Module 4A3 - TURBOMACHINERY 1
2024

1(a) From the Smith Chart look up $\psi = 1.85$ and $\Lambda = 0.5$ and this gives $\phi = 0.88$.

$$tan\alpha_1 = \frac{\left(1 - \frac{\psi}{2} - \Lambda\right)}{\phi} = \frac{(1 - 0.925 - 0.5)}{0.88} = -0.483$$

$$\alpha_1 = -25.8°$$

For a repeating stage

$$\alpha_1 = \alpha_3$$

From notes

$$\psi = \phi(tan\alpha_2 - tan\alpha_1)$$

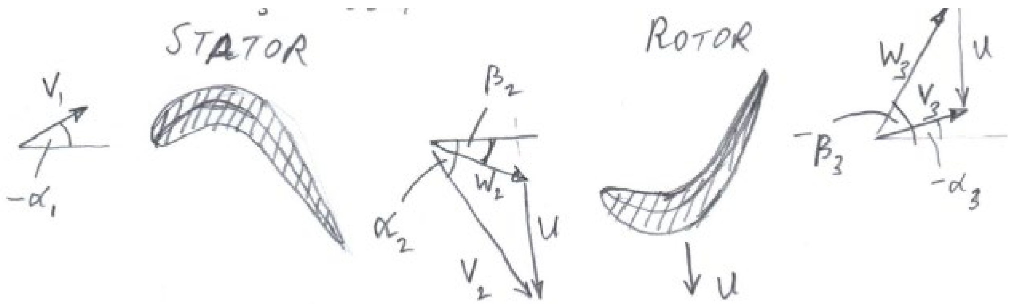
$$tan\alpha_2 = \frac{\psi}{\phi} + tan\alpha_1 = \frac{1.85}{0.88} - 0.483 = 1.619$$

$$\alpha_2 = 58.3°$$

The stage is 50% reaction so

$$-\alpha_1 = \beta_2$$

$$-\alpha_2 = \beta_3$$



(b)

$$U = \frac{2\pi \times rpm \times r}{60} = \frac{2\pi \times 20000 \times 0.1}{60} = 209.3\, m/s$$

$$\rho = \frac{p}{RT} = \frac{50 \times 10^5}{4140 \times 600} = 2.01$$

$$h = \frac{\dot{m}}{2\pi \times r \times \rho \times \phi \times U} = \frac{10}{2\pi \times 0.1 \times 2.01 \times 0.88 \times 209.3} = 0.04m$$

(c)

$$V_x = 0.88 \times 209.3 = 184.2m/s$$

$$V_2 = V_x \times tan\alpha_2 = 184.2 \times 1.591 = 293.1\ m/s$$

$$T_2 = T_{02} - \frac{V_2^2}{2c_p} = 600 - \frac{293.1^2}{2 \times 14.2 \times 10^3} = 597.0\ K$$

$$M_2 = \frac{V_2}{\sqrt{\gamma R T_2}} = \frac{293.1}{\sqrt{1.41 \times 4120 \times 597}} = 0.157$$

Speed of sound = $\sqrt{1.41 \times 4120 \times 597} = 1862\ m/s$

The speed of sound for hydrogen is much higher than for air and therefore for the same flow speeds the Mach number is low.

(d)

$$\eta = \frac{(T_{01} - T_{02})}{(T_{01} - T_{02s})}$$

$$\frac{T_{02s}}{T_{01}} = \left(\frac{p_{02}}{p_{01}}\right)^{\left(\frac{\gamma-1}{\gamma}\right)} = \left(\frac{1}{4}\right)^{\left(\frac{1.41-1}{1.41}\right)} = 0.6682$$

$$T_{02s} = \frac{T_{02s}}{T_{01}} \times T_{01} = 0.6682 \times 600 = 400.9\ K$$

$$T_{02} = T_{01} - 0.91 \times (T_{01} - T_{02s}) = 600 - 0.91 \times 600 \times (1 - 0.6682) = 418.8\ K$$

$$\Delta h_{stage} = \psi \times U^2 = 1.85 \times 209.3^2 = 81.0 \times 10^3\ kJ/kg$$

$$N_{stage} = \frac{c_p(T_{01} - T_{02})}{\Delta h_{stage}} = \frac{14.2 \times (600 - 418.8)}{81.0} = 31.7$$

So the turbine should have 32 stages.

The gamma for air is very similar to hydrogen and so the temperature at exit would be similar. However, the Cp for air is 14 times lower than for hydrogen and so a turbine operating on air would only have 2-3 stages.

(e) The design that has been considered has a stage loading of 1.85. On the Smith Chart this could be raised to 2.7. This would raise the flow coefficient to 1.1 and would drop the efficiency to 87%.

$$T_{02} = T_{01} - 0.87 \times (T_{01} - T_{02s}) = 600 - 0.87 \times 600 \times (1 - 0.6682) = 426.8\ K$$

$$\Delta h_{stage} = \psi \times U^2 = 2.7 \times 209.3^2 = 118.3 \times 10^3\ kJ/kg$$

$$N_{stage} = \frac{c_p(T_{01} - T_{02})}{\Delta h_{stage}} = \frac{14.2 \times (600 - 426.8)}{118.3} = 20.8$$

The number of stages could be reduced to 21. In practice stage loading can be increased to around 3.2 which would drop the number of stages even further to around 18. The limit is separation on the turbine suctions surface boundary layer due to the high loading.

It should also be noted that increasing the flow coefficient will also reduce blade height at inlet.

2 (a)

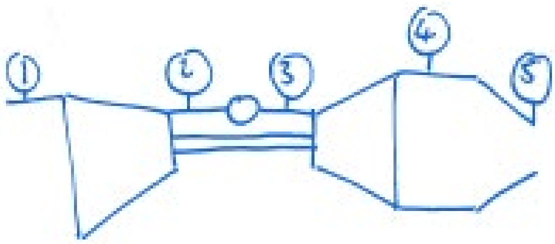
$$\frac{T_{02}}{T_{01}} = \left(\frac{p_{02}}{p_{01}}\right)^{\left(\frac{\gamma-1}{\eta_p \gamma}\right)}$$

$$1.53 = (3.8)^{\left(\frac{0.4}{\eta_p 1.4}\right)}$$

$$\log 1.54 = \left(\frac{0.4}{\eta_p 1.4}\right) log 3.8$$

$$\eta_p = 90\%$$

b)



$$\frac{\dot{m}\sqrt{c_{pe} T_{03}}}{A_3 P_{03}} = \frac{\dot{m}\sqrt{c_{pe} T_{05}}}{A_5 P_{05}} = \text{const} \quad \text{choked}$$

$T_{04} = T_{05}$ adiabatic

$P_{04} = P_{05}$ isentropic

① $$\frac{\sqrt{T_{03}}}{A_3 P_{03}} = \frac{\sqrt{T_{04}}}{A_5 P_{04}}$$

$$c_p (T_{02} - T_{01}) = c_{pe} (T_{03} - T_{04}) \quad \text{equal work}$$

② $$T_{02} - T_{01} = \frac{c_{pe}}{c_p} (T_{03} - T_{04}) = 1.218 (T_{03} - T_{04})$$

③ $$\frac{P_{02}}{P_{01}} = \left(\frac{T_{02}}{T_{01}}\right)^{\frac{\eta \gamma}{\gamma - 1}} = \left(1 - \frac{T_{02} - T_{01}}{T_{01}}\right)^{\frac{\eta \gamma}{\gamma - 1}}$$

① →
$$\frac{P_{04}}{P_{03}} = \left(\frac{T_{04}}{T_{03}}\right)^{\frac{1}{2}} \times \frac{A_3}{A_5}$$

$$\left(\frac{P_{04}}{P_{03}}\right)^2 = \frac{T_{04}}{T_{03}} \times \left(\frac{A_3}{A_5}\right)^2 \qquad \frac{P_{04}}{P_{03}} = \left(\frac{T_{04}}{T_{03}}\right)^{\frac{\gamma_e}{\eta(\gamma_e - 1)}}$$

$$\Downarrow \qquad = \left(\frac{T_{04}}{T_{03}}\right)^{\frac{2\gamma_e}{\eta(\gamma_e - 1)}}$$

$$\left(\frac{T_{04}}{T_{03}}\right)^{\frac{2\gamma_e}{\eta(\gamma_e - 1)} - 1} = \left(\frac{A_3}{A_5}\right)^2 = \left(\frac{T_{04}}{T_{03}}\right)^{7.896}$$

$$T_{04} = T_{03}\left(\frac{A_3}{A_5}\right)^{0.2533}$$

Into ②
$$T_{02} - T_{01} = 1.218\, T_{03}\left(1 - \left(\frac{A_3}{A_5}\right)^{0.2533}\right)$$

Into ①
$$\frac{P_{02}}{P_{01}} = \left(1 - 1.218\,\frac{T_{03}}{T_{01}}\left(1 - \left(\frac{A_3}{A_5}\right)^{0.2533}\right)\right)^{3.15}$$

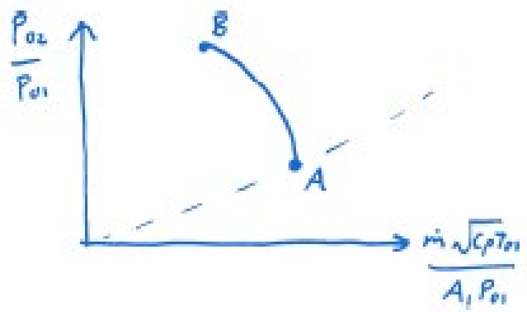
$B = 1.218 \quad C = 0.2533 \quad D = 3.15$

c)
$$\frac{\dot{m}\sqrt{c_p T_{01}}}{A_1 P_{01}} = \frac{\dot{m}\sqrt{c_{pe} T_{03}}}{A_3 P_{03}} \times \frac{P_{02}}{P_{01}} \times \overset{1 \text{ isentropic}}{\cancel{\frac{P_{03}}{P_{02}}}} \times \sqrt{\frac{T_{01}}{T_{03}}} \times \sqrt{\frac{c_p}{c_{pe}}}$$

$$= 1.3468 \times \frac{P_{02}}{P_{01}} \times \left(\frac{T_{03}}{T_{01}}\right)^{-\frac{1}{2}} \times 0.9061$$

$$= 1.22 \left(\frac{T_{03}}{T_{01}}\right)^{-0.5} \left(1 - 1.218 \frac{T_{03}}{T_{01}}\left(1 - \left(\frac{A_3}{A_5}\right)^{0.2533}\right)\right)^{3.15}$$

d) $$\frac{\dot{m}\sqrt{c_{pe} T_{03}}}{A_3 P_{02}} = \text{Const}$$

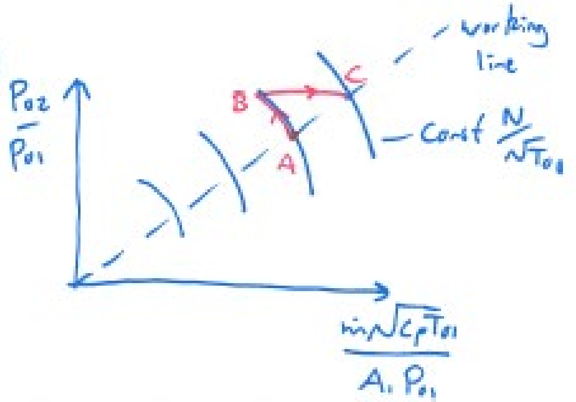


$$\frac{\dfrac{\dot{m}_A \sqrt{T_{03,A}}}{P_{02,A}}}{\dfrac{\dot{m}_B \sqrt{T_{03,B}}}{P_{02,B}}} = 1$$

$$\frac{\dot{m}_A}{\dot{m}_B} = \frac{P_{02,A}}{P_{02,B}} \times \sqrt{\frac{T_{03,B}}{T_{03,A}}}$$

$$1/0.9 = 1/1.3 \times \sqrt{\frac{T_{03,B}}{T_{03,A}}} \qquad \frac{T_{03B}}{T_{03A}} = 2.09$$

e)



The rotational speed will increase as the work extracted by the turbine is net positive.

Compressor flow rate increases + pressure ratio falls until it falls back onto the working line

3(a) From the lecture notes:

Applying Euler's equation to the impeller, and remembering the definition of slip factor we get

$$\Delta h_0 = U_2 V_{\theta 2} = U_2 \sigma (U_2 + V_{r2} \tan \chi_2)$$

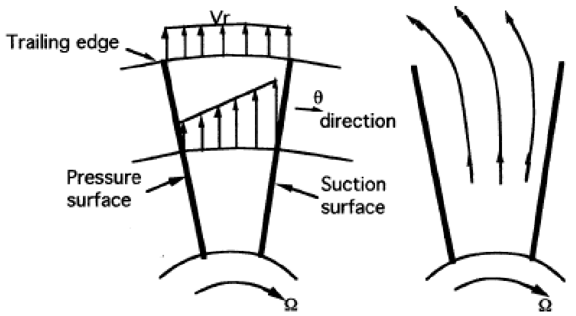
$$\text{Thus, } \psi = \frac{\Delta h_0}{U^2} = \sigma (1 + \phi \tan \chi_2)$$

where the flow coefficient is defined as $\phi = V_{r2}/U_2$.

For radial blades (which are common), $\chi_2 = 0$:

- the enthalpy rise is independent of flow rate
- the loading coefficient, $\psi = \sigma$ (around 0.85), which is much larger than the typical value of 0.4 for axial compressors.

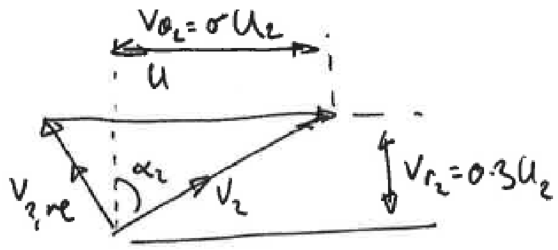
Slip is due to as the trailing edge is approached the blade loading must fall to zero and the radial velocity must become more uniform. This the flow leaving the blade inclines backwards, as shown below:



Slip in a Centrifugal Compressor

(b)

Exit velocity triangle



$V_{t2}=\sigma \times U_2$ and $V_{r2}=0.32 \times U_2$

$$\tan \alpha_2 = \frac{\sigma \times U_2}{0.32 \times U_2}$$

$$\sigma = 0.32 \times \tan 69 = 0.834$$

(c) From part (a)

$$\frac{\Delta h}{U^2} = \sigma = 0.834$$

$$U = \frac{50{,}000}{60} \times 2\pi \times 0.07 = 366.5 m/s$$

$$\Delta h = 0.834 \times U^2 = 0.834 \times 366.5^2 = 112.0 \times 10^3 \; KJ/kg$$

Air is a perfect gas with γ = 1.4 and c_p = 1005 J kg^{-1} K^{-1}.

$$\Delta T = \frac{1.12 \times 10^5}{1005} = 111.5 \; K$$

$$T_{02} = 300 + 111.5 = 412.5 \text{ K}$$

$$V_2 = \sqrt{{V_{r2}}^2 + {V_{t2}}^2} = U \times \sqrt{\sigma^2 + 0.32^2} = 366.5 \times 0.893 = 327.4 \text{ m/s}$$

$$T_2 = T_{02} - \frac{V_2^2}{2c_p} = 412.5 - \frac{327.4^2}{2 \times 1005} = 359.2 \; K$$

$$M_2 = \frac{V_2}{\sqrt{\gamma R T_2}} = \frac{327.4}{\sqrt{1.4 \times 287 \times 359.2}} = 0.861$$

(d)

$$T_{02s} = T_{01} + 0.92 \times (T_{02} - T_{01}) = 300 + 0.92 \text{ x } (111.5) = 402.6 \text{ K}$$

$$\frac{T_{02s}}{T_{01}} = \left(\frac{p_{02}}{p0_1}\right)^{\left(\frac{\gamma-1}{\gamma}\right)} = \left(\frac{293}{105}\right)^{\left(\frac{1.4-1}{1.4}\right)} = 1.34$$

$$T_{02s} = 1.34 \times 300 - 402 \; K$$

$$\eta = \frac{(T_{02s} - T_{01})}{(T_{02} - T_{01})} = \frac{(402 - 300)}{(111.5)} = 91.5\%$$

$$M_2 = 0.946$$

$$\dot{m}\sqrt{c_p T_{02}} \; / \; P_{02} A_2 = 1.278$$

$$\dot{m} = 1.278 \times A_2 \times \frac{p_{02}}{\sqrt{c_p \times T_{02}}} = 1.278 \times \pi \times 0.07 \times 0.01 \times \frac{293 \times 10^3}{1005 \times 412.5}$$

$$\dot{m} = 2.56 kg / s$$

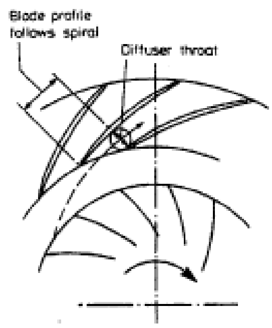
(e) From notes

Each individual passage of a vaned diffuser may be considered as a conventional diffuser for which numerous design rules exist.

A small angle of divergence is necessary for optimum pressure recovery but this leads to a long diffuser.

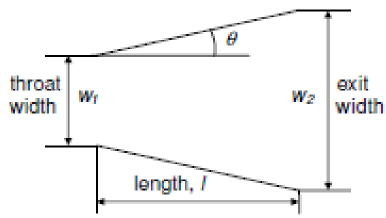
The most important parameter of the diffuser is its throat width, i.e. its minimum area. It must be sized to:

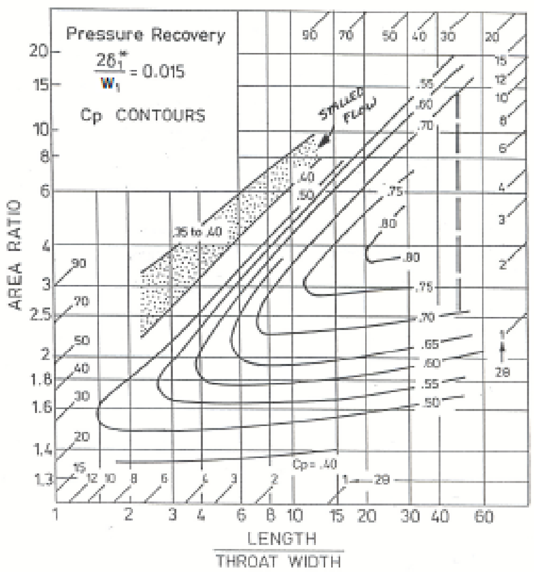
i. pass the required mass flow rate without choking
ii. limit the diffusion between the impeller exit and the throat.



Vaned diffuser geometry

Optimal pressure recovery occurs when half angle is optimised according to 1D diffusor charts.





Diffuser performance map for a plain rectangular diffuser