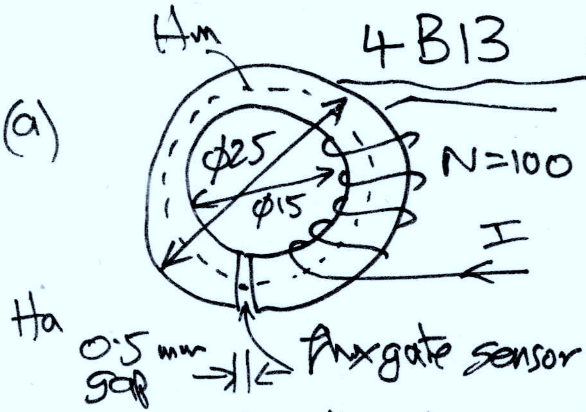
4B13 CRIB 2024

1(a)



$NI = \oint H \cdot dl$

$NI = 20\times10^{-3}.\pi.H_m + 0.5\times10^{-3} H_a$

$B_m = B_a \therefore \mu_0\mu_r H_m = \mu_0 H_a$

flux conservation

Avg. magnetic diameter = 20 mm, $\mu_r = 5000$

$$\therefore NI = \frac{\pi.20\times10^{-3}}{\mu_r} H_a + 0.5\times10^{-3} H_a = H_a . 5.13\times10^{-4}$$

$$B_a = \mu_0 H_a = \frac{NI . 4\pi\times10^{-7}}{5.13\times10^{-4}} = 2.45\times10^{-3} NI$$

for $N=1$, $I=5A$ $\quad \underline{B = 12.2\,mT}$ $\quad \therefore I_{sat} = \frac{500}{11.8}.5 = \underline{205A}$

$$L = \frac{N\phi}{I} = \frac{NBA}{I} = 100^2 . 2.45\times10^{-3} . 40\times10^{-6}$$

$$\underline{L = 0.98\ mH}$$

(b) $V_{ind} = N\frac{d\phi}{dt} = NA\frac{dB}{dt} = NA\,2\pi f B$ for 5A, 1kHz:

$$V_i = 100 . 40\times10^{-6} . 2\pi.10^3 . 12.2\times10^{-3} = \underline{0.307V}$$

(c) with 1Ω connected across coil, Vi induces nulling current in coil as current transformer $\therefore V = \frac{5A}{\underset{N\rightarrow}{100}}.1\Omega = \underline{0.05V}$

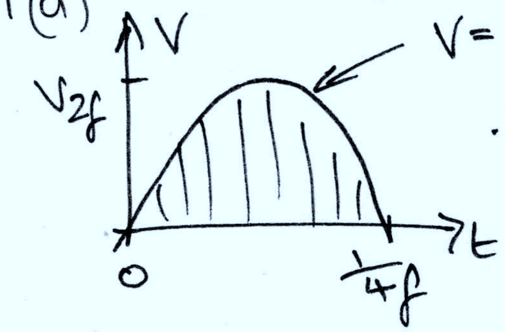
Min. frequency is determined by L/R time constant:

$f_{-3dB} = \frac{1}{2\pi L/R} = \underline{162\ Hz}$ (below this freq., the induced voltage is too low to produce a flux cancelling current large enough to null the flux produced from the central conductor I)

(d) Twin fluxgate cores give full 2f sine-wave, where $V_{2f} = n\,d\phi/dt \quad \therefore \int V_{2f}\,dt = nBA$

1(d) contd. $D_{cores} = (d/l)^2[\ln(2l/d) - 1] = 7.97\times10^{-3} = \frac{1}{125}$

$V = V_{2f} \sin 4\pi f t$



$$\therefore nBA = \left[\frac{-V_{2f}\cos 4\pi f t}{4\pi f}\right]_0^{1/4f} = \frac{V_{2f}}{2\pi f}$$

$$\therefore \underline{V_{2f} = 2\pi n f A B}$$

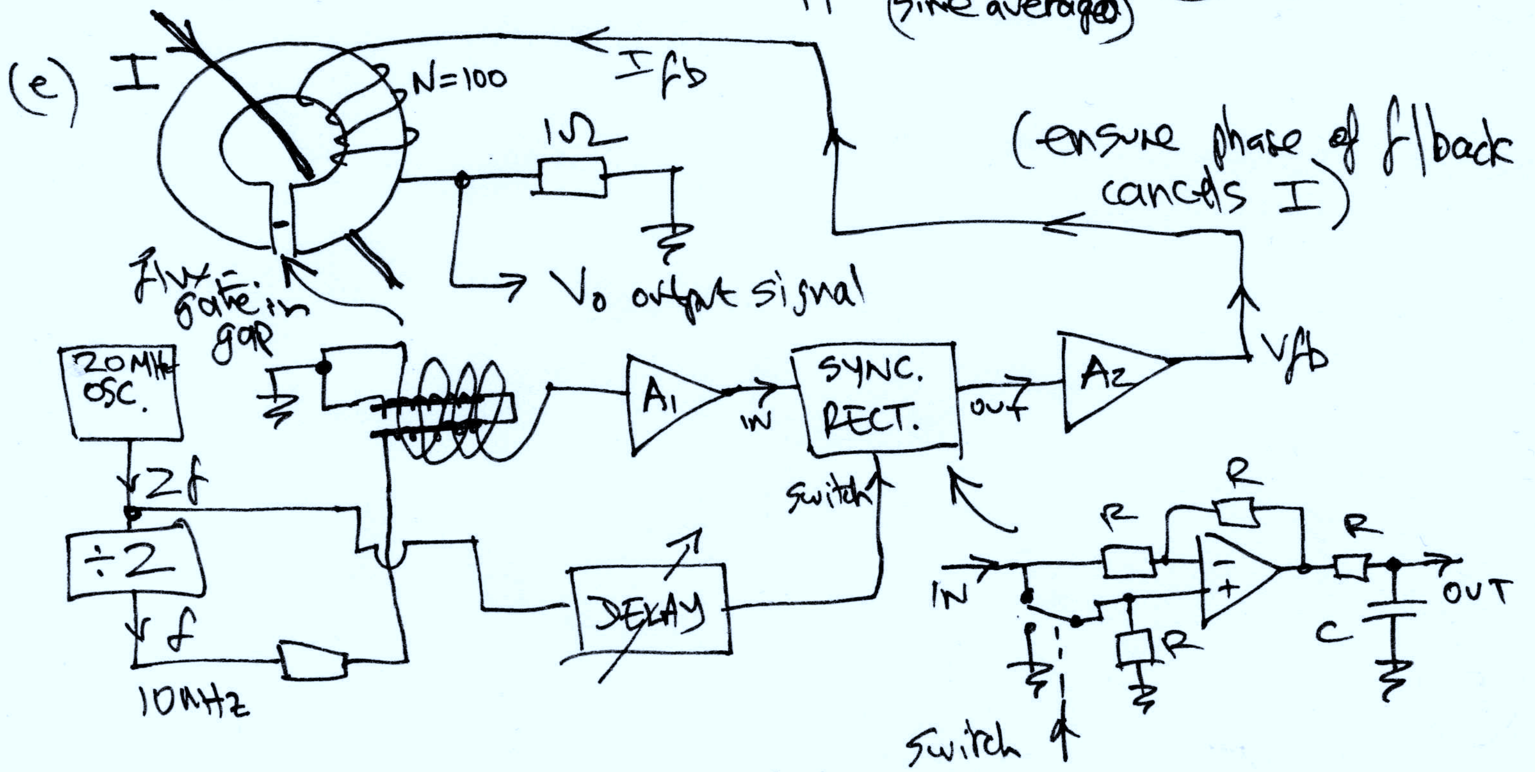
Here $n = 200$, $f = 10\times10^{6}$, $A = \frac{\pi(25\times10^{-6})^2}{4}$, $B = 12.2\times10^{-3}\times125$

Responsivity $= 2\pi n f A$ $= \underline{6.17 \text{ V/T}_{\text{(in core)}}} \Rightarrow \underline{771 \text{ V/T ext.}}$

for 5A current:

$V_{2f} = \underline{9.41 \text{ V}}$ @ sensor $\rightarrow \times\frac{2}{\pi}$ demodulated (sine averaged) $= \underline{5.99 \text{ V}}$

(e)



Advantages:
- response down to d.c.
- good linearity
- high current range (delayed satn)

(f) With $I = 5A$ and $N = 100$ feedback turns with 1Ω resistance

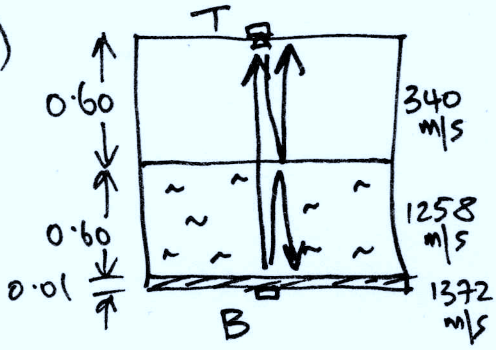
$V_{fb} = 0.05\text{V}$ from flux density @ 5A $\times 0.1\% = 12.2\times10^{-6}\times125$ T

$\times$ Flgate responsivity $= 6.17$ V/T $\times \frac{2}{\pi}$ demod. factor $= 5.99$ mV

$$\therefore \text{total gain reqd.} = \frac{0.05}{5.99\times10^{-3}} = \underline{\times 8.35}$$

[eg: $A_1 = A_2 = 2.9$ in pt.(e)]

2 (a)



Transit times:

$T \to B$, $B \to T$: $\frac{0{\cdot}01}{1372} + \frac{0{\cdot}60}{1258} + \frac{0{\cdot}60}{340} = \underline{2{\cdot}25\text{ ms}}$

$B \to B$: $\frac{0{\cdot}01}{1372} + \frac{1{\cdot}2}{1258} = \underline{0{\cdot}95\text{ ms}}$

$T \to T$: $\frac{1{\cdot}2}{340} = \underline{3{\cdot}53\text{ ms}}$

(b) $P_R = \frac{(Z_1 - Z_2)^2}{(Z_1+Z_2)^2}$, $P_T = \frac{4Z_1Z_2}{(Z_1+Z_2)^2}$, $Z = \rho v$

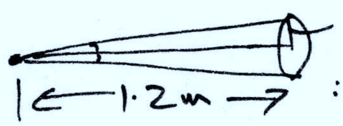
$Z_{air} = 415$, $Z_{PZT} = 30M$, $Z_{PTFE} = 3{\cdot}02M$, $Z_{acid} = 2{\cdot}31M$, $Z_{foam} = 24k$

$\therefore$ interfaces: PZT → PTFE   PTFE → ACID   ACID → FOAM   ACID → AIR   PZT → AIR

$P_T \Rightarrow$ 0·332   0·98   0·041 $7{\cdot}18 \times 10^{-4}$ $\frac{750}{}$ 0·041

also $P_R \Rightarrow$ — — — — → 0·959   ~1

Area of beam spread with ±5°



$r = 0{\cdot}105\text{ m}$

$\therefore A = 0{\cdot}0346\text{ m}^2$

Area of transducer $= \frac{\pi d^2}{4} = 7{\cdot}854 \times 10^{-5}\text{ m}^2$

ratio $= 1/441$

Attenuations: 0·6m ACID $= 10^{-\frac{0.6 \cdot 0.01}{10}} = \times 0{\cdot}9986$,

0·6m air $= 10^{-\frac{0.6 \cdot 1.2}{10}} = \times 0{\cdot}847$

$\underline{B \to T}$

$$Pr = \frac{v^2}{200} = \underset{P}{\frac{25^2}{200}} \cdot \underset{\eta^2}{0{\cdot}15^2} \cdot \underset{PZT \to PTFE}{0{\cdot}332} \cdot \underset{PTFE \to ACID}{0{\cdot}98} \cdot \underset{\text{acid atten.}}{0{\cdot}9986} \cdot \underset{\text{air atten.}}{0{\cdot}847} \cdot \underset{ACID \to AIR}{7{\cdot}18 \times 10^{-4}} \cdot \underset{AIR \to PZT\ (750)}{0{\cdot}041} \cdot \underset{\text{area ratio}}{\frac{1}{441}}$$

$\therefore v^2/200 = 1{\cdot}29 \times 10^{-9}$ $\therefore v = 50{\cdot}8$ μV loaded → ×2 o/cct.

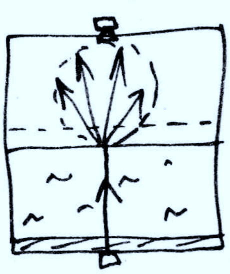
$\Rightarrow \underline{V_r = 1{\cdot}02\text{ mV}}$

$\underline{B \to B}$

$$Pr = \frac{v^2}{200} = \frac{25^2}{200} \cdot 0{\cdot}15^2 \cdot 0{\cdot}332^2 \cdot 0{\cdot}98^2 \cdot 0{\cdot}9986^2 \cdot \frac{1}{441} = 1{\cdot}68 \times 10^{-5}$$

$\therefore v = 58$ mV loaded or $\underline{V_r = 116\text{ mV}}$ o/c cct.

(c) P_T foam → air $= 0{\cdot}0668$



Scatter area ratio $= \times \frac{7{\cdot}854 \times 10^{-5}}{\pi\, 0{\cdot}6^2} = 6{\cdot}94 \times 10^{-5}$

(Lambertian intensity = 2× isotropic hemisphere)

Foam attenuation $= 10^{-\frac{0.05 \cdot 85}{10}} = \times 0{\cdot}376$

Air attenuation $= 10^{-\frac{0.55 \cdot 1.2}{10}} = \times 0{\cdot}859$

2(c) contd.

air atten. foam att. areas $\underset{\rightarrow air}{foam}$ $\underset{\substack{acid \rightarrow \\ foam\ 2}}{\overset{air \rightarrow PZT}{}}$

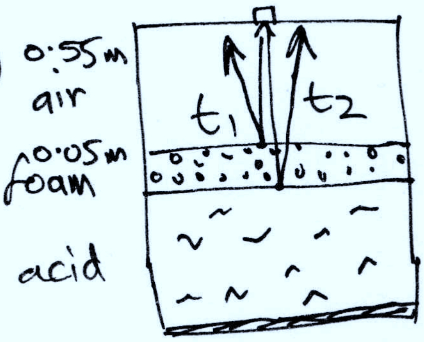
$$P_r = \frac{25^2}{200} \cdot 0.15^2 \cdot 0.332 \cdot 0.98 \cdot 0.9986 \cdot 0.859 \cdot 0.376 \cdot 6.94\times10^{-5} \cdot 0.0668 \cdot 0.041$$

$$P_r = 5.75\times10^{-11} = V^2/200 \quad \therefore V = 0.107\ mV$$

or 0.214 mV open cct.

(drop in signals × factor of ~5)

(d)



Measure reflected pulse-echo times from foam top surface and foam/acid interface

$$t_1 = \frac{0.55}{340} \times 2 = 3.24\ ms$$

$$t_2 = \left(\frac{0.55}{340} + \frac{0.05}{200}\right) \times 2 = 3.74\ ms$$

$\Delta t = 0.5\ ms$

P_R foam/air = 0.933

P_R foam/acid = 0.959

with Lambertian scatter from foam, ignore ±5° beam spread :

for t_1 pulse//

$$Pr_1 = \frac{25^2}{200} \cdot 0.15^2 \cdot 0.041^2 \cdot \underset{(0.738)}{10^{\frac{-0.55\times1.2\times2}{10}}} \cdot 0.933 \cdot \frac{7.854\times10^{-5}}{\pi\, 0.55^2} = V_1^2/200$$

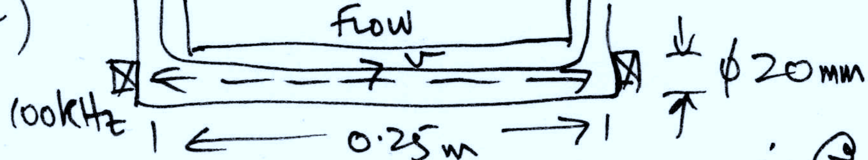
$V_1 = 1.16\ mV$ or $2.32\ mV_{o/cct}$

for t_2 pulse//

$$Pr_2 = \frac{25^2}{200} \cdot 0.15^2 \cdot 0.041^2 \cdot 0.738 \cdot \underset{(0.141)}{10^{\frac{-0.05\times85\times2}{10}}} \cdot \frac{7.854\times10^{-5}}{\pi\, 0.60^2} = V_2^2/200$$

$V_2 = 0.413\ mV$ or $0.826\ mV_{o/cct}$

(e)



$A = 3.14\times10^{-4}\ m^2$

$\therefore$ @ 1l/sec, $v_f = 3.18\ ms^{-1}$

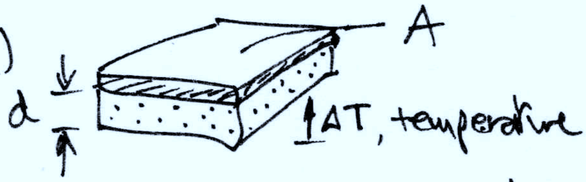
for v sound: $v = f\lambda$ $\quad 1258 = 100\times10^3\,\lambda \quad \therefore \lambda = 12.58\ mm$

$\therefore$ over 0.25m $\Rightarrow$ $19.87\,\lambda \equiv 7154°$ of delay with no flow

with $\pm 3.18\ ms^{-1}$: $\Delta\theta = \pm \frac{3.18}{1258} \cdot 7154 = \pm 18°$ phase shift

XOR gives 5V/180° $\therefore$ ΔV with ± flow $= \pm \frac{18}{180} \times 5 = \pm 0.50\ V_{output}$

3(a)(i)



$$W = \frac{kA\,\Delta T}{d} = \frac{0.02\,(5\times10^{-3})^2}{150\times10^{-6}}\,\Delta T$$

$$\therefore \text{Thermal rating} = \frac{\Delta T}{W} = \frac{d}{kA} = \left[\frac{0.02\,(5\times10^{-3})^2}{150\times10^{-6}}\right]^{-1}$$

$$= 300\ {}^\circ C/W$$

(a)(ii) Heat flux, F (or W) $= \frac{kA\,\Delta T}{d} = m c_p \dot{T}$ with $\Delta T = T_\infty - T$

$$\dot{T} = \frac{kA\,\Delta T}{m c_p d} \quad \text{with } \tau = \frac{m c_p d}{kA} \text{ and } T' = \frac{T - T_\infty}{\tau}$$

$$\therefore -T' = \tau \frac{dT'}{dt} \Rightarrow \text{soln. } \ln T' = \frac{-t}{\tau} + C \text{ with: }$$

$T = T_0$ @ $t = 0$

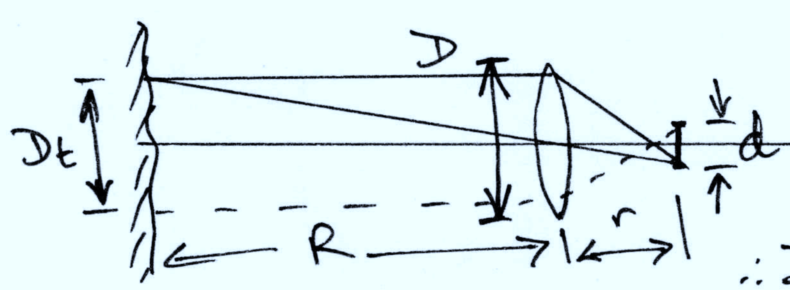
$T = T_\infty$ @ $t = \infty$

$$\therefore T = (T_0 - T_\infty)\,e^{-t/\tau} + T_\infty$$

$$t_{10-90\%} = 2.2\,\tau = \frac{2.2\,.(5\times10^{-3})^2.(3\times150\times10^{-6} + 2330\times40\times10^{-6}).150\times10^{-6}.700}{0.02\,.(5\times10^{-3})^2}$$

$$\therefore t_{10-90\%} = 1.08\ s \qquad (\tau = 0.49\ s,\ f_{-3dB} = 0.32\,Hz)$$

(a)(iii)



pixel size

$$\frac{D_t}{R} = \frac{d}{r} = \frac{D_t}{20} = \frac{5\times10^{-5}}{25\times10^{-3}}$$

$\therefore D_t$ pixel $= 0.04\,m$ on building

For good imaging, the drone should fly $< 0.04\,m$ in $1.08\ s$

$\therefore v < 37\ mm/s$

(a)(iv) Lambert's Law

$$\delta W = \frac{W\cos\theta\,A\,\delta\omega}{\pi} = \frac{W}{\pi}\cdot\frac{\pi D_t^2}{4}\cdot\frac{\pi D^2}{4R^2}, \quad \delta\omega = \frac{\frac{\pi D^2}{4}\cdot 4\pi}{4\pi R^2}$$

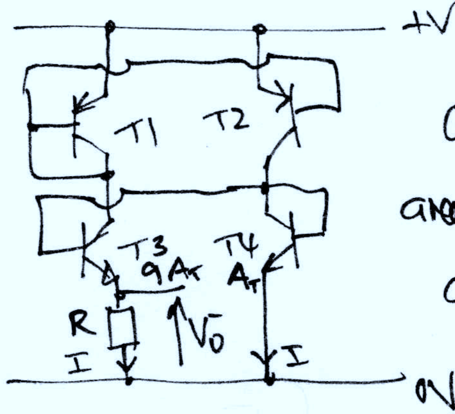
$$\therefore \delta W = \frac{W}{\pi}\cdot\frac{\pi^2}{16}\cdot\frac{d^2}{r^2}\cdot D^2 = \epsilon\,\sigma_{SB} T^4 \frac{\pi}{16}\frac{d^2 D^2}{r^2} \quad \text{with } T = 278 \text{ or } 288\,K$$

$$P_s = 2.6\,\frac{k}{q}\ln(2\times10^5 . 3\times10^{-2}) = 1.95\times10^{-3}\ V/K$$

$$\therefore \underset{278\to288}{\Delta V} = 1.95\times10^{-3} . 300 . 0.9 . 5.67\times10^{-8}(288^4 - 278^4)\frac{\pi}{16}\frac{(5\times10^{-3})^2(10\times10^{-3})^2}{(25\times10^{-3})^2}$$

$$= 2.34\times10^{-4}\,\Delta T^4 = 21.2\ \mu V$$

3(b) T_1, T_2 same const. current fed to transistors T_3, T_4



$$I_c = I_s e^{V_{BE}/kT/q}$$ Ebers-Moll.

$9\,J_{c3} = J_{c4}$ with area $T_3 = 9 \times$ area T_4

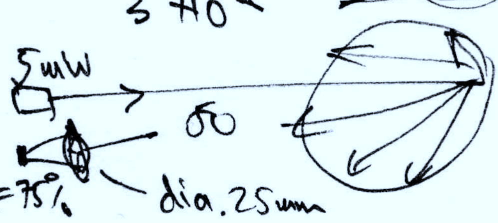
$$9 e^{V_{BE3}/kT/q} = e^{V_{BE4}/kT/q}$$

$$\frac{kT}{q} \ln 9 = \Delta V_{BE} = V_0 = T \cdot 1.895 \times 10^{-4}$$

$\therefore$ @ $T = 300K$, $V_0 = 0.0569V$

(c)(i) $t = \dfrac{2 \times 50}{3 \times 10^8} = 0.333\ \mu s$

(ii)



@ 850nm

$$P_{rec.} = 5 \times 10^{-3} \cdot \frac{2 \cdot 0.4}{2\pi \cdot 50^2} \cdot \frac{\pi (25 \times 10^{-3})^2}{4} = 1.25 \times 10^{-10}\ W$$

photon energy $= h\nu = \dfrac{hc}{\lambda} = 2.339 \times 10^{-19}\ J$

$\therefore$ Photocurrent $= I_p = \dfrac{1.25 \times 10^{-10}}{2.339 \times 10^{-19}} \cdot 0.75 \cdot 1.6 \times 10^{-19} = 64.3\ pA$

(iii) Reduce ambient light effects with:

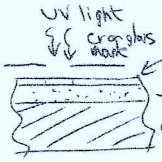
- λ matched, narrow band optical filter on detector
- enclosed optics + shielding
- A/R coatings on optics
- bandpass filter on electrical signal matched to pulse shape
- repeat pulses + average signals

4(a) Surface micro-machining – deposition of Si, SiO_xN_y
– photolithography
– wet chemical etching

(a)

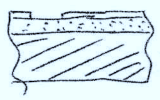
← deposit SiO_x from CVD
Silane + N_2O plasma

(b)



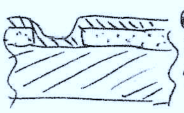
spin on photoresist, expose to UV through a mask pattern (Cr or glass)

(c) Develop photoresist by dissolving exposed areas in caustic solution eg: NaOH



then etch in HF solution to dissolve SiO_x or plasma etch in CF_4

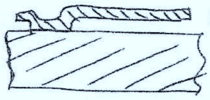
(d)



then deposit poly-Si from SiH_4 CVD heat/plasma deposition

(e) Poly-Si layer can then be patterned by dry plasma etching (leave holes for large area under-etching) of SiOx layer next by HF solution

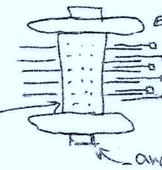
(f) SiOx layer is then removed, to leave free standing poly-Si structure



(g) can include Si Nitride insulation layers and metal layers (deposit by sputtering / evaporation).

(h) to form accelerometer or gyro. structures

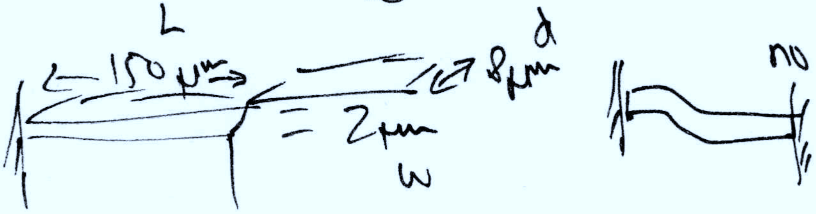
(i) use capacitive or strain gauges for spring readout.

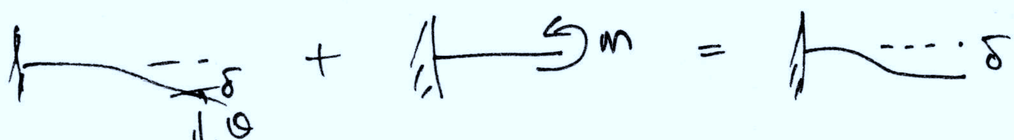


Differential capacitance read-out with a.c. bridge – see part 4(b)(v) for analysis of signal levels.

(b)(i) $C = \frac{A\epsilon_0}{d} = \frac{100 \times 800 \times 10^{-8} \times 8.854 \times 10^{-12} \times 8 \times 10^{-6}}{10^{-6}} = 3.54\ pF$

(ii)





δ	$WL^3/3EI$	$-\frac{ML^2}{2EI}$
θ	$WL^2/2EI$	$-\frac{ML}{EI}$

$\therefore \delta = \frac{WL^3}{3EI} - \frac{WL^3}{4EI} = \frac{WL^3}{12EI}$

$\therefore \frac{ML}{EI} = \frac{WL^2}{2EI} \qquad M = \frac{WL}{2}$

with 4 tethers per axis,

spring stiffness $k = \frac{W}{\delta} = \frac{48EI}{L^3}$

with $I = \frac{w^3 d}{12} = 5.33 \times 10^{-24}\ m^4$

$E_{Si} = 150\ GNm^{-2}$

$\rho Si = 2330\ Kgm^{-3}$

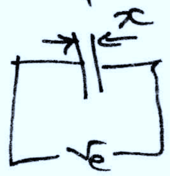
$\therefore k = 11.37\ Nm^{-1}$

(iii) Proof mass, $M = 350 \times 10^{-6} . 750 \times 10^{-6} . 8 \times 10^{-6} . 2330 = 4.893 \times 10^{-9}\ kg$

$\therefore f_{res} = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = 7672\ Hz$

4(b)(iv) Force reqd. for 0.5 μm with Q = 85 :

$$f = 11.37 \cdot 0.5 \times 10^{-6} / 85 = 6.69 \times 10^{-8}\ \text{N}$$



consider 2 capacitor plates : $E = \frac{1}{2} C v_e^2$, $C = \frac{A\epsilon_0}{x}$

$$E = \frac{A\epsilon_0 V_e^2}{2x} = \frac{1}{2}\epsilon_0 A v_e^2 x^{-1}$$

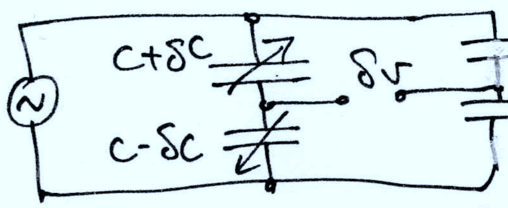
$$\therefore dE = F dx \quad \therefore \quad F = \frac{dE}{dx} = -\frac{A\epsilon_0 v_e^2}{2x^2} = -\frac{C V_e^2}{2x}$$

Taking 1/2 plates for excitation : $F = 6.69 \times 10^{-8} = \frac{3.54 \times 10^{-12} v^2}{4 \cdot 0.5 \times 10^{-6}}$

$\Rightarrow v_e = 0.194\ \text{V}$ amplitude @ 7.67 kHz

(2)

(v)

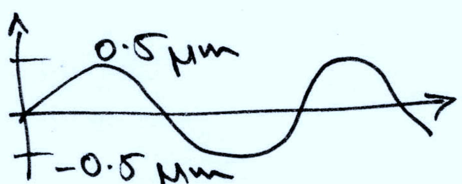


$$\delta v \simeq \frac{V_s}{2} \cdot \frac{\delta C}{C} = \frac{V_s}{2} \cdot \frac{\delta x}{x}$$

capacitive 1/2 bridge read-out.

Coriolis force $= 2\omega v_r m$ (radial velocity) with $90° s^{-1} \equiv \frac{\pi}{2}\ \text{rad s}^{-1}$

posn. x



@ 7672 Hz $= a \sin 2\pi f t$

0.5×10^{-6} m $= 0.024\ \text{ms}^{-1}$

$$\therefore \text{velocity} = \frac{dx}{dt} = 2\pi f a \cos 2\pi f t \quad \text{where } a = \delta x$$

32.6 pm $\qquad$ 4.893×10^{-9} kg.

$$\therefore \text{Coriolis deflection} = 2 \cdot \frac{\pi}{2} \cdot \frac{m}{k} \cdot 2\pi f \delta x = 2\pi^2 \cdot 7672 \cdot 0.5 \times 10^{-6} \cdot \frac{m}{k}$$

$$\text{and output signal, } \delta v = \frac{2}{2} \cdot \frac{2\pi^2 \cdot 7672 \cdot 0.5 \times 10^{-6} \cdot 4.893 \times 10^{-9}}{11.37 \times 10^{-6}}$$

$$\delta v = 3.26 \times 10^{-5}\ \text{V} \quad (32.6\ \mu\text{V})$$

(4)

Examiner's comments:

Q1 Current transformer and fluxgate sensor

The fluxgate section was generally quite well answered with candidates recalling the responsivity equation correctly. Flux density was occasionally incorrectly calculated due to assuming the current was flowing though the windings rather than though a conductor in the centre of the toroid. The system block diagram was also reasonably well attempted in most cases.

Q2 Ultrasonic level and flow sensor

A very popular question with generally good attempts. The complexity of the algebra for the foam layer reflections often resulted in arithmetic errors and the Lambertian scattering properties were neglected in a number of attempts. The final section on the flow sensor was fairly straightforward although a number of candidates made errors in deriving the phase shift.

Q3 Temperature measurement, LIDAR and pyrometer

A very popular question which attracted good quality attempts. The thermal time-constant was correctly calculated by many candidates although some incorrectly used the silicon thickness rather than the foam insulation thickness in their calculation. The current mirror section was generally recalled well and the LIDAR signal calculations were well attempted in many cases.

Q4 MEMs fabrication and device physics

A less popular question overall. The fabrication processes were reasonably well described in most cases although process details such as etchants and gases were sometimes omitted. The estimates of resonant frequency and capacitance force were often less well answered, due to confusing the beam dimensions, though there were some good attempts.

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