \frac{1}{\frac{p_{02}}{p_2}}} = 0.041$$

$$\frac{p_0{}^*}{p_{02}} = 0.041 \times \left(1 - \frac{1}{\frac{p_{02}}{p_2}}\right) + 1 = 0.041 \times \left(1 - \frac{1}{2.13}\right) + 1 = 1.022$$

Now from part (a)

$$\cos\alpha_2 = \frac{o}{s}\frac{f(1)}{f(M_2)}\frac{p_0{}^*}{p_{02}}$$

and

$$\frac{\dot{m}\sqrt{c_p T_{01}}}{hop_0{}^*} = f(1) = 1.281$$

$$\frac{o}{s} = \frac{\cos\alpha_2}{\frac{f(1)}{f(M_2)}\frac{p_0{}^*}{p_{02}}} = \frac{0.362}{\frac{1.281}{1.271} \times 1.022} = 0.351$$

If there was no loss downstream of the throat the

$$\cos\alpha_2 = \frac{o}{s}\frac{f(1)}{f(M_2)} = 0.351 \times \frac{1.281}{1.271} = 0.354$$

$$\alpha_2 = 69.3°$$

So the deviation caused by the loss downstream of the throat is $69.3° - 68.8° = 0.5°$

(e) The back pressure of the cascade can be lowered until the axial velocity component reaches the speed of sound. This condition is called the limit load and is the point where the pressure waves cannot travel upstream.

$$M_2 \cos\alpha_2 = 1$$

$\alpha_2 = 54°$

$$M_2 = \frac{1}{\cos 54°} = 1.70$$

$$\cos\alpha_2 = \frac{o}{s}\frac{f(1)}{f(M_2)}\frac{p_0{}^*}{p_{02}}$$

$$f(M_2) = 0.958$$

$$\frac{p_0{}^*}{p_{02}} = \frac{\cos\alpha_2}{\frac{o}{s}\frac{f(1)}{f(M_2)}} = \frac{\cos 54°}{0.351 \times \frac{1.281}{0.958}} = 1.2524$$

At $M_2 = 1.70$

$$\frac{p_{02}}{p_2} = \frac{1}{0.203} = 4.94$$

At the design condition

$$\frac{p_0{}^*}{p_{01}} = \frac{p_{02}}{p_{01}}\frac{p_0{}^*}{p_{02}}$$

$$\frac{p_0{}^*}{p_{01}} = \frac{1}{1.033}1.022 = 0.9894$$

So the

$$Y_{pT} = \frac{\frac{p_{01}}{p_{02}} - 1}{1 - \frac{1}{\frac{p_{02}}{p_2}}} = \frac{\frac{1}{0.9894}1.2524 - 1}{1 - \frac{1}{4.94}} = 0.333$$

A loss coefficient of 33.3%.

(f) There are three sources of entropy production:

(i) As the exit Mach number rises the strength of the shock increases. This results in an increase in the entropy production across the shock.

(ii) A complex shock structure occurs close to the trailing edge, as shown below. As the exit Mach number is increased this region becomes a significant source of entropy production. This is a larger source of entropy production than the shock itself.

(iii) As the shock strength increases the impingement of the shock on the suction surface boundary layer of the adjacent blade results in boundary layer separation. This results in a mixing process which creates entropy.

2. (a) Euler's work equation

$$\Delta h_0 = U(V_{\theta 2} - V_{\theta 1})$$

For a repeating stage $V_{x1} = V_{x2} = V_{x3}$

$$\Delta h_0 = UV_x(tan\alpha_2 - tan\alpha_1)$$

$$\psi = \frac{\Delta h_0}{U^2} = \phi(tan\alpha_2 - tan\alpha_1) \quad \text{or} \quad tan\alpha_2 = tan\alpha_1 + \frac{\psi}{\phi}$$

$$\Lambda = \frac{\Delta h_{rotor}}{\Delta h_{stage}} = 1 - \frac{\Delta h_{stator}}{\Delta h_{stage}} = 1 - \frac{1}{2}\frac{(V_2^2 - V_3^2)}{\Delta h_{stage}} = 1 - \frac{1}{2}\frac{\left(V_{\theta 2}^2 - V_{\theta 3}^2\right)}{\Delta h_{stage}} = 1 - \frac{1}{2}\frac{V_x^2(tan\alpha_2{}^2 - tan\alpha_3{}^2)}{\Delta h_{stage}}$$

NB $h_{02} = h_{03}$ and so $h_3 - h_2 = V_2^2 - V_3^2$

$$\Lambda = 1 - \frac{1}{2}\frac{\phi^2(tan\alpha_2{}^2 - tan\alpha_3{}^2)}{\psi}$$

Now because it is a repeating stage $\alpha_3 = \alpha_1$. Subbing in from above

$$\Lambda = 1 - \frac{1}{2}\frac{\phi^2\left(tan\alpha_1{}^2 + \left(\frac{\psi}{\phi}\right)^2 + 2\frac{\psi}{\phi}tan\alpha_1 - tan\alpha_1{}^2\right)}{\psi} = 1 - \frac{1}{2}\psi - \phi tan\alpha_1$$

$$\Lambda = 1 - \frac{1}{2}\psi - \phi tan\alpha_1$$

$$\psi = 2(1 - \Lambda - \phi tan\alpha_1)$$

(b)

$$\Lambda = 1 - \frac{1}{2}\psi - \phi tan\alpha_1$$

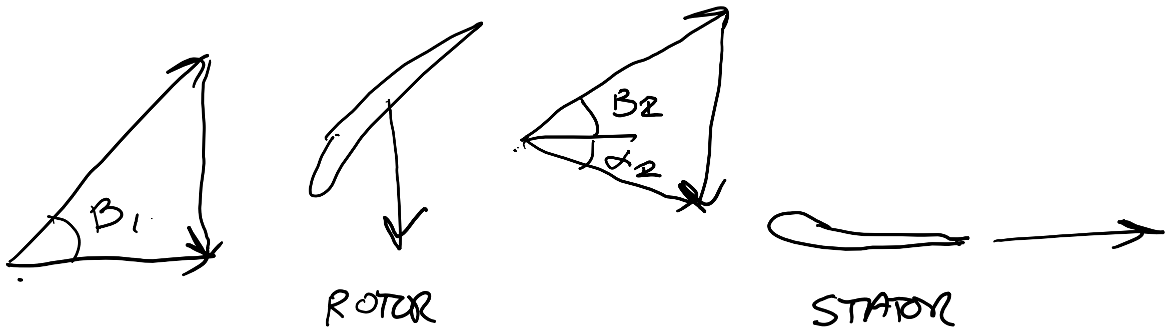
$tan\alpha_1 = 0$

$$\Lambda = 1 - \frac{1}{2}\psi = 1 - 0.2 = 0.8$$

So the zero inlet swirl results in a high reaction design.

$$tan\alpha_2 = \frac{\psi}{\phi} = \frac{0.4}{0.6} = 0.6667$$

$$\alpha_2 = 33.7°$$



$$tan\beta_1 = -\frac{1}{\phi} = -\frac{1}{0.6} = -1.667$$

$$\beta_1 = -59.0°$$

$$V_x tan\ \beta_2 = U - V_x tan\ \alpha_2$$

$$tan\ \beta_2 = tan\ \alpha_2 - \frac{1}{\phi} = tan33.7° - \frac{1}{0.6} = -1$$

$$\beta_2 = -45°$$

If you design with 0° interstage swirl then you don't need an inlet guide vane. This reduces the loss associated with including an extra blade row. However, for a high reaction design the inlet velocity of the rotor and stator are not equal. Because boundary layer loss scales with the relative inlet velocity cubed a high reaction design will be of higher loss. In practice a compressor often has a high reaction In the front stages and then drops the reaction to closer to 50% in the central stages. This avoids the use of an inlet guide vane but benefits from a balanced inlet velocity between the rotor and stator.

(c) At the inlet of the rotor

$$\frac{V_1}{\sqrt{c_p T_{01}}} = \frac{\phi U}{\sqrt{c_p T_{01}}} = \frac{0.6 \times 250}{\sqrt{1005 \times 300}} = 0.273$$

M_1 =0.440

$$M_{1rel} = \frac{M_1}{cos\beta_1} = \frac{0.440}{cos(59.0°)} = 0.854$$

Across the rotor

$$T_{02} = T_{01} + \frac{\psi U^2}{c_p} = 300 + \frac{0.4 \times 250^2}{1005} = 324.9\ K$$

$$\frac{V_2}{\sqrt{c_pT_{02}}}=\frac{\phi U/_{\cos\alpha_2}}{\sqrt{c_pT_{02}}}=\frac{0.6\times250/\cos33.7°}{\sqrt{1005\times324.9}}=0.316$$

M_2 =0.551

$$M_{2rel}=M_2\frac{cos\alpha_2}{cos\beta_2}=0.551\frac{cos(33.7°)}{cos(45°)}=0.648$$

(d) The

$$Y_p=\frac{p_{01,rel}-p_{02,rel}}{p_{01,rel}-p_1}=\frac{1-\frac{p_{02,rel}}{p_{01,rel}}}{1-\frac{p_1}{p_{01,rel}}}=0.06$$

For $M_{1rel}=0.854$

$$\frac{p_1}{p_{01,rel}}=0.620$$

Therefore

$$\frac{p_{02,rel}}{p_{01,rel}}=1-0.06\times\left(1-\frac{p_1}{p_{01,rel}}\right)=1-0.06\times(1-0.620)=0.977$$

$$\frac{\dot{m}\sqrt{c_pT_{01,rel}}}{A_{x1}cos\beta_1p_{01,rel}}=f(M_{1,rel})=\frac{\dot{m}\sqrt{c_pT_{01,rel}}}{A_{x2}cos\beta_2p_{02,rel}}\times\frac{A_{x2}p_{02,rel}cos\beta_2}{A_{x1}p_{01,rel}cos\beta_1}$$

$$f(M_{1,rel})=1.2557$$

$$f(M_{2,rel})=1.128$$

$$\frac{A_2}{A_1}=\frac{f(M_{1,rel})}{f(M_{2,rel})}\times\frac{1}{\frac{p_{02,rel}}{p_{01,rel}}}\times\frac{cos\beta_1}{cos\beta_2}=\frac{1.2557}{1.128}\times\frac{1}{0.977}\times\frac{cos59°}{cos45°}=0.830$$

So the there is an 17% reduction in area across the rotor.

(e) The

$$T_{0exit,s}=T_{01}\times(p_{ratio})^{\frac{\gamma-1}{\gamma}}=300*(5)^{\frac{0.4}{1.4}}=475.1\,K$$

$$T_{0exit}=T_{01}+\frac{(T_{0exit,s}-T_{01})}{\eta}=300+\frac{175.1}{0.9}=494.6\,K$$

$$\frac{T_{0exit}}{T_{01}}=1+\frac{N\,\psi U^2}{c_pT_{01}}$$

$$N = \frac{c_p T_{01}}{\psi U^2}\left(\frac{T_{0exit}}{T_{01}} - 1\right) = \frac{1005 \times 300}{0.4 \times 250^2}\left(\frac{494.6}{300} - 1\right) = 7.8$$

So you would select 8 stages.

3.a)

$$\dot{W}_f = \dot{m}_f c_p (T_{02} - T_{01}) = \dot{m}_c c_{pe} (T_{045} - T_{05})$$

$$w_f = c_p (T_{02} - T_{01}) = \frac{\dot{m}_c}{\dot{m}_f} c_{pe} (T_{045} - T_{05})$$

$$w_f = \frac{1}{BPR + 1} c_{pe} (T_{045} - T_{05}) = \frac{1}{BPR + 1} c_{pe} T_{045}\left(1 - \frac{T_{05}}{T_{045}}\right)$$

Because A_{45} and A_9 are choked

$$\frac{\sqrt{c_{pe} T_{045}}}{A_{45} p_{045}} = \frac{\sqrt{c_{pe} T_{05}}}{A_9 p_{05}}$$

So

$$\frac{p_{05}}{p_{045}} = \frac{A_{45}}{A_9}\sqrt{\frac{T_{05}}{T_{045}}}$$

$$\frac{T_{05}}{T_{045}} = \left(\frac{p_{05}}{p_{045}}\right)^{\frac{\gamma-1}{\gamma}\eta_p}$$

So

$$\frac{T_{05}}{T_{045}} = \left(\frac{A_{45}}{A_9}\right)^{\frac{\gamma-1}{\gamma}\eta_p}\left(\frac{T_{05}}{T_{045}}\right)^{\frac{\gamma-1}{2\gamma}\eta_p}$$

$$\left(\frac{A_{45}}{A_9}\right)^{\frac{\gamma-1}{\gamma}\eta_p} = \left(\frac{T_{05}}{T_{045}}\right)^{1-\frac{\gamma-1}{2\gamma}\eta_p} = \left(\frac{T_{05}}{T_{045}}\right)^{\frac{2\gamma-(\gamma-1)\eta_p}{2\gamma}}$$

$$\frac{T_{05}}{T_{045}} = \left(\frac{A_{45}}{A_9}\right)^{\frac{\gamma-1}{\gamma}\eta_p\frac{2\gamma}{2\gamma-(\gamma-1)\eta_p}} = \left(\frac{A_{45}}{A_9}\right)^{\frac{2(\gamma-1)\eta_p}{2\gamma-(\gamma-1)\eta_p}}$$

Sub in above give

$$w_f = \frac{1}{BPR + 1} c_{pe} T_{045}\left(1 - \left(\frac{A_{45}}{A_9}\right)^{\frac{2(\gamma-1)\eta_p}{2\gamma-(\gamma-1)\eta_p}}\right)$$

So the constant C_1 is

$$C_1 = \frac{1}{BPR+1} c_{pe} \left(1 - \left(\frac{A_{45}}{A_9} \right)^{\frac{2(\gamma-1)\eta_p}{2\gamma-(\gamma-1)\eta_p}} \right)$$

The assumption is made that 100% of the power is transmitted. i.e. no mechanical losses. No exhaust losses.

As the BPR rises the magnitude of C_1 drops. This means that for the same fan power the temperature in core must rise. This in turn will change c_{pe}.

(b) (i)

$$\frac{p_{02}}{p_{19}} = \frac{p_{02}}{p_a} = \left(1 + \frac{\gamma - 1}{2} M_{19}^2 \right)^{\frac{\gamma}{\gamma-1}}$$

$$M_{19}^2 = \left[\left(\frac{p_{02}}{p_{19}} \right)^{\frac{\gamma-1}{\gamma}} - 1 \right] \frac{2}{\gamma - 1} = \left[(1.35)^{\frac{0.4}{1.4}} - 1 \right] \frac{2}{0.4} = 0.4476$$

$M_{19} = 0.669$ NB $p_{01} = p_a$

$p_{01} = 101330\ Pa$ $\quad$ $p_{02} = 1.35 \times p_{01} = 136796\ Pa$

$T_{01} = 288K$ $\quad T_{02} = \left(\frac{p_{02}}{p_{01}} \right)^{\frac{\gamma-1}{\gamma \eta_p}} \times T_{01} = 315.2\ K$

At 1 $\quad f(0.8) = \frac{\dot{m}_f \sqrt{c_p T_{01}}}{A_1 p_{01}}$

At 19 $\quad f(0.669) = \frac{\dot{m}_b \sqrt{c_p T_{02}}}{A_{19} p_{02}}$

$$\frac{\dot{m}_f}{\dot{m}_b} = \frac{A_1}{A_{19}} \sqrt{\frac{T_{02}}{T_{01}}} \frac{p_{01}}{p_{02}} \frac{f(0.8)}{f(0.669)} = 1.2776 \sqrt{\frac{315.2}{288}} \frac{1}{1.35} \frac{1.2338}{1.145} = 1.0668$$

$$\frac{\dot{m}_f}{\dot{m}_b} = \frac{BPR + 1}{BPR} = 1.0668$$

$$BPR = \frac{1}{0.0668} = 15.0$$

(ii)

$$A_1 = 1.5 m^2$$

$$f(0.8) = \frac{\dot{m}_f \sqrt{c_p T_{01}}}{A_1 p_{01}}$$

$$\dot{m}_f = \frac{f(0.8) A_1 p_{01}}{\sqrt{c_p T_{01}}} = \frac{1.2338 * 1.5 * 101330}{\sqrt{1005 * 288}} = 348.6\ kg/s$$

$\dot{W}_f = \dot{m}_f w_f = \dot{m}_f w c_p (T_{02} - T_{01})$ =348.6*1005*27.2=9.53MW

(iii) At 12 km height from CUED data book

$$\frac{T}{T_{sl}} = 0.7519 \quad \frac{p}{p_{sl}} = 0.1915$$

$$T_{sl} = 288\ K \qquad p_{sl} = 1.01325\ bar$$

$$T = 216.5\ K \qquad p = 0.194\ bar$$

At Mach 0.8

$$\frac{p_1}{p_{01}} = 0.656$$

So $p_{01} = \frac{0.194}{0.656} = 0.2957\ bar$

Because the design point of the fan is unchanged its pressure ratio is unchanged.

$$p_{02} = 1.35 \times p_{01} = 0.399\ bar$$

The exit throttle Mach number is

$$\frac{p_1}{p_{02}} = \frac{0.194}{0.399} = 0.486$$

From compressible gas tables

$$M_{19} = 1.07$$

Because the non-dimensional operating point of the fan has not changed the stability margin of the fan will not have changed.