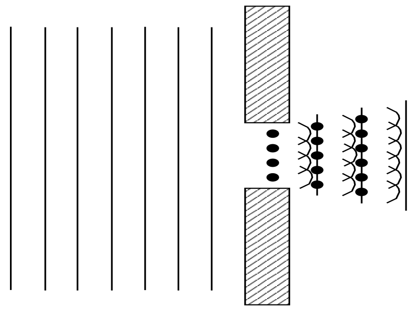
4B11 exam cribs 2025: Q1 a) [25%] If we consider an infinitely small differential area of the aperture, *dS*, we can model this as a point source of light emitting spherical 'Huygens' wavelets with an amplitude of *A(x,y)dS*. The wavelet acts as a radiating point source, so we can calculate its field at the point *P*, a distance *r* from *dS*. The point source *dS* can be considered to radiate a spherical wave front of frequency ω. As soon as the wavelets propagate from the aperture, the edges (where adjacent wavelets are missing) begin to propagate with reduced energy. This changes the summation of the phases and after a few wavelengths of propagation, the plane waves are distorted and wrinkled. This is the near field region.



Assumptions: Plane wave coherent (laser) illumination of the aperture. Only consider forward propagating waves. Aperture is larger than wavelength x2.

b) [40%] In order to understand and analyse the propagating wavelets, a series of approximations and assumptions must be made. If we consider only the part of the wavelets which are propagating in the forward (+*z*) direction and are contained in a cone of small angles away from the z axis, then we can evaluate the change in field *dE* at the point *P*, due to *dS*. As the wavelet *dS* acts as a point source, we can say that the power radiated is proportional to $1/r^2$ (spherical wavefront), hence the field *dE* will be proportional to *1/r*. We can see that for a real propagating wave of frequency ω and wave number *k*, ($k = 2\pi/\lambda$) we have the cosine component of a complex wave. The full complex field radiating from the aperture can be written in terms of exponentials as the cosine is just the real part of the complex exponential.

$$dE = \frac{A(x,y)}{r} e^{jwt} e^{-jkr} dS$$

Now, we need to change coordinates to the plane containing the point *P*, which are defined as [α,β].

$$r = R\sqrt{1 - \frac{2\alpha x + 2\beta y}{R^2} + \frac{x^2 + y^2}{R^2}}$$

The final full expression in terms of *x* and *y* ($dS = dxdy$) for *dE* will now be.

$$dE = \frac{A(x,y)e^{j\omega t} e^{-jkR\sqrt{1-\frac{2\alpha x+2\beta y}{R^2}+\frac{x^2+y^2}{R^2}}}}{R\sqrt{1-\frac{2\alpha x+2\beta y}{R^2}+\frac{x^2+y^2}{R^2}}} dxdy$$

Such an expression can only be solved directly for a few specific aperture functions. To account for an arbitrary aperture, we must approximate, simplify and restrict the regions in which we evaluate the diffracted pattern.

If the point *P* is reasonably coaxial (close to the *z* axis, relative to the distance *R*) and the aperture *A(x,y)* is small compared to the distance *R*, therefore $r = R$. The similar expression in the exponential term in the top line of the original equation is not so simple. It can not be considered constant as small variations are amplified through the exponential. To simplify this section we must consider only the far field or Fraunhofer region where.

$$R^2 >> x^2 + y^2$$

In this case, the final term in the exponential ($(x^2 + y^2)/R^2$) can be considered negligible. To further simplify, we use the binomial expansion,

$$\sqrt{(1-d)} = 1 - \frac{d}{2} - \frac{d^2}{8} \ldots$$

and keep the first two terms only to further simplify the exponential expression.

$$dE = \frac{A(x,y)}{R} e^{j(\omega t - kr)} e^{jk\left(\frac{\alpha x + \beta y}{R}\right)} dxdy$$

The total effect of the dS wavelets can be integrated across *dE* to get an expression for the far field or Fraunhofer diffraction pattern.

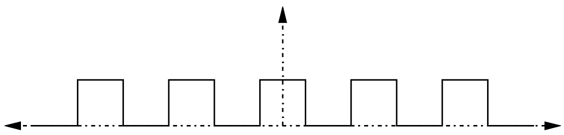
$$E(\alpha,\beta) = \frac{1}{R} e^{j(\omega t - kR)} \iint_{Aperture} A(x,y) e^{jk(\alpha x + \beta y)/R} dxdy$$

The initial exponential term $e^{j(\omega t - kR)}$ refers the wave to an origin at $t = 0$, but we are only interested in the scaling of relative points at P with respect to each other, so it is safe to normalise this term to 1. Thus, our final expression for the far field diffraction pattern becomes:

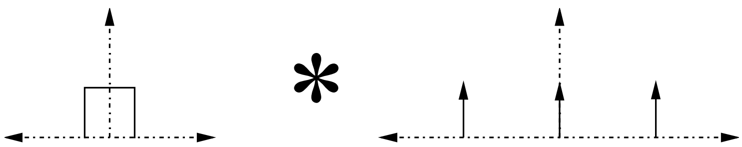
$$E(\alpha,\beta) = \iint_A A(x,y)e^{jk(\alpha x+\beta y)/R}dxdy$$

Hence the far field diffraction pattern at the point P is related to the aperture function $A(x,y)$, by the Fourier transform.

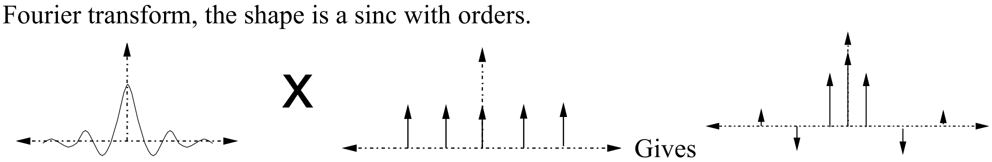
c) [25%] The pixel pitch and shape governs of the hologram defines the envelope function of the replay field and therefore define its overall physical size. The shape of the envelope is related to the shape of the pixel via the FT, hence a square pixel gives a sinc envelope. The pixel pitch also means that the hologram is effectively sampled, hence there will be an ordered harmonic structure via the FT. Ie a regular array of pixels gives a regular array of orders in the replay field.



Which can be expressed as a convolution of two functions, pixel shape and its pitch.



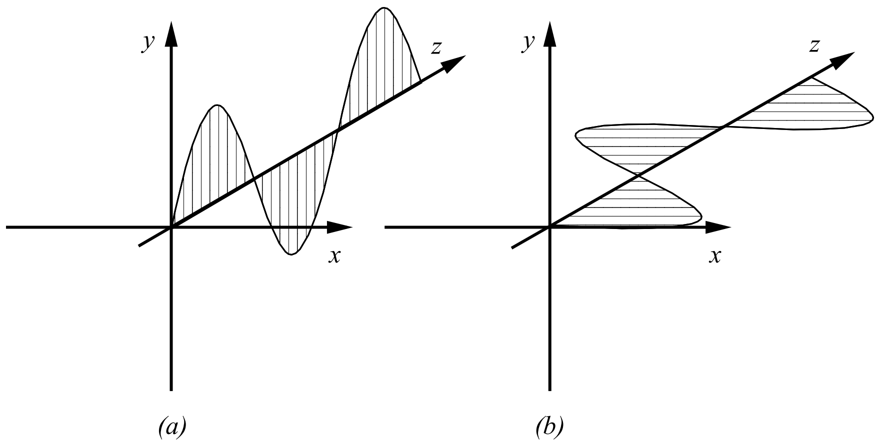
After the Fourier transform, the shape is a sinc with orders.



d) [10%] If the aperture were hexagonal in shape, then the hologram would be made from an array of hexagonally packed apertures. This means the far-field diffraction pattern would depend on the hexagonal shape and packing. The man effect would be the fact that the orders of the far-field would now be on a hexagonal grid as well. The envelope of the first order would still have an overall sinc shape, but it would now be hexagonal rather than square.

Mostly well answers which is not surprising considering it was mostly bookwork. For part (a) a lot of people did not mention plane waves and quite a few got the idea of Huygens wavelets wrong. (b) was better answered with the main problem being missing out the 1/r dependency. (c) was well answered with the derivation (not asked for) of the sinc function and coordinates. (d) was well answered with mots getting the idea of a hexagonal sinc.

Q2 a) [30%] Any light source can be represented in terms of an orthogonal set of propagating eigenwaves which are usually aligned to the x and y axes in a coordinate system with the direction of propagation along the z axis.



Vertically (a) and Horizontally (b) polarised light

If the light is polarised in the direction of the *y* axis, then we have linearly polarised light in the *y* direction of amplitude V_y or <u>vertically</u> polarised light.

$$V = \begin{pmatrix} 0 \\ V_y \end{pmatrix}$$

If we have an electromagnetic wave propagating in the *z* direction along the *x* axis then the light is classified as linearly polarised in the *x* direction or <u>horizontally</u> polarised. This wave can be represented as a Jones matrix, assuming an amplitude V_x .

$$V = \begin{pmatrix} V_x \\ 0 \end{pmatrix}$$

We can now combine these two eigenwaves to make any linear state of polarisation we require.

$$V = \begin{pmatrix} V_x \\ V_y \end{pmatrix}$$

We can also represent more complex states of polarisation such as circular states. We can also introduce a phase difference ϕ between the two eigenwaves which leads to circularly polarised light. In these examples, the phase difference ϕ is positive in the direction of the z axis and is always measured with reference to the vertically polarised eigenwave (parallel to the *y* axis), hence we can write the Jones matrix.

$$V = \begin{pmatrix} V_x \\ V_y e^{j\phi} \end{pmatrix}$$

There are two states, which express circularly polarised light. If ϕ is positive, then the horizontal component leads the vertical and the resultant director appears to rotate to the right around the *z* axis in a clockwise manner and is right circularly polarised light. Conversely, of the horizontal lags the vertical then the rotation is counter clockwise and the light is left circularly polarised. In the case of pure circularly polarised light, $\phi = \pi/2$ for right circular and $\phi = -\pi/2$ for left circular.

$$Circular\ V = V_x \begin{pmatrix} 1 \\ j \end{pmatrix},\ elliptical\ V = \begin{pmatrix} V_x \\ jV_y \end{pmatrix}$$

b) [30%] The three components can be combined using Jones matrices by forming the product of the light input with the quarter waveplate followed by the polarisor. Horizontally polarised light:

$$V = \begin{pmatrix} V_x \\ 0 \end{pmatrix}$$

Vertically aligned polarisor

$$H = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

Using the generalised waveplate matrix

$$W = \begin{pmatrix} e^{-j\Gamma/2} cos^2 \psi + e^{j\Gamma/2} sin^2 \psi & -j\, sin\frac{\Gamma}{2} sin(2\psi) \\ -j\, sin\frac{\Gamma}{2} sin(2\psi) & e^{j\Gamma/2} cos^2 \psi + e^{-j\Gamma/2} sin^2 \psi \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix}$$

The product will be

$$HWV = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} V_x \\ 0 \end{pmatrix}$$
$$= \begin{pmatrix} 0 \\ CV_x \end{pmatrix} = \begin{pmatrix} 0 \\ -jV_x \sin\frac{\Gamma}{2} \sin(2\psi) \end{pmatrix}$$

For the quarter waveplate, $\Gamma = \pi/2$ aligned parallel to the *y* axis, $\Psi = 0$ which means that the final polarisation state will be zero (no light output).

c) [20%] If the quarter waveplate is rotated about the optical axis (z axis), then the parameter Ψ is changing in the expressions for HWV above. For Y = 45 degrees, the resulting polarisation state will be

$$HWV = \begin{pmatrix} 0 \\ -jV_x \sin\frac{\Gamma}{2} \sin(2\psi) \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ -jV_x \end{pmatrix}$$

For a quarter waveplate rotated by $\Psi = 90$ degrees then the polarisation state will again be zero.

d) [20%] An FLC is an in-plane optical effects, so it can modulate the light by rotating the axes of retardation about the optical axis (ie it can vary Ψ into two stable stated separated by the switching angle θ). From the analysis above we can see that the optimum choice of Ψ is 45 degrees, so this will set the value of $\theta = 45$ degrees for the FLC material that we use in to get the best binary intensity modulation. As it is an in-plane effect, the retardation Γ is not affected by the applied electric field. We want $\Gamma = \pi/2$ for a quarter waveplate.

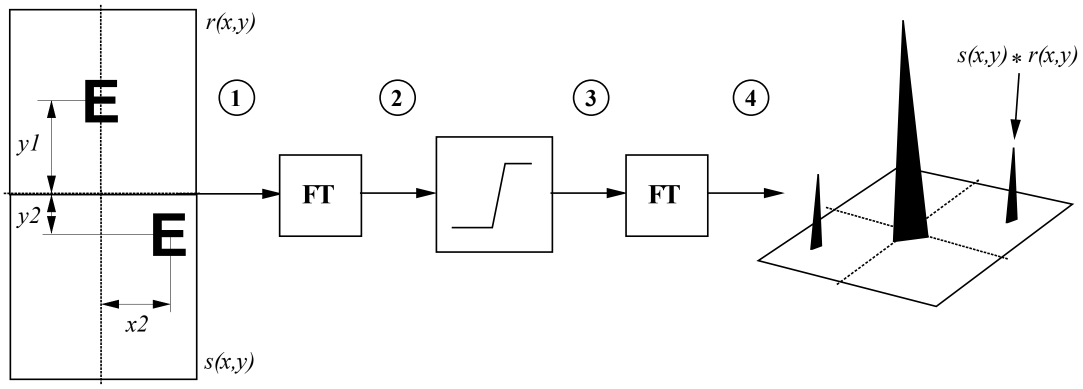
$$\Gamma = \frac{2\pi d(n_e - n_o)}{\lambda}$$

The choice of liquid crystal will dictate both θ and the birefringence of the FLC used in the system, hence the only variable that can then be set to guarantee the value of Γ is the cell thickness *d*.

The main advantage of this scheme is that FLCs are fast modulators, so response times of the order of microseconds could be achieved with the right FLC material and there are several to choose from that would have the right switching angle. The main disadvantage of this scheme is that there is an inherent loss of 30% of the filed which is equivalent to losing half of the light intensity.

Well answered overall, perhaps the question was too easy. Most did well explaining the principles of polarisation and Jones matrices, with a few missing out Vx and Vy as the two of the three parameters. Almost everyone got the matrix calculation right, but a few tried to take shortcuts and were over simplified in their answers. Part (d) has the most challenge, but most got the FLC correct but several used the binary amplitude scheme in the notes using $\Gamma = \pi$.

Q3 a) [30%]



Plane 1 is the input plane with reference and object place side by side. This is usually done optically using a liquid crystal spatial light modulator.
Plane 2 1s where the joint power spectrum (JPS) is formed via a Fourier transform. This is done optically with coherent illumination and a lens.
Plane 3 is the non-linearly processed JPS. This can be a simple square law detector like a CCD or CMOS camera and a second LC SLM. Or it can be an optically addressed SLM (OASLM).

Plane 4 is the output plane formed by another FT lens. This image is usually grabbed by a CCD or CMOS camera.

b) [20%] In plane 1, the input *s(x,y)* and reference *r(x,y)* are displayed side by side in an optical system and then transformed by a single lens into plane 2.

$$S(u,v)e^{-j2\pi(x_2u-y_2v)} + R(u,v)e^{-j2\pi y_1 v}$$

The non-linearity between planes 2 and 3 creates the correlation and in its simplest form can be modelled by a square law detector such as photodiode or CCD camera which takes the magnitude squared of the light falling upon it.

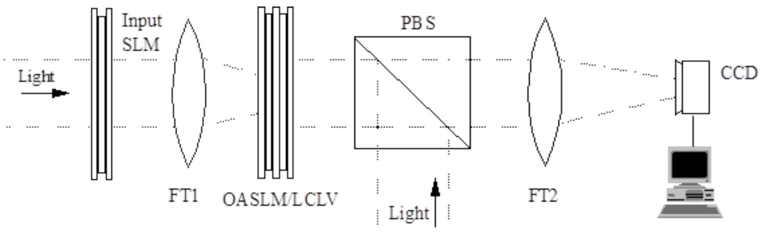
$$S^2(u,v) + R^2(u,v) + S(u,v)R(u,v)e^{-j2\pi(x_2u-(y_1+y_2)v)} + S(u,v)R(u,v)e^{-j2\pi(-x_2u+(y_1+y_2)v)}$$

The final plane 4 is after the second FT, with the central DC [0, 0] terms proportional to FT $[R^2 + S^2]$ and the two symmetrical correlation peaks spaced by $(x_2, -(y_1+y_2))$ and $(-x_2, y_1+y_2)$.

Assumptions – the lenses perform a perfect Fourier transform, there are no aberrations, the SLM is a perfect intensity modulator.

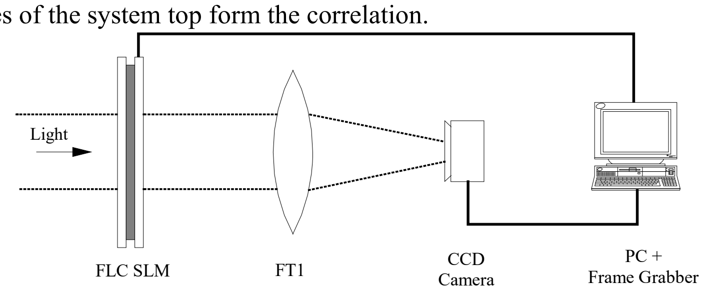
c) [30%] The square-law non-linearity will result in broad correlation peaks and very large DC terms which will dominate the cantal region of the correlation plane. This will make the process of detecting and locating the correlation peaks in the output plane more difficult and inaccurate.

i) Replace the square-law non-linearity with an optically addressed SLM which uses a liquid crystal with a more non-linear response such as a FLC. This will create sharper correlation peaks as well as add a degree of control, to the DC peaks. The use of an OASLM allows the JTC to be tuned to suit the data it is processing to give optimal performance. The use of an OALSM will make the architecture more complicated as it requires separate read and write illumination.



There is also a major problem in fabricating a suitable OASLM as the photoconductive layer is a very unreliable process, making the yield on such devices relatively poor.

ii) Add some electronic non-linear processing to the light captured from a square-law detector such as a CCD. This will create a much better set of correlation peaks and allows a whole range of electronic processes to be implemented to improve the JTC performance. The problem with approach is that a second SLM is required for the final stage of the JTC. The most extreme form of this is to create a 1/f JTC, where the same SLM is used twice in a two pass fashion. This is a very cheap and robust from of JTC architecture, however it is also twice as slow as it requires two passes of the system top form the correlation.

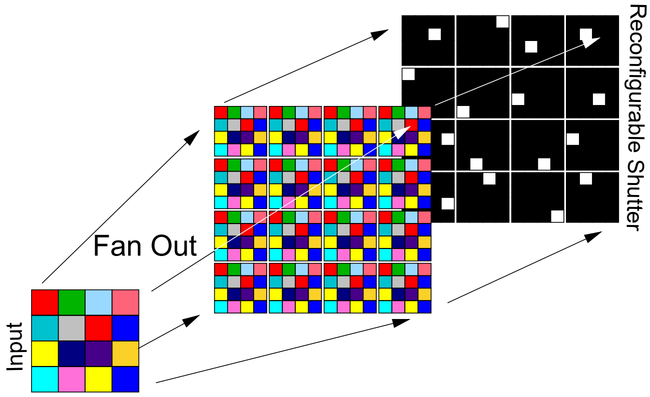


d) [20%] Shift invariance is an inherent property of correlators, however, other forms of invariance must be added by using additional functionality to the reference function *r(x,y)*. techniques. The most common forms of invariance are rotation invariance and scale (size) invariance, which can be included by the following techniques.

• Mathematical transformation: Circular harmonic function – allows rotation invariance, but reduces the information in the reference which will make the correlator les selective. Also requires more complicated modulation from the filter SLM
• Linear summation: Synthetic discriminant function – good invariance properties, but Also requires more complicated modulation from the filter SLM
• Nonlinear optimisation: Simulated annealing, convex sets – works better for the matched filter as the calculation of the reference for the JTC is complicated.

A fairly standard book work question that was answered pretty well by most candidates. a) was the standard introduction, answered well quite a few drew a whole OASLM based JTC which was not required. b) well answered by most with a few random minus signs. Some just quoted the answer without any proof c) well answered as a whole. d) well answered, with most getting the SDF. Quite a few said just run through a library in sequence which is not invariance. Some suggested a rotating input and even some AI.

Q4 a) [35%] The basic idea of optical fan-out is to fan out (or replicate) the input light source array, such that if there are n inputs (in a k x k array, $k^2 = n$) to the switch, then they would be replicated as an array of the input array k x k times. The fanned out inputs are replicated as an k x k array of the input k x k array, all of which are incident on the shadow logic SLM or shutter. This device operates as a shutter array to block or pass the desired light from the inputs.



Optical fan-out can be done by the illumination of a k x k spot CGH or Damman grating to replicate the source inputs. The use of the CGH to replicate the inputs comes from the CGH property that the spots in the replay field are the FT of input illumination. In past examples this has been Gaussian, but it could equally be an input structure or an array of input sources.

b) [25%] The fanned out inputs are replicated as an k x k array of the input k x k array, all of which are incident on the shadow logic SLM or shutter. This device operates as a shutter array to block or pass the desired light from the desired inputs as well as blocking the light from the undesired inputs. For each replicated input, there will be one open shutter in each replication which selects the input source to be routed to a particular output, hence there have to be k^2 x k^2 shutters on the SLM. The position of the shutter selects the input source and the replication position selects the output to be routed to.

The problem with this type of switch arise, when the characteristics of the interconnect are evaluated. It is obvious that the switch is lossy as it is based on the idea of blocking light. For the 1 to n switch (assuming ideal operating conditions), the output power P_{out} will be.

$$P_{out} = \frac{P_{in}}{n}$$

The loss becomes intolerably high for a large value of n. The second parameter to consider is the crosstalk through the switch. If the SLM operates as an ideal shutter, there will be no crosstalk between the outputs. However, such SLMs do not exist, so we must consider the effects of a finite contrast ratio, B on the possible crosstalk. If the output power P_{out} is as given above (hence the shutter has a transmission of 1 when open), then the closed shutters will each have a transmission of $1/B$. Hence the crosstalk will be.

$$C = B$$

The shutter could be implemented using a liquid crystal; material. The best choice would be an FLC based device as it has the fastest response time and is not limited by being binary. The LC response time effectively sets the overall switching speed of the optical switch. There are other LC effects that are faster which could also be used such as Smectic A or blue-phase.

c) [25%] In order for a fibre based switch to function, the switch must cause minimal disruption to the light as it passes through the switch. In order for the light to be launched into the output SM fibres, it much match the mode of the light inside the SM fibre as much as possible. Any aberration in the system will distort this launch condition and lose light and add crosstalk. As the number of fibres increase, the size of the replicated array also increase also with the size of the shutter array. The larger the area of these arrays, the wider the angles that the light has to pass through and the larger the aberrations will be. Hence this will be a limiting factor on the number of fibres that the switch can send light through. The two main limiting components are the lens that is used to perform the fan-out process and the quality of the SLM shutter array both in terns of flatness and cell gap uniformity.

There are two ways that these aberrations could be eliminated. Both will require a detailed analysis of the optical system in order to evaluate the aberrations. This could be done with ray tracing software such as Zemax as well as interferometric analysis of the flatness of the devices such as the shutter SLM. Once the aberrations are known, two techniques can be used.

1) add extra optical surfaces to the lenses to compensate for the aberrations in the system. This is the classical approach, however it adds more complexity to the optical systema and requires good quality alignment.

2) add the aberrations to the fan-out CGH design algorithm so that the hologram itself contains the conjugation of the aberrations and will cancel them out. This requires no further components and is very easy to implement.

Both techniques are limited in that the aberrations are fixed once corrected, so any other aberrations such as those due to misalignment will not be compensated for.

d) [15%] In the traditional n input by n output shuttering process, only one shutter is opened for each replication of the input array of fibres. This can be modified by opening more that one shutter per replication. This means that multiple copies of the inputs can be sent to different outputs in different combinations.

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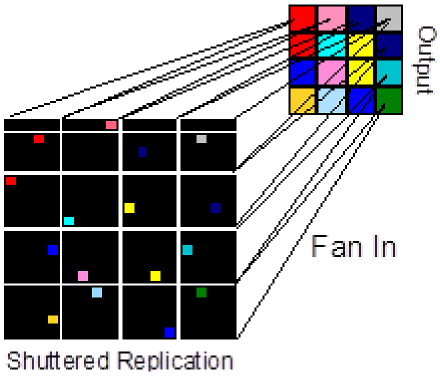

The problem with this process is that the output array is formed via the fan-in optics which only send one replicated array per output channel. In this system multiple outputs can be switched to by opening the same shutter is two different replications. If the output array contains more that n fibres, then the fan-in process could be adapted to accept multiple combinations of inputs, forms a limited broadcast capability.

This would require a sophisticated control algorithm as well as possible buffering to avoid the multiple switched inputs forming unwanted crosstalk.

This question was not well answered overall candidates making the wrong assumptions or just writing parts of the notes they thought were relevant. (a) most got some sort of fan-in arrangement ok and the use of a CGH. (b) Only a few realised the key property of a shutter is its contrast ratio as this sets both loss and crosstalk. A few derived the holographic switch equation. Most got the use of an FLC for speed. (c) was not well answered with most just qi=or=ting various parts of the AO notes such as Shack-Hartmann and closed loop AO. Only a couple got the fact that the fan-out CGH could be used to compensate for fixed aberrations. (d) Only one realised that the system only need to change the shutters to increate the number of outputs addressed.