# Module 4A3 - TURBOMACHINERY 1
# 2025

Q1. (a) Diffusion Factor is a measure of the diffusion from the peak suction of the suction surface to the trailing edge of the suction surface. As the number of blades is reduced the diffusion factor increases. The Diffusion Factor is fixed in the design process and this in turn then sets the blade number.

[10%]

(b)

$$DF = 1 - \frac{V_2}{V_1} + \frac{|V_{\theta 2} - V_{\theta 1}|}{2V_1}\frac{s}{c}$$

$V_x = V_2 \cos\alpha_2$, $V_x = V_1 \cos\alpha_1$, $V_{\theta 2} = V_x \tan\alpha_2$ and $V_{\theta 1} = V_x \tan\alpha_1$

$$DF = 1 - \frac{\cos\alpha_1}{\cos\alpha_2} + \frac{|\tan\alpha_2 - \tan\alpha_1|}{2}\frac{s}{c}\cos\alpha_1$$

$$\frac{s}{c} = \left(DF - 1 + \frac{\cos\alpha_1}{\cos\alpha_2}\right)\frac{2}{|\tan\alpha_2 - \tan\alpha_1|\cos\alpha_1}$$

DF=0.45 so

$$\frac{s}{c} = \left(0.45 - 1 + \frac{\cos 48^\circ}{\cos 30^\circ}\right)\frac{2}{|\tan 30^\circ - \tan 48^\circ|\cos 48^\circ} = 1.25$$

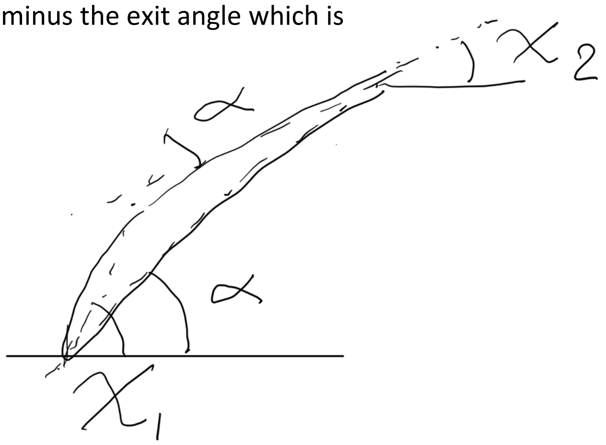
It was assumed that the AVDR of the cascade is one and so the axial velocity is constant across the cascade.

[15%]

(c)

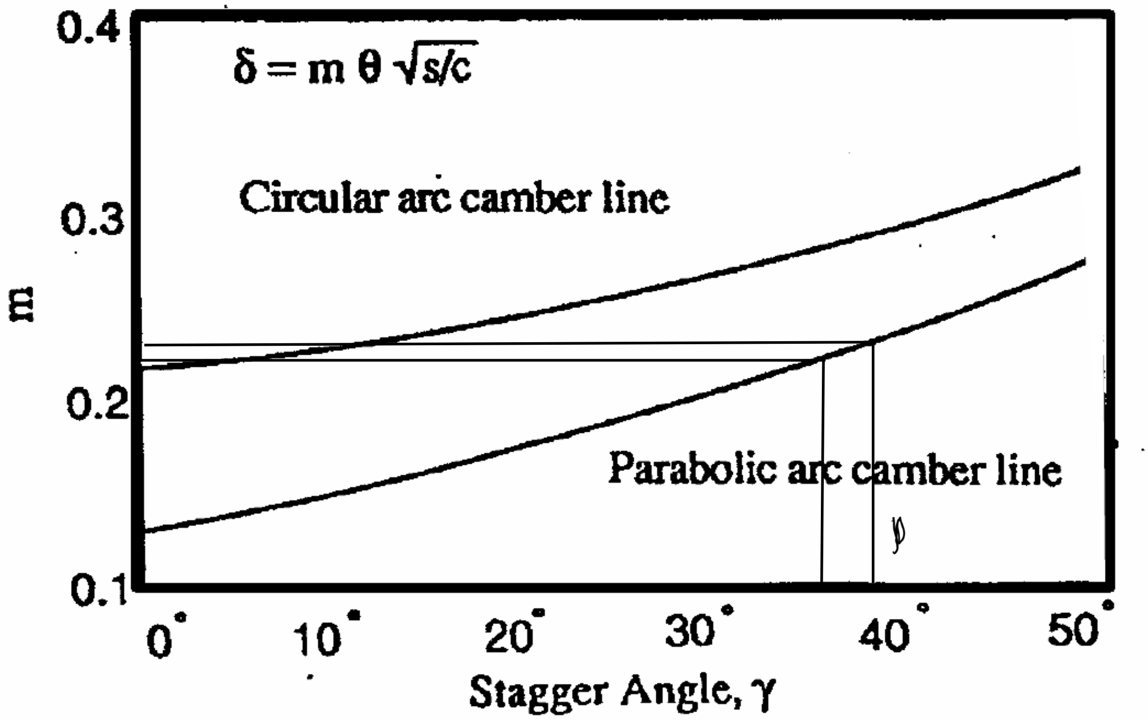
$$\delta = m\theta\sqrt{\frac{s}{c}}$$

Now use the line in Fig 1 for the parabolic blade. The first estimate of the stagger angle is the inlet angle minus the exit angle which is



The camber angle $\theta = (\chi_1 - \chi_2)$

The stagger $\gamma = \frac{1}{2}(\chi_1 + \chi_2)$




<u>Initial guess</u>

Start with an estimate using the flow angles $\theta = (\chi_1 - \chi_2) = 18^\circ$, $\gamma = \frac{1}{2}(48^\circ + 30^\circ) = 39^\circ$

From graph $m = 0.23$

$$\delta = 0.23 \times 18^\circ \sqrt{1.25} = 4.6^\circ$$

So $\chi_2 = 30^\circ - 4.6^\circ = 25.4^\circ$

<u>First iteration</u>

So $\theta = (\chi_1 - \chi_2) = 22.6^\circ$, $\gamma = \frac{1}{2}(48^\circ + 25.4^\circ) = 36.7^\circ$

From graph $m = 0.22$

$$\delta = 0.22 \times 22.6^\circ \sqrt{1.25} = 5.6^\circ$$

So $\chi_2 = 30^\circ - 5.6^\circ = 24.4^\circ$

<u>Second iteration</u>

So $\theta = (\chi_1 - \chi_2) = 23.6^\circ$

Change in m small so

$$\delta = 0.22 \times 23.6^\circ \sqrt{1.25} = 5.8^\circ$$

So $\chi_2 = 30^\circ - 5.8^\circ = 24.2^\circ$

So exit metal angle is ~24.2°

[25%]

(d) The stagnation pressure coefficient is defined as

$$Y_p = \frac{p_{01} - p_{02}}{p_{01} - p_1}$$

$$Y_p = \frac{\frac{p_{01}}{p_1} - \frac{p_{02}}{p_1}}{\frac{p_{01}}{p_1} - 1}$$

$$\frac{\dot{m}\sqrt{c_p T_{01}}}{hscos\alpha_1 p_{01}} = f(M_1) = \frac{\dot{m}\sqrt{c_p T_{01}}}{hscos\alpha_1 p_{01}} \times \frac{cos\alpha_2}{cos\alpha_1} \times \frac{p_{02}}{p_{01}}$$

$$f(0.45) = 0.8843$$

$$f(0.65) = 1.128$$

$$\frac{p_{02}}{p_{01}} = \frac{f(M_1)}{f(M_2)} \times \frac{cos\alpha_1}{cos\alpha_2} = \frac{1.128}{0.8843} \times \frac{cos48°}{cos30°} = 0.9856$$

At M=0.65

$$\frac{p_{01}}{p_1} = \frac{1}{0.7528} = 1.328$$

$$Y_p = \frac{\frac{p_{01}}{p_1} - \frac{p_{02}}{p_{01}} \times \frac{p_{01}}{p_1}}{\frac{p_{01}}{p_1} - 1} = \frac{1.328 - 0.9856 \times 1.328}{1.328 - 1} = 0.0583$$

[20%]

I Applying mass conservation between into and throat.

$$\frac{\dot{m}\sqrt{c_p T_{01}}}{hscos\alpha_1 p_{01}} = f(M_1) = \frac{\dot{m}\sqrt{c_p T_{01}}}{hop_0{}^*} \times \frac{o}{scos\alpha_1} \times \frac{p_0{}^*}{p_{01}}$$

$$f(1) = 1.281$$

Assume no loss between inlet and throat.

$$cos\alpha_1 = \frac{f(1)}{f(M_1)} \times \frac{o}{s} = \frac{1.281}{1.128} \times 0.66 = 0.7495$$

$$\alpha_1 = 41.5°$$

This means that the blade chokes at $41.5° - 48° = -6.5°$

[20%]

(f) At high incidence close to stall all the blades in the cascade are close to separation. At this point the bottom blade has the cascade end wall boundary layer entering it and so it tends to separate. This increases the incidence on the next blade up the cascade, which in turn separates. This makes achieving periodicity in compressor cascades at high incidence very difficult.

[10%]

Q2 (a) The compressor isentropic efficiency at design is 91%.

$$\frac{T_{02}}{T_{01}} = 1 + \frac{1}{\eta_{is}}\left[\left(\frac{p_{02}}{p_{01}}\right)^{(\gamma-1)/\gamma} - 1\right] = 1 + \frac{1}{0.91}\left[1.5^{0.4/1.4} - 1\right] = 1.13497$$

The polytropic efficiency is then given by:

$$\eta_p = \frac{\gamma-1}{\gamma}\frac{\ln\left(p_{02}/p_{01}\right)}{\ln\left(T_{02}/T_{01}\right)} = \frac{0.4}{1.4}\frac{\ln 1.5}{\ln 1.13497} = \underline{0.915} \quad (91.5\%)$$

[10%]

(b) Applying continuity between inlet ant exit,

$$\frac{\dot{m}\sqrt{c_p T_{01}}}{A_1 p_{01}} = \frac{\dot{m}\sqrt{c_p T_{02}}}{A_N p_{02}}\frac{A_N}{A_1}\frac{p_{02}}{p_{01}}\sqrt{\frac{T_{01}}{T_{02}}}$$

Applying the definition of polytropic fficiency

$$\frac{\dot{m}\sqrt{c_p T_{01}}}{A_1 p_{01}} = \frac{\dot{m}\sqrt{c_p T_{02}}}{A_N p_{02}}\frac{A_N}{A_1}\left(\frac{p_{02}}{p_{01}}\right)^{1-\frac{\gamma-1}{2\gamma\eta_p}}$$

Using the fact that the exit nozzle is choked, $\frac{\dot{m}\sqrt{c_p T_{02}}}{A_N p_{02}} = 1.281$, rearranging gives

$$\frac{p_{02}}{p_{01}} = \left(\frac{\dot{m}\sqrt{c_p T_{01}}}{A_1 p_{01}}\frac{A_1}{1.281 A_N}\right)^{\frac{2\gamma\eta_p}{(2\gamma\eta_p-\gamma+1)}} = \left(\frac{\dot{m}\sqrt{c_p T_{01}}}{A_1 p_{01}}\frac{A_1}{1.281 A_N}\right)^{\frac{2\times1.4\times0.915}{(2\times1.4\times0.915-0.4)}}$$

$$\underline{\frac{p_{02}}{p_{01}} = \left(\frac{\dot{m}\sqrt{c_p T_{01}}}{A_1 p_{01}}\frac{A_1}{1.281 A_N}\right)^{1.185}}$$

[20%]

(c) Using conditions at the design point, rearranging the above,

$$\frac{\dot{m}\sqrt{c_p T_{01}}}{A_1 p_{01}} = \frac{1.281 A_N}{A_1}\left(\frac{p_{02}}{p_{01}}\right)^{\frac{1}{1.185}} = \frac{1.281}{1.586}(1.5)^{\frac{1}{1.185}} = 1.1372$$

From tables, the fan inlet Mach number,

$$\underline{M_1 = 0.66}$$

The tip relative Mach Number,

$$M_{rel} = \sqrt{M_1^2 + \left(\frac{M_1}{\phi_{tip}}\right)^2} = \sqrt{0.66^2 + \left(\frac{0.66}{0.55}\right)^2} = \underline{1.37}$$

[20%]

(d) Design point working line:

| Normalised $\frac{\dot{m}\sqrt{T_{01}}}{p_{01}}$ | $\frac{\dot{m}\sqrt{c_p T_{01}}}{A_1 p_{01}}$ | $\frac{p_{02}}{p_{01}} = 1.288\left(\frac{\dot{m}\sqrt{c_p T_{01}}}{A_1 p_{01}}\right)^{1.185}$ |
|---|---|---|
| 1 | 1.1372 | 1.5 |
| 0.9 | 1.0235 | 1.324 |
| 0.8 | 0.9098 | 1.151 |

This working line intersects the 80% speed line where

$$\underline{\frac{p_{02}}{p_{01}} = 1.24}, \quad \frac{\dot{m}\sqrt{T_{01}}}{p_{01}} = 0.857 \Rightarrow \frac{\dot{m}\sqrt{c_p T_{01}}}{A_1 p_{01}} = 0.974$$

From tables, the fan inlet Mach number,

$$\underline{M_1 = 0.513}$$

(approximate answers accepted)

[20%]

(e) At instability on the 100% speed line,

$$\frac{p_{02}}{p_{01}} = 1.525, \quad \frac{\dot{m}\sqrt{T_{01}}}{p_{01}} = 0.818 \Rightarrow \frac{\dot{m}\sqrt{c_p T_{01}}}{A_1 p_{01}} = 0.930$$

Rearranging above equation, the area ratio at instability,

$$\frac{A_1}{A_N^S} = \frac{1.281}{\frac{\dot{m}\sqrt{c_p T_{01}}}{A_1 p_{01}}}\left(\frac{p_{02}}{p_{01}}\right)^{\frac{1}{1.185}} = \frac{1.281}{0.93}(1.525)^{\frac{1}{1.185}} = 1.966$$

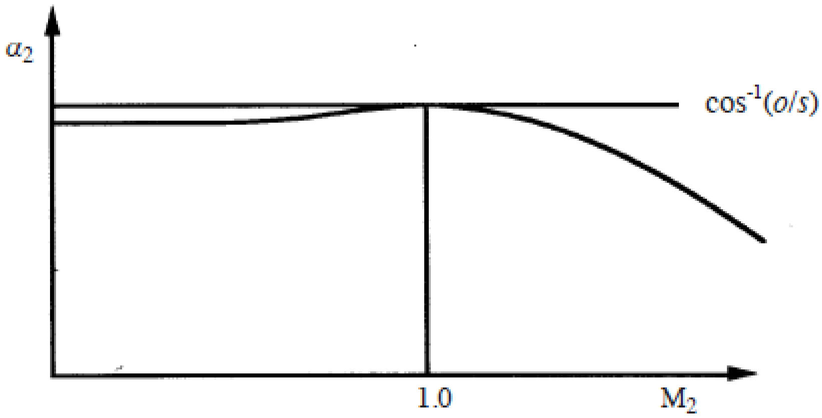
$$\frac{A_N^S}{A_N} = \frac{1.586}{1.966} = 0.806 \rightarrow$$ i.e. A nozzle area reduction of ~19% relative to the design point

[15%]

The cause of stall is the increasing incidence onto the fan rotor blades as the axial Mach number is reduced at constant speed. Eventually the flow around a blade section will separate and the separation will form a stall cell that propagates around the annulus. The stall in a fan is likely to be part span stall towards the rotor tip due to higher loading there and the low hub/tip ratio. The cells will extend axially from upstream of the rotor through the blade passages. To return the fan to unstalled operation, the exit nozzle area needs to be increased significantly from that at stall to overcome any hysteresis. Usually the fan will drop out of stall before reaching the design point working line.

[15%]

Q3. (a)




For supersonic flow the turbine passage is a convergent-divergent nozzle. For the divergent section the exit angle must reduce. The limiting exit Mach number occurs when the axial Mach number is one. At this point pressure waves cannot travel upstream and this therefore the exit Mach number cannot rise any further.

[20%]

(b)

$$Y_p = \frac{p_{01} - p_{02}}{p_{01} - p_2} = 0.08$$

$$Y_p = \frac{\frac{p_{01}}{p_{02}} - 1}{\frac{p_{01}}{p_{02}} - \frac{p_2}{p_{02}}} = 0.08$$

$$\frac{p_{01}}{p_{02}} - 1 = 0.08\left(\frac{p_{01}}{p_{02}} - \frac{p_2}{p_{02}}\right)$$

$$\frac{p_{01}}{p_{02}} 0.92 = 1 - 0.08\frac{p_2}{p_{02}}$$

$M_2 = 1.2$ $\frac{p_2}{p_{02}} = 0.4124$

$$\frac{p_{01}}{p_{02}} = \left(\frac{1 - 0.08\frac{p_2}{p_{02}}}{0.92}\right) = \left(\frac{1 - 0.08 \times 0.4124}{0.92}\right) = 1.0511$$

$$\frac{\dot{m}\sqrt{c_p T_{02}}}{hscos\alpha_2 p_{02}} = f(M_2) = \frac{\dot{m}\sqrt{c_p T_0}}{hop_0{}^*} \times \frac{o}{scos\alpha_2} \times \frac{p_0{}^*}{p_{02}}$$

$$1.2432 = f(M_2) = 1.2810 \times 0.34 \times \frac{1}{cos\alpha_2} \times 1.0511$$

$$cos\alpha_2 = \frac{1.2810}{1.2432} \times 0.34 \times 1.0511 = 0.3682$$

$$\alpha_2 = 68.4°$$

[20%]

(c)

$$\frac{\dot{m}\sqrt{c_p T_{01}}}{h_1 s p_{01}} = f(M_1) = \frac{\dot{m}\sqrt{c_p T_0}}{h^* o p_0{}^*} \times \frac{h^*}{h_1}\frac{o}{s} \times \frac{p_0{}^*}{p_{01}}$$

Because all loss occurs downstream of the throat

$\frac{p_0{}^*}{p_{01}} = 1$ and $\frac{h^*}{h_1} = 1$

$$\frac{\dot{m}\sqrt{c_p T_{01}}}{h_1 s p_{01}} = f(M_1) = 1.281 \times 0.34 \times 1 = 0.4355$$

$$M_1 = 0.20$$

[10%]

(d) The pitch to chord ratio of the turbine is set to 0.82. Calculate the Zweifel Loading Coefficient of the blade row. Comment on its value.

$$z = \frac{\dot{m}(V_{\theta 2} - V_{\theta 1})}{hC_x(p_{01} - p_2)}$$

$$V_{\theta 2} = V_2 sin\alpha_2 = f_1(M_2)\sqrt{c_p T_{02}} sin\alpha_2$$

$$V_{\theta 1} = V_1 sin\alpha_1 = f_1(M_1)\sqrt{c_p T_{01}} sin\alpha_1$$

$$\alpha_1 = 0°$$

$$f_1(M_2) = 0.6687$$

$$\dot{m} = \frac{f_2(M_1) h s p_{01}}{\sqrt{c_p T_{01}}}$$

$$f_2(M_1) = 0.4355$$

$$z = \frac{\left(\frac{f_2(M_1) h s p_{01}}{\sqrt{c_p T_{01}}}\right)\sqrt{c_p T_{01}}(f_1(M_2) sin\alpha_2)}{hC_x p_{01}\left(1 - \frac{p_2}{p_{01}}\right)}$$

$$z = \frac{f_2(M_1) s f_1(M_2) sin\alpha_2}{C_x\left(1 - \frac{p_2}{p_{01}}\right)}$$

$\frac{p_2}{p_{02}} = 0.4124$ and $\frac{p_{01}}{p_{02}} = 1.0511$

$$\frac{p_2}{p_{01}} = \frac{\frac{p_2}{p_{02}}}{\frac{p_{01}}{p_{02}}} = \frac{0.4124}{1.0511} = 0.3924$$

$$z = \frac{0.4355 \times 1.35 \times 0.6687 \times sin68.4°}{(1 - 0.3924)} = 0.60$$

This is a little low. The optimum value would be 0.8. The pitch to axial chord ratio could be raised.

[25%]

(e) At the limit load the axial Mach number is one.

$$M_{x,lim} = M_{2,lim} cos\alpha_{2,lim} = 1$$

$$cos\alpha_{2,lim} = \frac{1}{M_{2,lim}} = \frac{1}{1.7} = 0.5882$$

Conserving mass

$$\frac{\dot{m}\sqrt{c_p T_{02}}}{hscos\alpha_{2,lim} p_{02}} = f(M_2) = \frac{\dot{m}\sqrt{c_p T_0}}{hop_0{}^*} \times \frac{o}{scos\alpha_{2,lim}} \times \frac{p_0{}^*}{p_{02}}$$

At Mach 1.7 $f(M_2) = 0.9577$ $\frac{p_2}{p_{02}} = 0.2026$

$$0.9577 = 1.281 \times 0.34 \times \frac{1}{0.5882} \times \frac{p_0{}^*}{p_{02}}$$

$$\frac{p_0{}^*}{p_{02}} = \frac{0.9577 \times 0.5882}{1.281 \times 0.34} = 1.2934$$

$$Y_p = \frac{\frac{p_{01}}{p_{02}} - 1}{\frac{p_{01}}{p_{02}} - \frac{p_2}{p_{02}}} = \frac{1.2934 - 1}{1.2934 - 0.2026} = 0.269$$

The rise in loss is both due to the increase in shock loss and due to the strength of the shock separating the boundary layers.

[25%]