A.W. 4A3 2021

1 a) i) Two options $\phi = 1.12$, $\phi = 0.53$

$\psi = 1.7$, $\Lambda = 0.5$ $\rightarrow$ $\psi = 1 - 2\phi \tan\alpha_1$ — (1)

$\phi = 1.12$ [close to peak η_{TT} ridge]

from (1) $\underline{\alpha_1 = -17.35^\circ}$

$\psi = \phi [\tan\alpha_2 - \tan\alpha_1]$ (repeating stage) and $\psi = \frac{\Delta V_\theta}{U}$

$\rightarrow$ $\underline{\alpha_2 = 50.3^\circ}$

$\underline{\alpha_3 = \alpha_1 = -17.35^\circ}$

$V_\theta = U + W_\theta$ $\rightarrow$ $\tan\beta_2 = \tan\alpha_2 - 1/\phi$

$\underline{\beta_2 = 17.35^\circ}$

50% Reaction

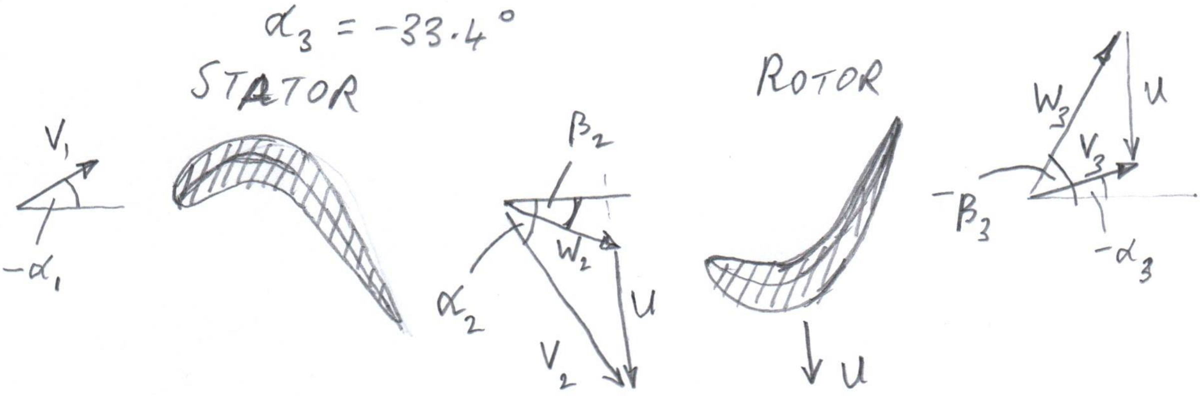
$\beta_2 = -\alpha_1$ $\qquad$ $\underline{\beta_3 = -50.3^\circ}$

$\beta_3 = -\alpha_2$

For $\phi = 0.53$ $\quad \alpha_1 = -33.4^\circ$, $\alpha_2 = 68.6^\circ$, $\beta_2 = 33.4^\circ$, $\beta_3 = -68.6^\circ$, $\alpha_3 = -33.4^\circ$

[30]

ii)



V = abs. vel

W = rel. vel

α = abs. angle

β = rel. angle.

[10]

iii) $(\Delta h_0)_{machine} = 148 \text{ kJ/kg}$

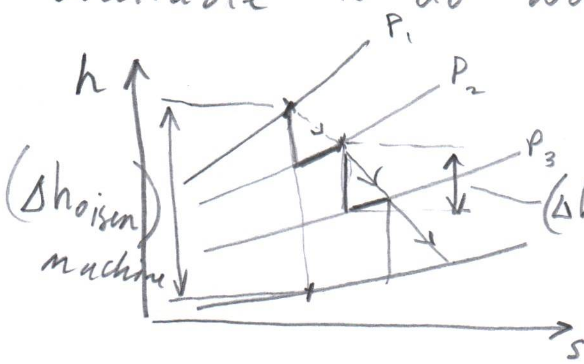
$(\Delta h_0)_{stage} = 1.7 \times U^2 = 1.7 \cdot \left(\frac{V_x}{\phi}\right)^2$

$$\frac{(\Delta h_0)_{machine}}{(\Delta h_0)_{stage}} = 7 \quad (\phi = 1.12)$$

7 stages $(\phi = 1.12)$ <u>OR</u> 2 stages $(\phi = 0.53)$

[10]

iv) $\eta_{machine} > \eta_{stage}$ because of the "reheat" effect. Some of the thermal energy created due to irreversibility is available to do work in subsequent stages.



$(\Delta h_{0isen})_{machine} < \sum (\Delta h_{0isen})_{stage}$

[10]

b) $$Z = \frac{\dot{m}(V_{\theta 2} - V_{\theta 1})}{C_x H (P_{01} - P_2)} = \frac{\rho V_x s H}{\rho V_x^2 C_x H} \cdot \frac{V_x [\tan\alpha_2 - \tan\alpha_1]}{\frac{1}{2(\cos\alpha_2)^2}}$$

$(P_{01} - P_2 = \frac{1}{2}\rho V_2^2)$ loss free, incompressible.

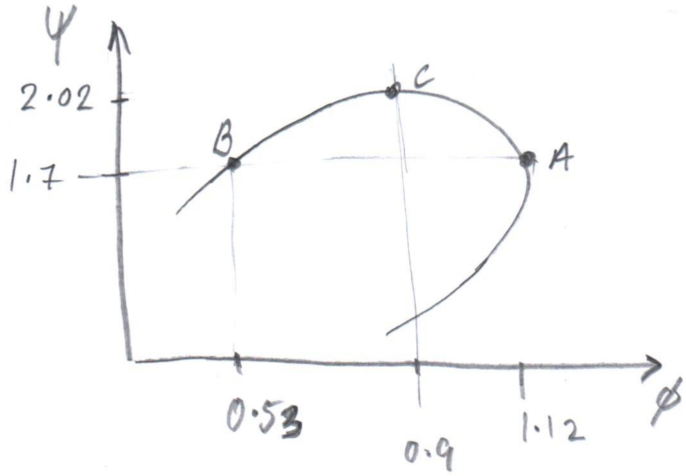
$$Z = 2(\cos\alpha_2)^2 [\tan\alpha_2 - \tan\alpha_1] \cdot s/C_x$$

$$s/C_x = \frac{0.8}{2(\cos\alpha_2)^2 [\tan\alpha_2 - \tan\alpha_1]}$$

Stator s/C_x = Rotor $s/C_x \Rightarrow$ 50 % Reaction

$\underline{s/C_x = 0.646}$ $(\phi = 1.12)$ $[0.934 \text{ for } \phi = 0.53]$ [20]

c) Consider Smith Chart ($\eta_{tt} = 90\%$ isobar)



For aeroengine turbine need to consider

- Size, weight, cost
- Compatability with compressor or fan

<u>Answers may vary</u>

Potential options to consider at A, B, C

Initial design at A or B $\left(s/c_x \propto \frac{1}{\cos^2\alpha_2 (\tan\alpha_2 - \tan\alpha_1)} \right)$

Stage	s/c_x	$\cos^2\alpha_2$	$(\tan\alpha_2 - \tan\alpha_1)$	$\Delta\psi$
A	0.646	0.408	1.518	—
B	0.934	0.133	3.207	—
C	0.680	0.262	2.244	+20%

Choice C ($\psi = 2.02$, $\phi = 0.9$) increases stage loading by 20%.

- Compared to A, reduction in stages and higher pitch to chord s/c_x. Although higher turning, larger acceleration ($\alpha_2 \uparrow$) means $\cos^2\alpha_2$ term wins out.

- Compared to B, higher loading and lower acceleration means smaller s/c_x. Benefit of higher ψ not clear because still require 2 stages. Increased ϕ for compatability with fan or compressor. (reduce U)

[20]

2. i)

$$\dot{W}_f = \dot{W}_{LPT}$$

$$c_p(\dot{m}_b + \dot{m}_c)(T_{02} - T_{01}) = \dot{m}_c \times 264.3$$

$$(BPR + 1)(T_{02} - T_{01}) = \frac{264.3}{1.005}$$

$$\underline{T_{02} - T_{01} = 23.9 \text{ K}}$$

$$T_{02}/T_{01} = \frac{23.9}{288} + 1 = 1.083$$

$$T_{02}/T_{01} = (\pi_f)^{\frac{\gamma-1}{\eta_p \gamma}}$$

$$\log(T_{02}/T_{01}) = \frac{\gamma-1}{\eta_p \gamma} \cdot \log(\pi_f)$$

$$\underline{\eta_p = 94.0\%}$$ [20]

ii)

$$\frac{\dot{m}_b \sqrt{c_p T_{02}}}{A_{19} P_{02}} = 1.281 \quad (\text{choked})$$

$$\underline{T_{02} = 311.9 \text{ K}}$$

$$\underline{P_{02} = 131.3 \text{ kPa}}$$

$$\Rightarrow \dot{m}_b = 210.3 \text{ kg/s}$$

$$\dot{m}_f = \dot{m}_c + \dot{m}_b = 210.3 + 21.03 = \underline{231.3 \text{ kg/s}}$$

$$\text{Fan Power} = \dot{m}_f \cdot c_p(T_{03} - T_{02}) = \underline{5.56 \text{ MW}}$$ [20]

iii) $A_1 / A_{19} = 1.43$, $A_{19} = 1.0 m^2$

$$\frac{\dot{m}\sqrt{C_p T_{01}}}{A_{19} P_{01}} = 1.232$$

from tables , $\underline{M_1 = 0.8}$

[20]

b) <u>Choked flow</u>

$$\rightarrow \quad \frac{\dot{m}_c\sqrt{C_{pe} T_{045}}}{A_{45} P_{045}} = \frac{\dot{m}_c\sqrt{C_{pe} T_{05}}}{A_9 P_{05}} \Rightarrow \frac{P_{05}}{P_{045}} = \frac{A_{45}}{A_9} \cdot \sqrt{\frac{T_{05}}{T_{045}}}$$

$$W_f = \frac{C_{pe}(T_{045} - T_{05})}{(BPR+1)} \quad (\text{from}(a))$$

$$W_f = \left(\frac{C_{pe}}{BPR+1}\right) T_{045} \left[1 - \frac{T_{05}}{T_{045}}\right]$$

$$T_{05}/T_{045} = \left(P_{05}/P_{045}\right)^{\frac{(\gamma-1)\eta_p}{\gamma}}$$

$$\frac{T_{05}}{T_{045}} = \left(\frac{A_{45}}{A_9}\right)^{\frac{\gamma-1}{\gamma}\eta_p} \times \left(\frac{T_{05}}{T_{045}}\right)^{\frac{\gamma-1}{2\gamma}\eta_p}$$

$$1 - \left(\frac{\gamma-1}{2\gamma}\right)\eta_p = \frac{2\gamma - (\gamma-1)\eta_p}{2\gamma}$$

$$\left(\frac{T_{05}}{T_{045}}\right)^{\frac{2\gamma-(\gamma-1)\eta_p}{2\gamma}} = \left(\frac{A_{45}}{A_5}\right)^{\frac{\gamma-1}{\gamma}\eta_p}$$

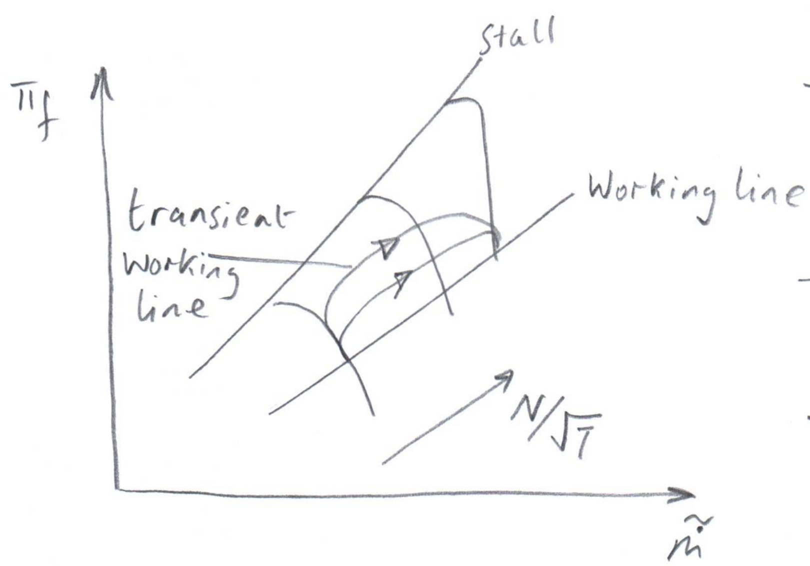
$$\left(\frac{\gamma-1}{\gamma}\eta_p\right)\cdot\frac{2\gamma}{2\gamma-(\gamma-1)\eta_p} = \frac{2(\gamma-1)\eta_p}{2\gamma-(\gamma-1)\eta_p}$$

$$\Rightarrow \quad W_f = \frac{c_{pe}\,T_{045}}{BPR+1}\cdot\left[1-\left[\frac{A_{45}}{A_9}\right]^{\frac{2(\gamma-1)\eta_p}{2\gamma-(\gamma-1)\eta_p}}\right]$$

Assume → no losses in nozzle, or mechanical

$$\Rightarrow \quad C_1 = \frac{c_{pe}}{1+BPR}\left[1-\left(A_9/A_{45}\right)^{\frac{2(\gamma_e-1)\eta_p}{(\gamma_e-1)\eta_p-2\gamma}}\right] \qquad [30]$$

c)



→ Sudden rise in TET

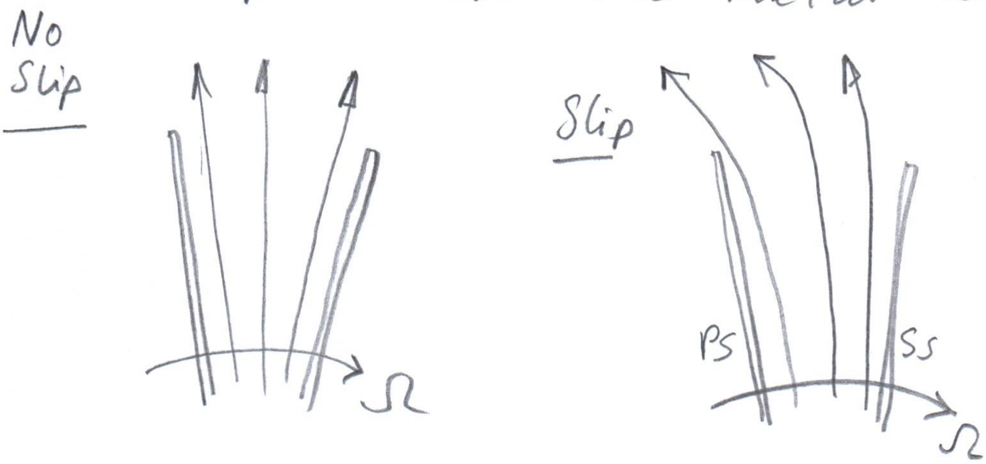
→ Drop in $\dot{m}$ in Compressor

→ Pushed towards stall

→ depends on rate of $\dot{m}_f$ rise

[10]

3 a) Slip is the phenomenon whereby the flow does not leave the impeller at the metal angle.



Loading must go to zero at T.E

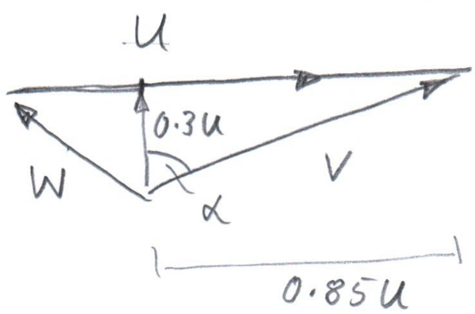
→ flow on P.S. accelerates } streamlines curve in relative frame

→ flow on S.S. decelerates

→ slip proportional to loading $\propto N_{blade}$ [10]

b) $\chi_2 = 0$, $\sigma = \psi = 0.85 = V_\theta / U$

i) $\Rightarrow N = \left(\frac{1}{0.15}\right)^{1/0.7} = 15$



$\alpha = \operatorname{atan}\left(\frac{0.85}{0.30}\right)$

$\underline{\alpha = 70.56^\circ}$

[20]

ii) $(\Delta h_0)_{isen} = \frac{\Delta P_0}{\rho}$ (incompressible)

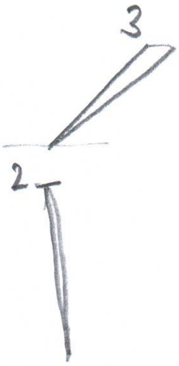
$\Delta h_0 = U^2 \psi$

$$\eta_{tt} = \frac{\Delta h_{0isen}}{\Delta h_0} = \frac{\Delta P_0}{\rho U^2 \psi} = 0.9$$

$$\Rightarrow \frac{\Delta P_0}{\rho \Omega^2 (D/2)^2} = \underline{\underline{0.765}} \quad \left(= \frac{\Delta P_0}{\rho U^2} \right)$$ [10]

$$\left(\frac{\Delta P_0}{\rho \Omega^2 D^2} = \underline{\underline{0.191}} \right)$$

iii)



$$P_3 - P_2 = 0.7 \, (P_{02} - P_2)$$

$$\eta_{ts} = \frac{P_3 - P_{01}}{\rho U^2 \psi}$$

$$= \frac{1}{\psi} \left[\frac{(P_3 - P_2)}{\rho U^2} - \frac{(P_{02} - P_2)}{\rho U^2} + \frac{(P_{02} - P_{01})}{\rho U^2} \right]$$

$$\eta_{ts} = \frac{1}{\psi} \left[0.7 \frac{[P_{02} - P_2]}{\rho U^2} - \frac{[P_{02} - P_2]}{\rho U^2} + 0.765 \right]$$

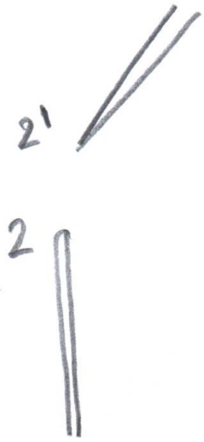
$$\frac{P_{02} - P_2}{\rho U^2} = \frac{\rho V^2}{2 \rho U^2} = \frac{0.3^2 + 0.85^2}{2} = 0.40625$$

$$\eta_{ts} = \frac{1}{0.85} \left[-0.3 \times 0.40625 + 0.765 \right]$$

$$\eta_{ts} = \underline{75.7\,\%}$$

[30]

iv)



angular momentum, mass conservation

$$V_\theta \propto \frac{1}{r}, \quad V_r \propto \frac{1}{r}$$

$$\Rightarrow \quad V \propto \frac{1}{r}$$

$$P_{2'} - P_2 = \frac{0.7}{3} \times 0.40625 \times \rho U^2$$

$$P_{0_{2'}} = P_{0_2} \quad (\text{loss free})$$

$$P_{2'} + \frac{\rho V_{2'}^2}{2} = P_2 + \frac{\rho V_2^2}{2}$$

$$P_{2'} - P_2 = \frac{\rho}{2}\left[V_2^2 - V_{2'}^2\right]$$

$$0.09479\, \rho U^2 = \frac{\rho}{2}\left[V_2^2 - V_{2'}^2\right]$$

$$0.09479 \times 2 \times \left(\frac{U}{V}\right)^2 = 1 - \left(\frac{V_{2'}}{V_2}\right)^2$$

$$\Rightarrow \quad \left(V_{2'}/V_2\right)^2 = 0.767$$

$$V_{2'}/V_2 = 0.876$$

$$\Rightarrow \quad r_{2'}/r_2 = 1.142$$

$$r_{2'} = 1.142\, r_2$$

$$r_{2'} - r_2 = 0.142\, r_2$$

$$\Rightarrow \quad \underline{\Delta r = 0.071 D}$$

[30]

The statistics below are based on the IIB marks only.

Q1: Repeating-stage turbine: 33/35 attempts mean=60.2%. This was the most popular question. Nearly all students were able to determine the flow angles and draw the blade shapes and velocity triangles required for the first part of the problem. Most students were able to estimate the required number of stages. Students were generally not able to give a good description of reheat and the effect of this on multi-stage efficiency. The second part required students to determine the pitch-to-chord from the Zwiefel coefficient. This was less well answered. While a good number of students were able to use the suggestion to consider incompressible, loss free flow, some struggled to apply Bernoulli and there were even some unsuccessful attempts to use compressible flow relationships to solve this part. The final part of the problem was to discuss alternative stage designs for an aeroengine application. Students tended not to link the choices of stage parameters to pitch-to-chord which they derived in the earlier part of the question.

Q2: Turbo-fan: 28/35 attempts, mean=72.5%. Another popular question and generally very well answered. Common mistakes in the first part of the problem were to not recognize that the mass flow through the fan included both the bypass flow and the core flow. The derivation of the fan specific work was also generally well answered, with a few sign errors. For the last part of the problem, most students recognized that an impulse of fuel would affect the compressor stability but many failed to give an adequate explanation of why this is the case.

Q3: Low-speed compressor: 9/35 attempts, mean=50.6%. This was the least popular question. The first part of the problem was generally well answered, with most students giving a reasonable description of slip. Students on the whole were able to relate this to stage loading and the rotor inlet velocity triangle without much problem. Students tended to struggle when deriving the rotor pressure-rise coefficient and total-to-static efficiency. Some students were not able to link the total-pressure rise to the isentropic work. Others were confused by the definition of the total-to-static efficiency. A good number spotted that the velocity was inversely proportional to radius in the vaneless diffuser, but then often struggled to relate this to the radial length of the vaneless space.

AW. 19.05.21