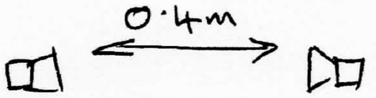
4B13 2019

1(a)



$$t_e = \frac{0.4}{340} = 1.18\ \text{ms}, \quad t_m = \frac{0.4}{240} = 1.667\ \text{ms}$$

$$\therefore\ \Delta t = 0.49\ \text{ms}$$

(b) $Z_e = \rho v = 408$ Ryls, $Z_m = 21.6$ Ryls

On Earth: $$P_t = \underbrace{\frac{V^2/2}{R}}_{0.0333} \cdot \underbrace{\frac{4 Z_t Z_e}{(Z_t+Z_e)^2}}_{0.895} = 0.0298\ \text{W}$$

Received pwr @ transducer $= \underset{0.2}{\eta} \cdot \left[\frac{\pi d^2}{4} \Big/ \pi\,(0.4 \tan 10^\circ)^2 \right] \cdot P_t \cdot \underbrace{10^{\frac{-0.4\times 2.5}{10}}}_{0.794}$ (transducer ϕ)

$$P_r = 5.37 \times 10^{-5}\ \text{W}$$

Power coupled to transducer and converted to electrical signal

$$P_e = P_r \cdot \frac{4 Z_t Z_e}{(Z_t+Z_e)^2} \cdot \eta = 9.61 \times 10^{-6}\ \text{W} = \frac{V_r^2}{R}$$

with $R = 1500\,\Omega$, $V_r = 0.120\ V_{rms} = 0.170$ V ampd loaded

$\times 2$ for open-cct $=$ 0·34 V amplitude

On Mars: $$\frac{4 Z_t Z_m}{(Z_t + Z_m)^2} = 0.102 \quad \therefore \text{ on Mars of Earth}$$

power in sig. $= \times\,(0.102/0.895) \cdot 10^{\frac{0.4}{10}(2.5-2.1)}$ $\times\ 0.118$ $=$ so voltage

ampd $= \times \sqrt{0.118} = \times 0.344 \Rightarrow$ 0·117 V amplitude

(c) On Mars, 10 m/s @ 45° $\Rightarrow$ 7·07 m/s along axis

Zero windspeed phase shift, $\phi = 0.4 \cdot \frac{150\times 10^3}{240} \cdot 360^\circ = 9\times 10^4\,^\circ$

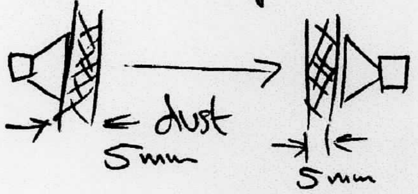
$\therefore$ with $\pm$ 7·07 m/s $\quad \Delta\phi = \pm \frac{7.07}{240} \cdot 9\times 10^4 = \pm 2650^\circ$

1 (d) $-63°C \pm 80°C \equiv 210K \pm 80K$ as the speed of sound $\propto \sqrt{T}$ then v changes by approx. $\pm 20\%$

Hence Z changes by $\frac{1}{T}.\sqrt{T} = \frac{1}{\sqrt{T}}$ $T\uparrow, v\uparrow$
(ρv) $T\uparrow, Z\downarrow$

So, the measured windspeed will be less than actual with increasing temperature by $\sim 20\%$
The coupled powers due to changes in Z_m change by 20% each coupling hence as there are 2, the received power changes by 40% and so the signal voltages by 20%.
Higher temperature $\rightarrow$ lower signals.

(e)



$Z_d = 120 \times 500 = 6\times10^4$ Rayls

Transit time $= \frac{0.39}{240} + \frac{0.01}{500} = 1.645\text{ ms}$

(vs. 1.667 ms)

Hence windspeed will be undermeasured by 1.3%

Extra coupling losses thro' dust layers $Z_t \rightarrow Z_d$, $Z_d \rightarrow Z_m$

$Z_t \rightarrow Z_d$: $\frac{4 \cdot 800 \cdot 6\times10^4}{(800 + 6\times10^4)^2} = 0.0519$

$Z_d \rightarrow Z_m$: $\frac{4 \cdot 21.6 \cdot 6\times10^4}{(21.6 + 6\times10^4)^2} = 1.44\times10^{-3}$

Attenuation thro' dust $= 66 \times 0.01 = 0.66\text{ dB}$
$\Rightarrow \times 10^{-\frac{0.66}{10}} = \times 0.859$

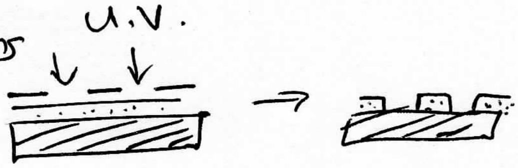
$\therefore$ power coupling loss $= 0.0519^2.(1.44\times10^{-3})^2.0.859$
$= \times 4.80\times10^{-9}$! (a lot)

So signal amplitude $\times\sqrt{4.8\times10^{-9}} = \times 6.93\times10^{-5}$

Hence $0.117\text{ V} \rightarrow 8.1\ \mu\text{V}$ (still measurable though).

2(a) MEMs processes:

- photolithography to define patterns
- expose spun-on photoresist to u.v. through mask



- exposed areas dissolve away in NaOH developer
- then etch underlayer/substrate: plasma or KOH
- hinge formed by boron doping, slowing down hot KOH etch. silicon xtal plane etches @ 35° to give tapered edges.
- glass etched with HF solution to form recess
- electrode patterned in evaporated or sputtered Al layer, etched with acidic soln.
- glass and silicon bonded with pressure + temperature to fusion bond @ 500°C

(b) (i) zero def: $C = \frac{A\epsilon}{d} = \frac{10^{-6} \cdot 8.854 \times 10^{-12}}{100 \times 10^{-6}} = \underline{0.088\ pF}$

5° of tilt: mid-point deflection $= \delta$ 0.5mm 5°

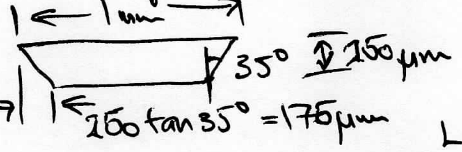
$= 0.0437$ mm

$\therefore$ average gap, d is now $(0.1 - 0.0437) = 0.056$ mm

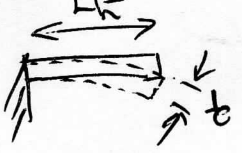
$\therefore C = \frac{0.1}{0.056} \times 0.088 = \underline{0.156\ pF}$ (assuming fringe fields compensate for edge angles)

(ii) Mass of mirror: $\Rightarrow 2.5(1-0.175)^2 . 10^{-6} . 10^{-7} . 2330$ kg

$= 3.96 \times 10^{-10}$ kg



Flexure hinge:



$w = 10^{-3}$ m

$L_h = 50 \times 10^{-6}$ m

$t = 2.5 \times 10^{-6}$ m

Hinge beam Deflection, $\delta_b = \frac{F L_h^3}{3EI} + \frac{M L_h^2}{2EI}$

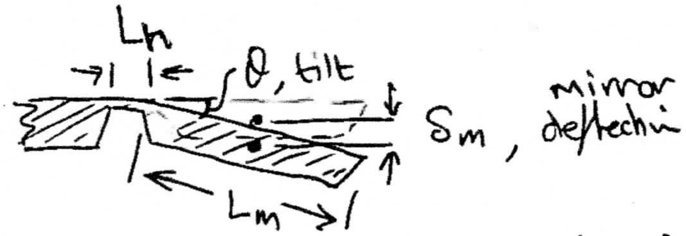
$I = \frac{1}{12} w t^3 = 1.30 \times 10^{-21}\ m^4$

$E = 110 \times 10^9\ Nm^{-2}$

Assume $M = F \times \underbrace{0.5 \times 10^{-3}}_{L_m/2}$ (centre of force is ½ way along mirror)

F = force on mirror

2(b)(ii)



Mirror deflection, $\delta_m = \delta_b + \frac{L_m}{2}\theta$

$$\theta = \frac{F L_h^2}{2EI} + \frac{F \frac{L_m}{2} L_h}{EI}$$

$$\therefore \delta_m = \frac{F L_h^3}{3EI} + F\frac{L_m}{2}\frac{L_h^2}{2EI} + \frac{F L_h^2 L_m}{2EI\,2} + \frac{F L_m^2 L_h}{2EI\,2}$$

$$\delta_m = \frac{F L_h}{EI}\left(\frac{L_h^2}{3} + \frac{L_m L_h}{4} + \frac{L_h L_m}{4} + \frac{L_m^2}{4}\right)$$

dominant term (90%)

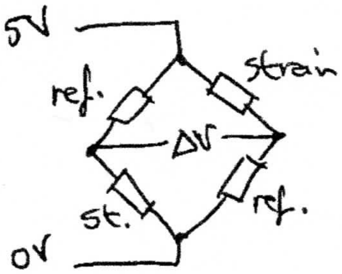
with $L_h = 50\times10^{-6}$ m, $L_m = 10^{-3}$ m, $E = 110\times10^{9}$ Nm^{-2}, $I = 1.3\times10^{-21}$ m^4

$\delta_m = F\,.\,0.096$ hence spring constant $k = \frac{1}{0.096} = 10.4$ N/m

$\therefore$ resonant freq. $= \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{10.4}{3.96\times10^{-7}}} = \underline{816 \text{ Hz}}$

$\therefore$ settling time $\simeq Q/f_{res} = \underline{49 \text{ ms}}$

(c)(i)



$$\Delta V = \frac{2}{4}\times\epsilon\times G.f.\times V_s \qquad g.f. = 35,\; V_s = 5V$$

$$\frac{\sigma}{y} = \frac{M}{I} = \frac{\epsilon E}{y} \qquad E = 110\times10^{9}\text{ Nm}^{-2},\; y = 1.25\times10^{-6}\text{ m},\; L_h = 50\times10^{-6}\text{ m}$$

@ max. deflection of 5°, $\theta = \frac{M L_h}{EI} = \frac{5\pi}{180}$

$\therefore M = \frac{5\pi}{180}\cdot\frac{EI}{L_h} = \frac{\epsilon E I}{y} \quad \therefore \epsilon = \frac{5\pi}{180}\frac{y}{L_h} = 2.18\times10^{-3}$

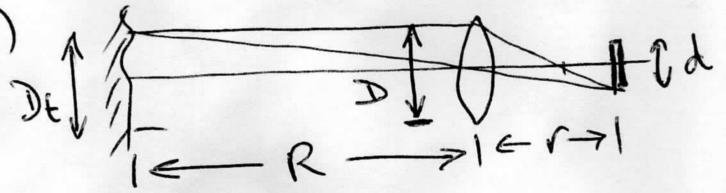
$\therefore \Delta V = 5\times35/2\,\epsilon = 87.5\,\epsilon = \underline{0.191\text{ V}}$

Noise voltage $v_n = \sqrt{4kTRB}$ with $R = 10^4\,\Omega$, $B = \frac{816}{40}$ Hz

$v_n = 0.058\ \mu V_{rms}$ for S/N of say 10:1 then min. deflection resolution $= 5°\times\frac{0.058\times10^{-6}\times10}{0.191} = \underline{1.5\times10^{-5}\text{ degrees}}$

(ii) with capacitive drive, the tilt is non-linear with voltage as force is prop. to V^2/x^2 so strain gauges can provide linear feedback signal – also to improve dynamic response + damping.

3(a)



Stephan Law

$W = \epsilon \sigma_{SB} T^4$ $\quad \epsilon = 0.85$

$T = 100 - 400\ K$

Lambert's Law, $\delta W = \frac{W}{\pi} \cos\theta\, A\, \delta\omega = \frac{\omega}{\pi} \frac{\pi D_t^2}{4} \frac{\frac{\pi D^2}{4} 4\pi}{4\pi R^2}$

From similar Δ's $\frac{D_t}{R} = \frac{d}{r}$ $\therefore \delta W = \frac{\epsilon \sigma_{SB} T^4}{\cancel{\pi}} \frac{\cancel{\pi} d^2}{4 r^2} \frac{\pi D^2}{4}$

Power on detector, $\delta W = \epsilon \sigma_{SB} T^4 \frac{\pi}{16} \frac{d^2 D^2}{r^2}$ $= 6.73 \times 10^{-6}\ W$ @ $T = 100K$ $= 1.72 \times 10^{-3}\ W$ @ $T = 400K$

$\epsilon = 0.85$, $\sigma_{SB} = 5.67 \times 10^{-8}$, $d = 4 \times 10^{-3}$, $D = 50 \times 10^{-3}$, $r = 75 \times 10^{-3}$

$\Delta i_{ph} =$

$\therefore$ Change in power $\leq 1.72 \times 10^{-3}\ W \times$ photodiode A/W $\times$ photodiode response

with Q.E. = 70% and $\lambda = 10\mu m$, $E_{phton} = \frac{hc}{\lambda} = 1.99 \times 10^{-20}\ J$

$\therefore A/W = \frac{1.6 \times 10^{-19} . 0.7}{1.99 \times 10^{-20}} = 5.63 \Rightarrow \Delta i_{ph} = 9.68 \times 10^{-3}\ A$

(b)(i) $R_T = A e^{\beta'/T}$ $\therefore \frac{dR_T}{dT} = \frac{-\beta'}{T^2} A e^{\beta'/T} = \frac{-\beta'}{T^2} R_T$

If $R = 100$ @ $T = 298K$ and $\beta' = 4200$:

$100 = A e^{4200/298}$ $\therefore A = 7.57 \times 10^{-5}$ and $R_{-130°C,\ 143K} = 431\ M\Omega$

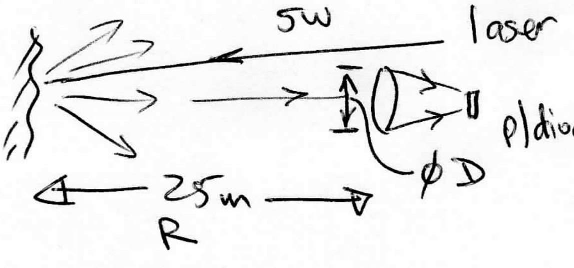
(ii) Hence $\frac{dR}{dT}$ @ $283K$ (10°C) $= -\frac{4200}{283^2} \cdot 7.57 \times 10^{-5} e^{\frac{4200}{283}} = -11.1\ \Omega/K$

Actual Resistance @ 10°C = $211\ \Omega$ $\qquad$ Rlin

—— " —— @ 20°C = $127\ \Omega$ whereas $211 - 11.1 \times 10 = 100\ \Omega$

$\therefore$ Non-linearity $= \frac{R_{actual} - R_{lin}}{\text{Linear change in } R} \times 100\% = \frac{127 - 100}{111} \times 100 = 24\%$ non-linearity.

(c)



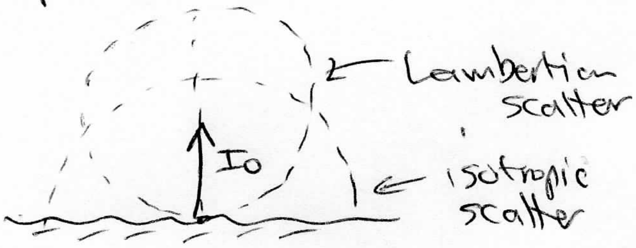
Fraction of scattered light collected $= \frac{\frac{\pi D^2}{4}}{\frac{4\pi R^2}{2}} = 5 \times 10^{-7}$

3 (c) cont'd.

Scattered light $= 35\% \times 5W = 1.75W$

$\therefore$ photocurrent $= 0.8 \times 1.75 \times 5\times10^{-7} = \underline{7\times10^{-7}\ A}$

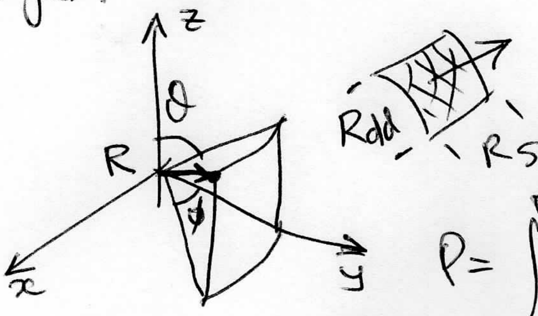
(d)



$I = I_{0i}\cos\theta$

$I = I_{0i}$

For same total emitted power, integrate $I_0\cos\theta$ over hemisphere to determine total power back-scattered over all angles.



$$P = \int_0^{\pi/2}\int_0^{2\pi} I_0 \cos\theta\, R^2 \sin\theta\, d\phi\, d\theta$$

$$\therefore P = I_0 R^2 2\pi \int_0^{\pi/2} \sin\theta\cos\theta\, d\theta = I_0 R^2 2\pi \int_0^{\pi/2} \tfrac{1}{2}\sin 2\theta\, d\theta$$

$$\therefore P = I_0 R^2 2\pi \left[-\tfrac{1}{2}\frac{\cos 2\theta}{2}\right]_0^{\pi/2} = I_0 \pi R^2$$

So, at 45° then Lambertian intensity $= \dfrac{P\cos 45^\circ}{\pi R^2} = \dfrac{1/\sqrt{2}\, P}{\pi R^2} = I_L$

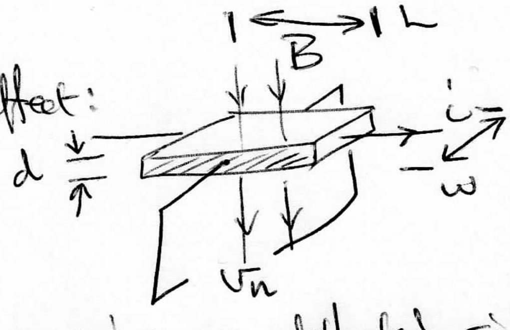
whereas in the isotropic case $I_i = \dfrac{P}{2\pi R^2}$ $\therefore \underline{\dfrac{I_L}{I_i} = \sqrt{2}}$

(signal is larger in Lambertian case)

(e) To ensure immunity to ambient light:

- use a narrow-band optical filter on the photo-diode, matched to pass the laser light only
- modulate the laser and use synchronous detection so that the signal only responded to the modulation freq.
- shade the photo-diode field of view to avoid ambient light being seen eg: avoid direct view of the sun (reduces shot noise)

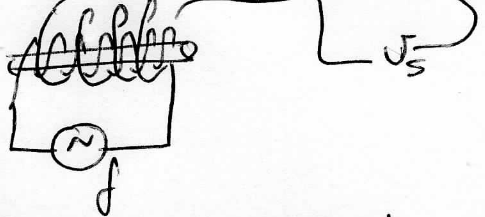
4(a) Hall effect:



semi-conductor slice through which current passes. Orthogonal magnetic field deflects moving carriers.

Current carriers are deflected sideways by Lorentz force $f = Bev_d$ when moving at drift velocity v_d. The carriers accumulate on edge faces to build up transverse electric field which balances Lorentz force in equilibrium state so $Ee = Bev_d$, $E = \frac{V_{hall}}{w}$, $v_d = \mu \frac{V_s}{L}$

Fluxgate:



One (or two) cores are driven into saturation by coil with a.c. excitation.
This periodically saturates the core in +ve and -ve direction - if an external field also couples into the core, then this introduces an offset in the timings of +ve and -ve saturation sweeps. These transitions effectively gate the external field into/out of the core as the permeability is switched high/low. A pick-up coil around the core is induced with a $2f$ signal proportional to the applied external field. Using 2 cores inside the pick-up coil, driven in anti-phase, cancels direct coupling of f drive - hence a clearer $2f$ signal is seen. The $2f$ is detected and demodulated by a synchronous detection circuit set to $2f$, with phase shift to maximise signal.

(b) Cores ϕ 1mm, $L = 40$mm $\quad D = \frac{[\ln 80 - 1]}{40^2} = \frac{1}{473}$

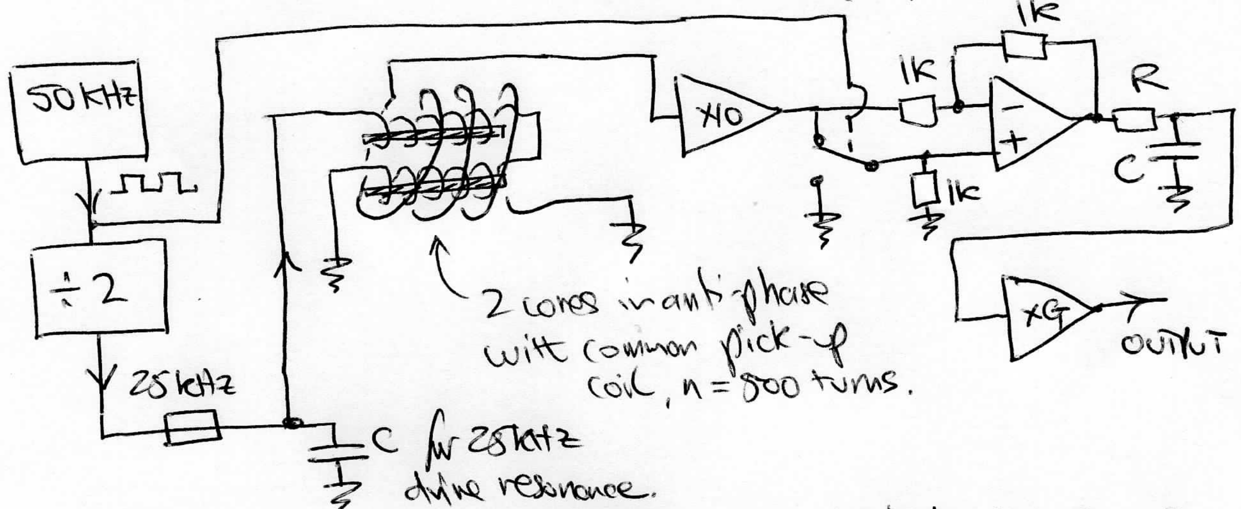
$\therefore$ internal flux density $B = B_{ext} \times 473 = 0.71$ mT

Induced 2nd harmonic voltage $V = V_2 \sin 4\pi f t$, so $\int_0^{1/4f} V\,dt = BAn$

$\therefore \int V\,dt = BAn = \left[\frac{V_2 \cos 4\pi f t}{4\pi f}\right]_0^{1/4f} = \frac{V_2}{2\pi f}$ so signal ampl^{d}

$V_2 = 2\pi n f BA$, with $n = 500$, $f = 25 \times 10^3$ Hz, $A = \frac{\pi 10^{-6}}{4}$

4(b) contd. $V_2 = 0{\cdot}0438$ V @ 50kHz (amplitude with 2-cores)



The RC will average the rectified v_2 amplitude to $\frac{2}{\pi} \times v_2$

Hence final gain $G = \dfrac{1}{0{\cdot}0438 \times 10 \times 2/\pi} = \underline{3{\cdot}59}$

Choose R & C for 100Hz B/W eg. $100 = \dfrac{1}{2\pi RC}$ $\underline{R = 10k\Omega \quad C = 16\mu F}$

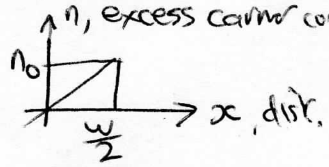
(c)(i) with cores end-to-end $\frac{L}{d} = 80$, $D = \frac{1}{1570}$ $\therefore$ flux density is concentrated by $\underline{\times 1570 \Rightarrow 2{\cdot}36 \text{ mT}}$

(ii) $B \phi v_d = \phi \dfrac{v_h}{w} = \dfrac{B \mu V_s}{L}$ $\therefore \dfrac{v_h}{B} = \dfrac{w \mu V_s}{L} = \underline{0{\cdot}8 \text{ V/T}}$ (μ: 0·16, V_s: 5)

$\therefore V_h = 2{\cdot}36 \times 0{\cdot}8 \times 10^{-3}$ V $= \underline{1{\cdot}89 \text{ mV}}$

(iii) Hall sensor resistance $= \dfrac{\rho L}{w d} = 1000\ \Omega$

Assuming liner excess carrier conc. $n = \dfrac{2 n_0 x}{w}$



Diffusion flux across centre-line $F = -D \dfrac{dn}{dx}$, $D = \mu kT/q$

where $\dfrac{dn}{dx} = \dfrac{2n_0}{w}$ $\therefore$ total carriers diffusing back $\dfrac{dN}{dt} = -D \dfrac{2n_0}{w} Ld$, $n_0 = \dfrac{4N}{wLd}$

as total $N = \displaystyle\int_0^{w/2} Ld \frac{2n_0}{w} x\, dx = \frac{n_0 w L d}{4}$; $\dfrac{dN}{dt} = -\dfrac{8DN}{w^2}$ 1st order D.E.

Soln of form $N = N_0 e^{-t/\tau}$ with $\tau = w^2/8D$

$\therefore f_{-3dB} \simeq \dfrac{1}{2\pi\tau} = \dfrac{4D}{\pi w^2} = \dfrac{4\mu kT}{\pi q w^2} = \underline{58{\cdot}6 \text{ kHz}}$

and thermal noise voltage, $v_n = \sqrt{4kTRB} = \underline{0{\cdot}99\ \mu V_{rms}}$