Module 4A3 - TURBOMACHINERY 1
2023

1(a) When the blade throat is choked the average Mach number across the throat is choked.

$$\frac{\dot{m}\sqrt{c_p T_{01}}}{h o p_0{}^*} = f(1) = 1.2810$$

Where h is height and The Mach number at blade exit is 0.95. $T_{01} = T_{02}$

$$\frac{\dot{m}\sqrt{c_p T_{01}}}{hs\, cos\alpha_2\, p_{02}} = f(M_2) = 1.2783$$

Relating to the exit condition

$$\cos\alpha_2 = \frac{o}{s}\frac{f(1)}{f(M_2)}\frac{p_0{}^*}{p_{02}}$$

Now

$$Y_p = \frac{\frac{p_{01}}{p_2} - \frac{p_{02}}{p_2}}{\frac{p_{02}}{p_2} - 1} = 0.035$$

$$\frac{p_{01}}{p_2} = 0.035\left(\frac{p_{02}}{p_2} - 1\right) + \frac{p_{02}}{p_2}$$

ForM_2=0.95

$$\frac{p_{02}}{p_2} = \frac{1}{0.5595} = 1.7873$$

So substituting in above gives

$$\frac{p_{01}}{p_2} = 0.035(1.7873 - 1) + 1.7873 = 1.8149$$

No loss upstream of throat so $p_0{}^* = p_{01}$

$$\frac{p_0{}^*}{p_{02}} = \frac{p_{01}}{p_2} \times \frac{p_2}{p_{02}} = 1.8149 \times \frac{1}{1.7873} = 1.0154$$

Substituting above gives

$$\cos\alpha_2 = \frac{o}{s}\frac{f(1)}{f(M_2)}\frac{p_0{}^*}{p_{02}} = 0.3 \times \frac{1.2810}{1.2783} \times 1.0154 = 0.3053$$

$$\alpha_2 = 72.22°$$

(b) Inlet angle is 30° but need to calculate inlet Mach number

$$f(M_1) = \frac{cos\alpha_2}{cos\alpha_1} \times \frac{p_{01}}{p_{02}} \times f(M_2) = \frac{cos(72.22°)}{cos(-30°)} \times 1.0154 \times 1.2783 = 0.4577$$

From the table the Mach number at inlet is

$$M_1 = 0.213$$

From table

$$V_1 \Big/ \sqrt{c_p T_{01}} = 0.133$$

The Zweifel loading coefficient is

$$Z = \frac{\dot{m}|V_{\theta 2} - V_{\theta 1}|}{(p_{01} - p_2)c_x h} = \frac{\dot{m}\sqrt{c_p T_{01}}}{hscos\alpha_1 p_{01}} \times \frac{\left| V_2 sin\alpha_2 \Big/ \sqrt{c_p T_{01}} - V_1 sin\alpha_1 \Big/ \sqrt{c_p T_{01}} \right|}{\left(1 - \frac{p_2}{p_{01}}\right) c_x h} \times hscos\alpha_1$$

$$Z = \frac{\dot{m}\sqrt{c_p T_{01}}}{hscos\alpha_1 p_{01}} \times \frac{\left| \left(V_2 \Big/ \sqrt{c_p T_{01}} \right) sin\alpha_2 - \left(V_1 \Big/ \sqrt{c_p T_{02}} \right) sin\alpha_2 \right|}{\left(1 - \frac{p_2}{p_{01}}\right)} \times \frac{s}{c_x} cos\alpha_1$$

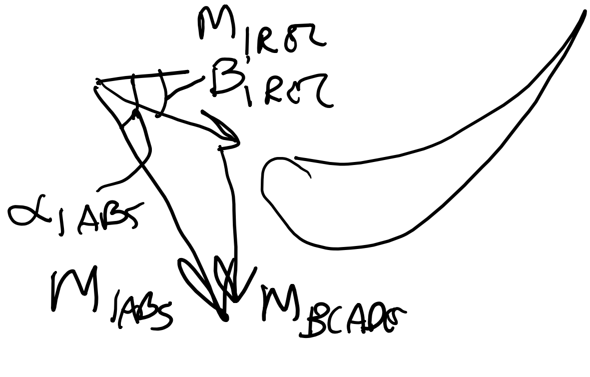
$$V_2 \Big/ \sqrt{c_p T_{01}} = 0.5530$$

$$Z = 0.4577 \times \frac{|0.133 \times sin(-30°) - 0.5530 \times sin(72.22°)|}{\left(1 - \frac{1}{1.8149}\right)} \times 1.0 \times cos(-30°)$$

$$Z = 0.5236$$

The optimal Zweifel loading coefficient is 0.8 and so this blade has a pitch to axial chord ratio which is too low for minimum loss. This means that the wetted area is two high.

(c)



$M_{1rel} = \ 0.213, \beta_{1rel} - 30°$

$$M_{1,abs} = \frac{M_{1,rel} \times cos\beta_{1,rel}}{cos\alpha_{1,abs}} = \frac{0.213 \times cos(-30°)}{cos(-75°)} = 0.713$$

$$T_1 = T_{01} \times \left(1 + \frac{\gamma - 1}{2} M_{1,abs}{}^2\right)^{-1} = 1000 \times \left(1 + \frac{0.4}{2} 0.713^2\right)^{-1} = 907.7K$$

$$M_{blade} = M_{1,abs} \times sin\alpha_{1,abs} - M_{1,rel} \times sin\beta_{1,rel} = 0.736 \times sin(75°) - 0.213 \times sin(30°)$$

$$M_{blade} = 0.604$$

$$U_{blade} = M_{blade} \times \sqrt{\gamma R T_1} = 0.604 \times \sqrt{1.4 \times 287.15 \times 907.7} = 364.9m/s$$

(d) The radius

$$r_{blade} = \frac{U_{blade}}{\Omega_{blade}} = \frac{364.9}{6000 * 2\pi/60} = 0.581m$$

$$N_{blade} = \frac{2\pi r_{blade}}{s} = \frac{2\pi r_{blade}}{c_x \times \frac{s}{c_x}} = \frac{2\pi \times 0.581}{0.05 \times 1} = 73\ blades$$

2. (a) Euler's work equation

$$\Delta h_0 = U(V_{\theta 2} - V_{\theta 1})$$

For a repeating stage $V_{x1} = V_{x2} = V_{x3}$

$$\Delta h_0 = UV_x(tan\alpha_2 - tan\alpha_1)$$

$$\psi = \frac{\Delta h_0}{U^2} = \phi(tan\alpha_2 - tan\alpha_1) \quad \text{or} \quad tan\alpha_2 = tan\alpha_1 + \frac{\psi}{\phi}$$

$$\Lambda = \frac{\Delta h_{rotor}}{\Delta h_{stage}} = 1 - \frac{\Delta h_{stator}}{\Delta h_{stage}} = 1 - \frac{1}{2}\frac{(V_2^2 - V_3^2)}{\Delta h_{stage}} = 1 - \frac{1}{2}\frac{\left(V_{\theta 2}^2 - V_{\theta 3}^2\right)}{\Delta h_{stage}} = 1 - \frac{1}{2}\frac{V_x^2(tan\alpha_2{}^2 - tan\alpha_3{}^2)}{\Delta h_{stage}}$$

NB $h_{02} = h_{03}$ and so $h_3 - h_2 = V_2^2 - V_3^2$

$$\Lambda = 1 - \frac{1}{2}\frac{\phi^2(tan\alpha_2{}^2 - tan\alpha_3{}^2)}{\psi}$$

Now because it is a repeating stage $\alpha_3 = \alpha_1$. Subbing in from above

$$\Lambda = 1 - \frac{1}{2}\frac{\phi^2\left(tan\alpha_1{}^2 + \left(\frac{\psi}{\phi}\right)^2 + 2\frac{\psi}{\phi}tan\alpha_1 - tan\alpha_1{}^2\right)}{\psi} = 1 - \frac{1}{2}\psi - \phi tan\alpha_1$$

$$\Lambda = 1 - \frac{1}{2}\psi - \phi tan\alpha_1$$

$$\psi = 2(1 - \Lambda - \phi tan\alpha_1)$$

To increase loading either

- Use low reaction
- Make the interstage swirl angle α_1 =α_3 large and negative.

(b)(i) $\Lambda = 0.5$ and $\alpha_1 = 0$.

$$\psi = 2(1 - 0.5) = 1$$

$$\Delta h_{stage} = \psi U_{blade}{}^2 = 1 \times 200^2 = 40\ kJ/kg$$

So the number of stages is

$$N_{stage} = \frac{400}{40} = 10$$

(ii) $N_{stage} = 6$

$$\Delta h_{stage} = \frac{400}{6} = 66.67\ kj/kg$$

$$\psi = \frac{\Delta h_{stage}}{{U_{blade}}^2} = \frac{66.67\ kJ/kg}{200^2} = 1.667$$

$$\psi = 2(1 - \Lambda - \phi tan\alpha_1)$$

So

$$tan\ \alpha_1 = \frac{\left(-\frac{\psi}{2} + 1 - \Lambda\right)}{\phi} = \frac{\left(-\frac{1.667}{2} + 1 - 0.3\right)}{0.4} = -0.3338$$

$$\alpha_1 = -18.46°$$

(iii) The 50% reaction machine would have a higher efficiency because the peak velocities in the stator and rotor would be balanced and loss scales with V cubed. The 30% reaction machine would be lighter weight and smaller. The choice between the two would therefore be made based on the application. Whether higher efficiency or greater compactness was required.

(c)

$$Power = \dot{m} \times N_{stage} \times \psi \times {U_{blade}}^2$$

Therefore

$$\bar{r}^2 = \frac{Power}{\dot{m} \times N_{stage} \times \psi \times \Omega^2} = \frac{400\ kJ/kg}{6 \times 1.667 \times (3000 \times 2\pi/60)^2} = 0.4052$$

$$\bar{r} = 0.637m$$

$$\dot{m} = \rho A_x V_x = \rho \times 2\pi h\bar{r} \times \phi\bar{r}\Omega$$

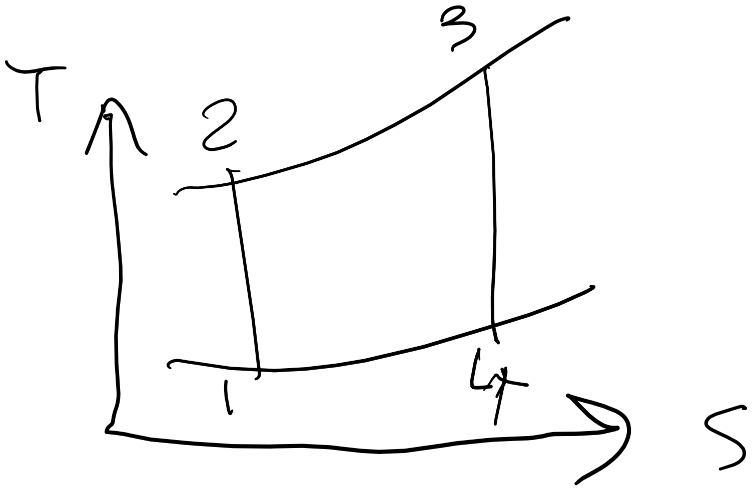
$$h = \frac{\dot{m}}{\rho \times 2\pi\bar{r} \times \phi\bar{r}\Omega} = \frac{30}{1.7 \times 2\pi \times 0.637 \times 0.4 \times 0.637 \times (3000 \times 2\pi/60)} = 0.0551$$

So hub to tip ratio is

$$hub\ to\ tip\ ratio = \frac{\left(\bar{r} - \frac{h}{2}\right)}{\left(\bar{r} + \frac{h}{2}\right)} = \frac{\left(0.4052 - \frac{0.0551}{2}\right)}{\left(0.4052 + \frac{0.0551}{2}\right)} = 0.8727$$

The hub to tup ratio is close to 1 and therefore the blade speed does not change up the blade span. This means that the velocity triangle of the stator does not vary significantly across the blade span.

3. (a)



$$\dot{W}_x = \dot{m}c_p\big((T_{03} - T_{04}) - (T_{02} - T_{01})\big)$$

(b)

$$\frac{M_3}{M_1} = 1 = \frac{\dfrac{\dot{m}\sqrt{c_p T_{03}}}{A_3 p_{03}}}{\dfrac{\dot{m}\sqrt{c_p T_{01}}}{A_1 p_{01}}} = \sqrt{\frac{T_{03}}{T_{01}}}\frac{p_{01}}{p_{03}}\frac{A_1}{A_3}$$

NB $p_{02} = p_{03}$

$$\frac{p_{02}}{p_{01}} = \sqrt{\frac{T_{03}}{T_{01}}}\frac{A_1}{A_3}$$

Now

$$\frac{T_{04}}{T_{03}} = \left(\frac{p_{04}}{p_{03}}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{p_{04}}{p_{03}}\right)^{0.286} = \left(\frac{p_{01}}{p_{02}}\right)^{0.286}$$

$$\frac{T_{04}}{T_{01}} = \frac{T_{04}}{T_{03}}\frac{T_{03}}{T_{01}} = \left(\frac{p_{01}}{p_{02}}\right)^{0.286} \times \frac{T_{03}}{T_{01}}$$

So

$$\frac{T_{04}}{T_{01}} = \left(\sqrt{\frac{T_{01}}{T_{03}}}\frac{A_3}{A_1}\right)^{0.286} \times \frac{T_{03}}{T_{01}} = \left(\frac{A_3}{A_1}\right)^{0.286} \times \left(\frac{T_{03}}{T_{01}}\right)^{0.857}$$

(c)

$$\dot{W}_x = \dot{m}c_p T_{01}\left(\left(\frac{T_{03}}{T_{01}} - \frac{T_{04}}{T_{01}}\right) - \left(\frac{T_{02}}{T_{01}} - 1\right)\right)$$

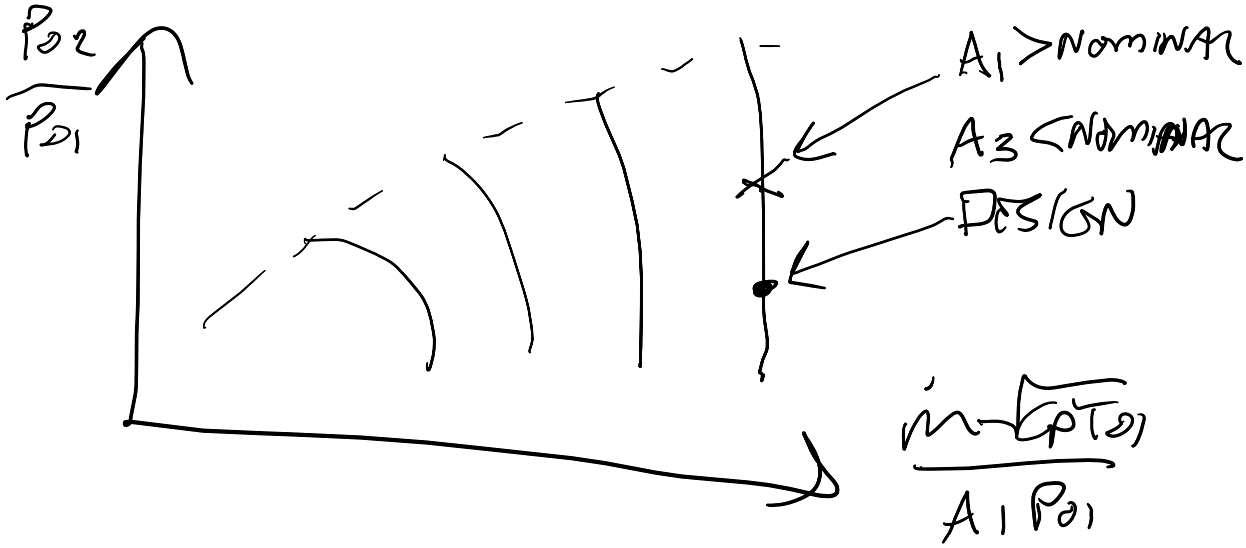
$$\frac{T_{02}}{T_{01}} = \left(\frac{p_{02}}{p_{01}}\right)^{\frac{\gamma-1}{\gamma}}$$

$$\frac{\dot{m}\sqrt{c_p T_{01}}}{A_1 p_{01}} = 1.281$$

So

$$\dot{W}_x = 1.281 \times A_1 \times p_{01} \times \sqrt{c_p T_{01}} \left(\frac{T_{03}}{T_{01}} - \left(\frac{A_3}{A_1}\right)^{0.286} \times \left(\frac{T_{03}}{T_{01}}\right)^{0.857} - \left(\frac{A_1}{A_3}\right)^{0.286} \times \left(\frac{T_{03}}{T_{01}}\right)^{0.143} + 1 \right)$$

(d)



Change schedule of variable guide vanes. To implement this the first few stages of the compressor would have to have variable guide vanes. This involves implementing a variable guide vane mechanism and scheduling the vanes correctly at each operating point.

Change bleed schedule. The bleed would be implemented in the mid or rear stages in order to remove mass flow from the compressor. The bleed would again have to be scheduled correctly at each operating point.

(e) The power drops when A_1 < nominal and A_3 > nominal. To rectify T_{03} must be raised by increasing the fuel flow rate. The first practical implication is that cooling mass flow has to be increased. The increased cooling flow would reduce cycle efficiency at a cycle level, and it would also decrease the efficiency of the turbine. This is due to the irreversibilities introduced by the mixing of the cooling flows.The second practical implication is that the life of the turbine might be reduced due to the increased in thermal stresses and the increase in peak temperatures in the first turbine stage.