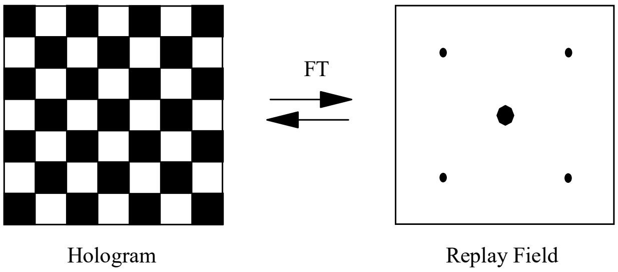
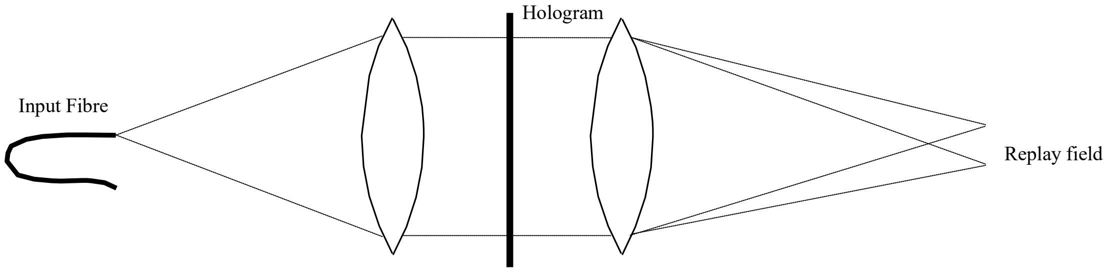
4B11 – Exam cribs 2023 (answers are more verbose than those expected from the candidates).

Q1 a) [20%] A CGH is a 2D array of pixels which form an image in the replay field via the process of diffraction. By combining an array of these pixels at various positions on a regular grid, it is possible to generate a complex amplitude function in the far field. Such a 2-D combination of these pixels in various positions is defined as a Hologram and the pattern generated by the hologram if the far field is the Replay Field. The translation between the two is the Fourier transform. Important properties are the size of the pixel, its shape, its pitch in the CGH and the type of modulation.





b) [30%] The far field of a single square pixel is its Fourier transform: $F(u,v) = A\Delta^2 \operatorname{sinc}(\pi\Delta u)\operatorname{sinc}(\pi\Delta v)$

The original rectangular aperture is defined as a single pixel. By combining an array of these pixels at various positions on a regular grid, it is possible to generate a complex amplitude function in the far field. By altering the value of the amplitude A of each pixel, centred on a grid of interval b (in the example of two pixels above Δ was equal to b, but may not always be so), it is possible to add up the 2-D sinc functions and create an arbitrary 2-D distribution in the far field region. By superimposing all the exponential phase terms due to the shift and varying the amplitude A, it is possible to create useful patterns in the far field. In general terms, the broader the feature or combination of pixels, the smaller or more delta function-like the replay object. The exact structure of this distribution depends on the shape of the 'fundamental' pixel and the number and distribution of these pixels in the hologram. The pattern we generate with this distribution of pixels is repeated in each lobe of the sinc function from the fundamental pixel. The lobes can be considered as spatial harmonics of the central lobe which contains the desired 2-D pattern.

The spatial coordinates (u,v) are related to the original absolute coordinates used earlier in the diffracted aperture (α,β) by the relation.

$$u = \frac{k\alpha}{2\pi f} \qquad v = \frac{k\beta}{2\pi f}$$

This function is common for all the apertures which make up the hologram, only the phase changes as they get shifted about. Hence it forms the envelope function for the replay field of the hologram. The useful information of the replay field is contained in the central first lobe of the sinc function, so we can calculate the width of the replay field as where the first zero of the sinc function occurs ($\pi\Delta u = \pi$, $\pi\Delta v = \pi$). We want the coordinates in terms of $[\alpha,\beta]$, so we use the above transformation to get.

$$\alpha_M = \frac{f\lambda}{\Delta} \qquad \beta_M = \frac{f\lambda}{\Delta}$$

α_M and β_M tell us the width of the central lobe of the sinc envelope, due to the pixel pitch Δ. From this we can assume that an NxN pixel hologram will generate NxN spatial frequency 'pixels' in the replay field. This is an approximation, as the FT actually generates a continuous function in the replay field. Hence, the replay field will have a spatial frequency pixel of pitch.

$$\alpha_0 = \frac{f\lambda}{N\Delta} \qquad \beta_0 = \frac{f\lambda}{N\Delta}$$

Note that as shown before, there are other orders appearing in the replay field due to the orders of the first suppressed zero in the sinc envelope. This effectively limits the useable area in the replay field to $[\alpha_M/2, \beta_M/2]$ if overlapping hologram replay patterns are to be avoided.

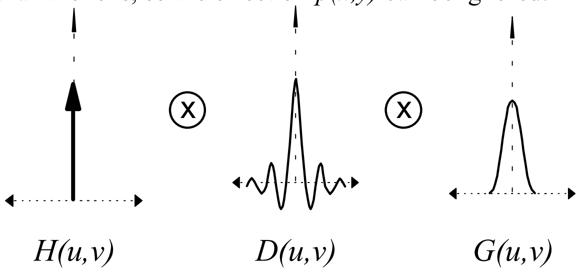
c) [30%] In the examples given the illumination of the hologram is uniform and that the hologram and lens extend to infinity. This is not the case in the real world, as there are a finite number of hologram pixels creating a aperture over the hologram and the light used to illuminate it will not be uniformly distributed. In all the examples we are assuming that the illumination source is a collimated monochromatic laser which generates high quality parallel wavefronts with a wider diameter than the hologram or the lenses. Such a source will usually have an intensity distribution which can be expressed as a Gaussian beam profile or function.

$$g(x,y) = A_G e^{l\left(x^2+y^2\right)}$$

The entire illumination system (apodisation) can be modeled as a sequence of multiplied functions. The input illumination distribution *g(x,y)* times the hologram aperture *d(x,y)* times the total aperture of the FT lens (if it has a smaller diameter than the hologram) *p(x,y)*. Hence effect of the FT on these functions results in a convolution of their transforms.

$$F(u,v) = G(u,v) \otimes D(u,v) \otimes P(u,v)$$

The ideal hologram replay field *H(u,v)* is designed as an array of delta functions in desired positions. The lens aperture *p(x,y)* is a large circular hole, so the FT *P(u,v)* will be a first order Bessel function (like a circular sinc function). The hologram aperture is a large square of size *NΔ* and its FT, *H(u,v)* will be a sharp sinc function. The effect of the FT of the illumination *G(u,v)* is to add a Gaussian profile. Hence, the profile of the spots in the hologram replay field will not be delta functions, they will be delta functions convolved with a Bessel function convolved with a sinc function convolved with a Gaussian function. This means that the replay field will not look exactly as expected, spots which are placed next to one another will interfere due to the tails of the Gaussian, sinc and Bessel functions and the individual desired sharp 'spots' become ringed blobs with finite width. In most cases, the hologram aperture will be smaller than the lens, so the effect of *p(x,y)* can be ignored.

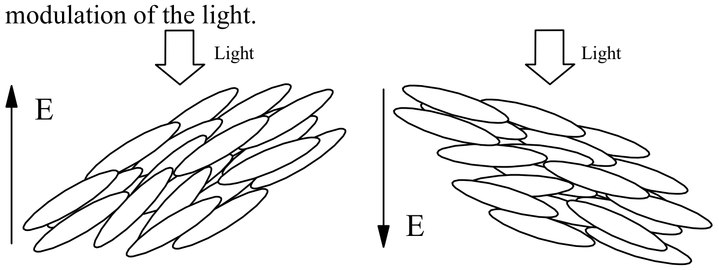


This can be a severe limitation to crosstalk in optical switches as the sidelobes are picked up by the neighbouring channels.

d) [20%] All holograms are shift invariant, so when a hologram is replicated, then multiple copies of the replay field are superimposed onto each other with associated phase shifts. This means that the replications interfere with one another. This can be seen in the pixels of the final replay field as the spacing between then changes. K replications of an NxN pixel hologram will have effectively KN pixels in total which increases the size of the aperture and reduces sidelobes in the replay field which helps reduce the crosstalk between pixel in the RPF. The final replay field appears as if it has been sampled K times which created a sparse distribution of the hologram orders. Hence if the spacings of the fibre channels are carefully chosen, then the fibre cores will appear in the gaps of the sidelobes greatly reducing the crosstalk.

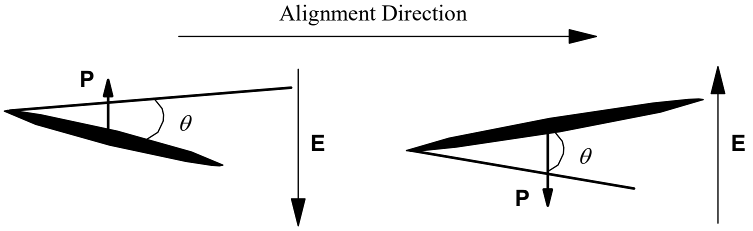
Generally well answered, but not as well as expected as it is a fairly common structured question. A lot just quoted the results with no derivation. a) answered well, although quite a few forgot the definitions and a few derived unnecessary equations. b) well answered overall with most getting a good derivation of the dependencies, but a few were unstructured and not very clear. c) also well answered but again messy. A few missed the relevance of a finite number of pixels. d) Not well answered with most not getting point of the repetition and its effect on the RPF. Only 2 candidates mentioned that the replication would give a phase shift on the overlapping RPF and would reduce the effects of apodisation.

Q2 a) [35%] One of the commonest calimatic liquid crystal mesophases is the nematic phase. This is the least ordered mesophase before the isotropic. Here the molecules have only long range order and no longitudinal order. This means that the molecules retain a low viscosity, like a liquid, and are prone to flow. This can greatly effect the speed at which nematic LCs can modulate the light. The existence of dielectric anisotropy means that we can move the molecules around by applying an electric field across them. This combined with the flow properties means that a nematic molecule can be oriented in any direction with the use of an electric field. This is a very desirable feature as it leads to their ability to perform greyscale modulation of the light.



The calimatic molecular shape also leads to an optical anisotropy in nematic LCs, with the two axes of the molecule appearing as the refractive index. The refractive index along the long axis of the molecules is often referred to as the extraordinary n_e (or fast n_s) and the short axis the ordinary n_o (or slow n_s) axis. The difference between the two is the birefringence. $\Delta n = n_e - n_o$. One of the most useful smectic mesophases is the smectic C (SmC) phase as the molecules are highly ordered and form layers with the molecules tilted within each layer. The smectic C structure can be improved by adding chirality to the molecular structure which adds an extra dipole perpendicular to the molecular axis of the LC material. This is often is referred to as chiral smectic C or SmC*.

If the FLC is restricted to a cell thickness of 2-5μm then the helix of along the cell is suppressed and the molecules are bounded into two stable states either side of the director cone. The angle between these two states is defined as the switching angle θ. This is referred to as a surface stabilised FLC geometry an creates a high degree of ferroelectricity and creates a large birefingent electro-optical effect. The penalty for doing this is that the molecules are only stable in the two states an therefore the modulation will only be binary. The up side to this binary modulation is that it can be very fast (~10μsec) and that the stability can lead to the molecules remaining in the two states in what is known as bistable switching.

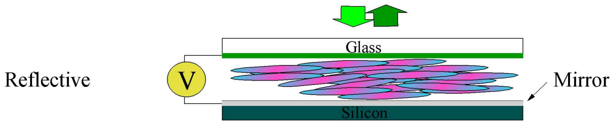


b) [20%] i) An optical correlator – must be fast hence FLC is best. Binary phase is ok, but has no asymmetry in the recognised objects. To break symmetry, must use more then binary phase levels. Polarisation sensitivity is not an issue.

ii) A single mode fibre to fibre optical switch – Speed is important, but not as important as loss and crosstalk. Therefore need to use a nematic LC for multi-level phase modulation. But this is polarisation dependent, hence we need an integrated quarter wave plate device.

ii) An optical packet switch – speed is everything. Need to modulate below 1usec, hence an electoclinic modulator is Ok. Optical loss is high, but this is Ok for speed

c) [30%]



LCOS device has integrated circuitry and pixel mirrors on the silicon backplane. This means the device can be very small, compact and high speed as well as possible intelligence. All of the drive electronics can be integrated into the backplane which makes for a very compact microdisplay. The mirror quality can also be optimised using planarisation and cold evaporated aluminium.

A LCOS backplane is designed for a particular type of modulation or material. FLCs are binary materials which require very simple circuitry to drive the pixels. This means that the circuits can implement complex binary logic, memory, registers and counters which makes the implementation very flexible and sophisticated. Allows for DC balancing and bitplane greyscale type operations.

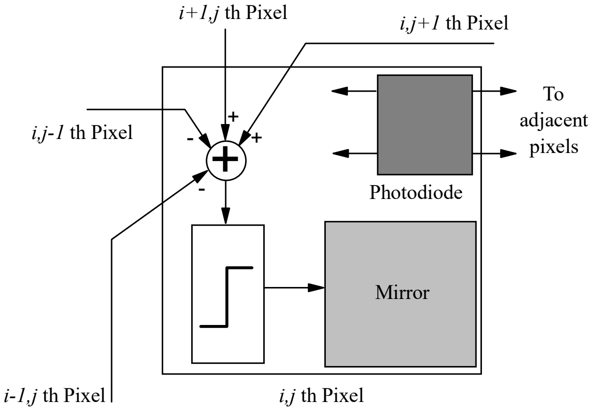
An LCOS backplane used with nematics is more complicated as it must maintain an analogue RMS voltage across the pixels. This needs bulky analogue/mixed circuitry and also includes circuits such as D to A converters and analogue multiplexers.

d) A logical advance in the design of a SLM is to combine the optical properties of the SLM with the processing ability of electronics. This is especially important when VLSI silicon backplane SLMs are used as the VLSI processes are identical to those used to create a standard silicon chip. For this reason, electronics is combined with the SLM to create smart pixels capable of processing information presented optically and then modulating light for further processing. Such pixels are very useful for both image processing and for optical neural networks hence they combine the modulating qualities of the LC material with the electronics and the use of silicon photodetectors at the pixel level.

One type of smart pixel is the isophote or intelligent camera, which performs spatial image processing. In this case, the pixel is fed by the four adjacent pixels to the left, right, above and below. The photodiodes from the adjacent pixels are connected to the central pixel in the fashion shown. The functionality of the summation of these signal is set such that we have the left pixel subtracted from the right added to the top subtracted from the bottom.

$$P'_{i,j} = \left|P_{i,j+1} - P_{i,j-1}\right| + \left|P_{i-1,j} - P_{i+1,j}\right|$$

This performs the function of a Roberts Cross filter which is a means of achieving edge enhancement. More complex processes such as Sobel filtering (a combination of all eight nearest neighbours) could also be implemented.



Well answered overall which was pleasing as it was the a more challenging question in the exam. a) was NLC and FLC bookwork with a few silly errors. Many forgot that the indicatrix is in fact a 3D structure and that it was polarisation sensitive. b) mixed answers overall. A lot did not chose FLC for part i) at the though it was intensity modulation one spotted that it could be both. ii) was not well answered as the performance is more important than speed. iii) was answered well. c) was surprisingly badly answered with many suggesting a transmissive SLM or an OASLM. Only one pointed out the impact on the backplane for NLC (analogue) vs FLC (digital) LC effects. d) better answered with most thinking of an OASLM, but a few pointed our a potential smart pixel.

Q3 a) [25%] The term *loss* refers to the amount of optical power which is launched down the optical fibre at the output end of the switch. It is normally the ratio of the optical power launched into each output fibre and the optical power at each input fibre. The ratio of the launched power into the output fibre to the input power is defined as the *efficiency* of the hologram (η). input If the switch is configured to route light to the *k*th fibre in an output array of *n*, then the *crosstalk* is the ratio of light launched down the desired fibre to the light launched down one of the other fibres which are not being routed. Both are normally expressed as power ratios in decibels.

Fan-in loss arises in a holographic switch because the only fibre in the output which is on the main optical axis is the one in the centre of the array (zero order). When light is steered to the outermost fibres it is at an angle to the central axis which no longer satisfies the perfect launch condition of a single mode fibre. Hence there is a loss which depends on this angle and therefore the position of the output port.

b) [25%] The total input power which appears in the output plane is P_{in}. The total power which is routed into a desired fibre by the CGH is P_{sp} and the remaining power is dissipated into the whole plane as the background or noise power P_{bk}.

$$P_{in} = 2P_{sp} + P_{bk}$$

The factor of 2 is due to the symmetry of the pattern due to binary phase. We can define the CGH efficiency η as the ratio between the power in the spot, P_{sp} and the input power P_{in}.

$$\eta = \frac{P_{sp}}{P_{in}}$$

For n fibres in the output array of a 1 to n switch, the power into a single fibre will be ηP_{in}. If the CGH has NxN pixels, then the replay field can also be assumed to contain NxN 'spatial frequency pixels'. If we assume that the background power is uniformly distributed over the N^2 spatial frequency pixels in the replay field then the background power at each pixel will be.

$$P_{bpix} = \frac{(1-2\eta)P_{in}}{N^2}$$

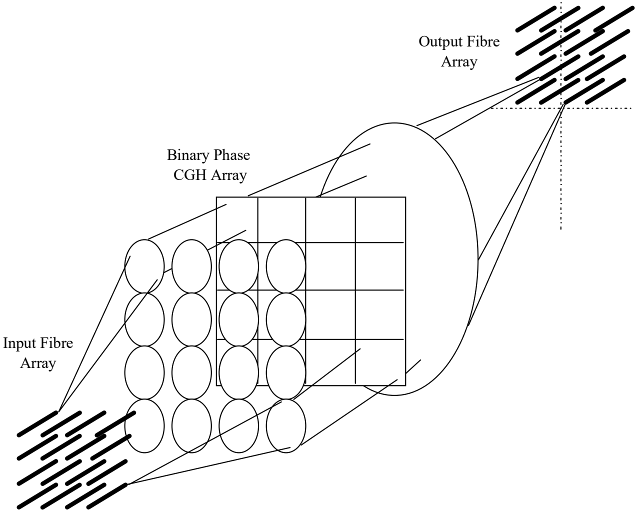
Hence the crosstalk is the ratio of the light routed to a fibre to P_{bpix}.

$$C = \frac{\eta}{1-2\eta}N^2$$

Assumptions:

- The distribution of the background power is uniform.
- The number of CGH pixels is infinite.
- The pixel pitch is effectively zero, hence no sinc envelope.
- The SLM used to display the CGH inevitably has no deadspace.
- The physical alignment of the fibres in the output array is perfect.
- Ferfect optics with no limitations or distortions.
- No fan in loss

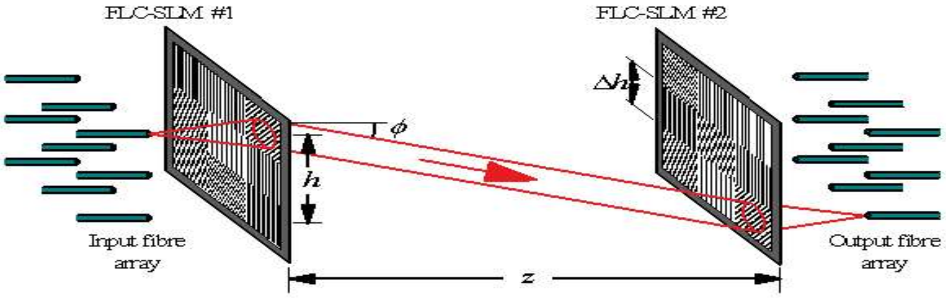
c) [25%] The n x n switch is basically an array of n 1 x n switches with a shared output lens



The n x n analysis for the crosstalk is the same except that we now have the background noise from each of the other n - 1 input fibres appearing at the each output fibre along with the ηP_{in} from the routed input. Hence the crosstalk will be.

$$C = \frac{\eta}{1-2\eta}\frac{N^2}{(n-1)}$$

d) [25%] The single hologram *n* x *n* switch is limited in scalability as it can only diffract light over a limited angle. Also the crosstalk boundary becomes probative. We can rectify this by using two holograms to steer the light. The first steer light into the switch, whilst the second steers light out of the switch back onto the output fibre's axis. The most efficient combination for routing is if the second hologram is the complex conjugate of the first routing hologram.

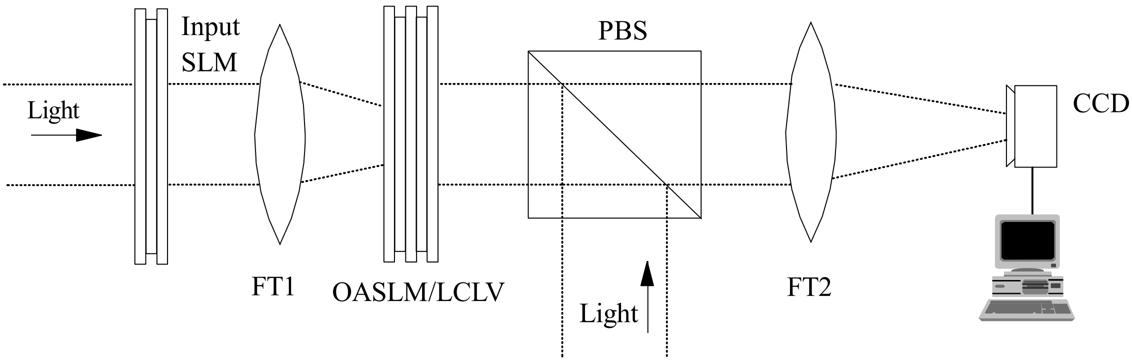


The two hologram switch can be scaled to any size and the loss through the switch does not scale with the number of input and output ports, it does however increase the loss as there are now two holograms routing the same beam. The only parameters which scale with the number of ports are the crosstalk and the physical length *z*. The crosstalk of the two hologram switch is greatly improved as the crosstalk of the first hologram is multiplied by the cross talk of the second hologram.

$$C = \left(\frac{\eta}{1-2\eta}\frac{N^2}{(n-1)}\right)^2$$

A more popular question on a fairly common theme and well answered overall. a) mostly book work but quite a few did not know the difference between loss and efficiency but did well in fan-in loss. b)almost all got the derivation correctly but many forgot to state the assumption made which are pretty important. c) most did this well and described the effects on crosstalk although nobody commented in the need for extra SLM pixels. d) very well answered, most got the effect on crosstalk and the impact of fan-in loss.

Q4 a) [40%]



The two images *r(x,y)* and *s(x,y)* are displayed side by side on the input SLM. Lens FT1 forms the Fourier transform of the input SLM image in its focal plane. This is then detected by the photoconductive surface of the OASLM and transferred to the liquid crystal (LC) side via the internal electric field. The LC side of the OASLM is then read by a second laser to form the image for the second Fourier transform lens FT2. The final output plane is captured by a camera and the correlation peaks analysed to see if the two images match.

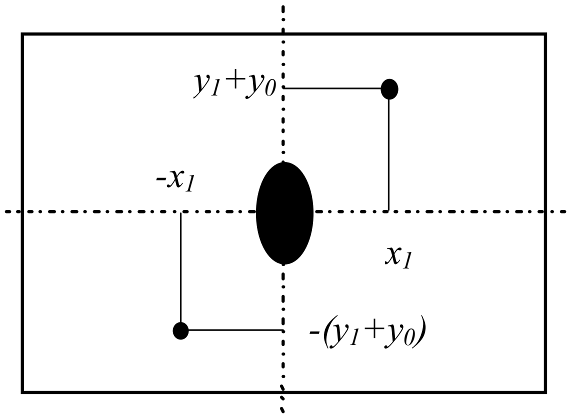
In plane 1, the input *s(x,y)* and reference *r(x,y)* are displayed side by side in an optical system and then transformed by a single lens into plane 2. This gives the spectrum of each image, with its associated phase shift due to their positions in the input plane.

$$S(u,v)e^{-j2\pi(x_1u-y_1v)} + R(u,v)e^{-j2\pi y_0v}$$

The nonlinearity between planes 2 and 3 creates the correlation and in its simplest form can be modelled by a square law detector such as photodiode or CCD camera which takes the magnitude squared of the light falling upon it.

$$S^2(u,v) + R^2(u,v) + S(u,v)R(u,v)e^{-j2\pi\left(x_1u-\left(y_0+y_1\right)v\right)} + S(u,v)R(u,v)e^{-j2\pi\left(-x_1u+\left(y_0+y_1\right)v\right)}$$

The final plane is after the second FT, with the central DC terms proportional to FT [R^2 + S^2] and the two symmetrical correlation peaks spaced by $(x_1,-(y_1+y_0))$ and $(-x_1,(y_1+y_0))$.



The central DC term is an unwanted source of noise, which degrades the optical system. There are always a symmetric pair of correlation peaks, so only half the output plane needs to be considered but the position of the correlation peak has to be decoded to gain the position of the reference object in *r(x,y)* if it correlates with *s(x,y)*.

b) [20%] The JTC works on the basis of a non-linearity working on the spectrum of the input objects to create the product of the two Fourier transforms. This was modelled as a simple square law detector, but this gives undesirable broad peaks as well as significant DC terms. A much better correlation peak is obtained when the degree of non-linearity is increased as high as possible. A simple square root function on the spectrum gives good narrow peaks, but the best performance is when the spectrum is thresholded to a purely binary function. The JTC was originally built using an optically addressed SLM base on a nematic LC, but this was too slow and had a lower degree of non-linearity compared to a FLC layer which gives a purely binary modulation and the thresholding is set by the optical response of the LC layer.

c) [20%] There are two effects at play. The thickness of the LC layer will change the modulation of the LC and therefore will change either the uniformity of the intensity or phaser modulation. The variation in the thickness of the glass used to make the OASLM changes the optical pathlength either side of the LC layer and appears like a extra phase term on top of the meditation of the LC layer. The overall effect of both defects it a bit like apodisation in holograms.

Both effects have a similar effect on the quality of the correlation peaks formed by the JTC. The appear like an optical aberration on the path of the light through the OASLM and therefore will broaden the width of the correlation peaks formed in the final output plane. It is effectively the convolution of the ideal correlation peaks with the Fourier transform of the aberration function r-that has been caused by the errors on device fabrication.

d) [20%] The AO approach is based on the concept of phase conjugation, so to remove the effects of the fabrication error, the inverse of the aberration must be presented to the light after the modulation of the LC layer on the OASLM. There are two possible approaches

1) Given the imperfections in the OASLM could be measured, it would be possible to create a fixed optical element that would have the conjugate phase of the OALSM imperfections. This would work well for the glass faults, but not so well for the LC thickness variation as this would be polarisation dependent and would require a birefringent correction element which is much mor difficult to make.
2) The aberration could be characterised and then the input SLM could be used to display the Fourier transform of the conjugate. This is not as feasible as it might appear as the input SLM would have to able to modulate both the input images (usually intensity) as well as the conjugate Fourier transform (usually phase). This could be possible, but difficult. The approach would still not work as the photodetector in the OALSM will eliminate any phase in the spectrum of the input and therefore nullify the phase conjugation. It may be possible using some form of global optimisation to eliminate the effects of the aberration.

Was very well answered by most as it was the least speculative. Part a) was well answered with a few giving a system diagram rather than an optical layout, most got the derivation. b) some interesting answers, but not many spotted that the FLC was in fact a binary threshold and created a high order non-linearity automatically. c) not so well answered with most realising the impact of the distortions, but not tying it to the shape of the correlation peaks as it is a multiplicative distortion which would give a convolution in the output plane. d) some interesting answers with most using an AO approach. A few suggested encoding the aberration into the hologram, but the input is not a hologram so would be difficult to conjugate.