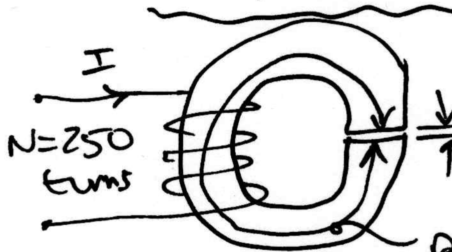
4B13 CRIB 2023

1(a)



$A = 50\,\mathrm{mm}^2$ $\quad g = 0.50\,\mathrm{mm}$

$\phi_i = 50\,\mathrm{mm}$

$\phi_{ext} = 60\,\mathrm{mm}$

$t = 10\,\mathrm{mm}$

path $l \approx 55\pi\,\mathrm{mm} - 0.5\,\mathrm{mm} = 172.3\,\mathrm{mm}$

$\mu = 2500$

Under no load, $g = 0.5 \times 10^{-3}\,\mathrm{m}$ $\qquad \int H \cdot dl = \oint J \cdot ds$

$NI = H_0 (g + l/\mu_r)$ as $B_0 = B = B_m$ for flux conservation with $B_m = \mu_0 \mu_r H_m$, $B_0 = \mu_0 H_0$

$\therefore B = \dfrac{\mu_0 N I}{(g + l/\mu_r)}$ and $\phi = BA$ with $L = \dfrac{N\phi}{I} = \dfrac{NBA}{I}$

Hence $L_{no\,load} = \dfrac{\mu_0 N^2 A}{(g + l/\mu_r)} = 6.908\,\mathrm{mH}$

$L_{1000N} = \dfrac{3.93 \times 10^{-6}}{10^{3}\left(0.60 + \frac{172.3}{2500}\right)} = 5.875\,\mathrm{mH}$

$\therefore \Delta L = -1.033\,\mathrm{mH}$

(b)

$B_{no\,load} = \dfrac{LI}{NA} = \dfrac{6.908 \times 10^{-3} \cdot 1}{250 \cdot 50 \times 10^{-6}} = 0.553\,\mathrm{T}$

$B_{1000N} = 0.470\,\mathrm{T}$ $\qquad \therefore \Delta B = -0.083\,\mathrm{T}$

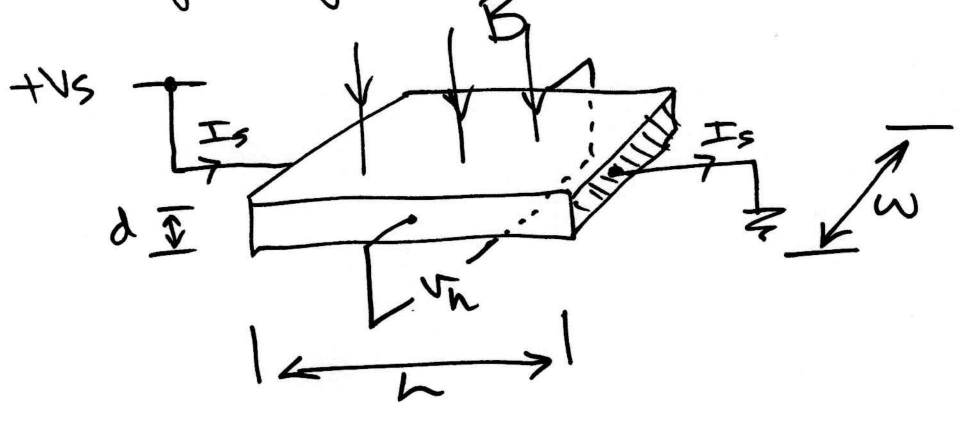
(c) Consider stored energy $\frac{1}{2} L I^2$ @ no load $= 3.454\,\mathrm{mJ}$

@ 1000N $= 2.938\,\mathrm{mJ}$

$\therefore$ a 0.1 mm displacement causes a change in stored energy of 0.516 mJ. Assuming virtual-work δE = force × distance: gives a force of $0.516/0.1 = 5.16\,\mathrm{N}$

1(c) contd. This force is only 0.5% of the mechanical load, and the flux density is around 0.5 T – perhaps about 1/3 of the saturation value. So, current-force feedback is not feasible for large forces around 1000 N. A few 10's of N would be practical limit.

(d) $\Delta B = -0.083T$ from part (b). For the Hall sensor responsivity:



Force balance on moving charges: $Bqv_d = \frac{qV_h}{w}$ where

$v_d = \frac{\mu V_s}{L}$ (drift velocity) $\therefore \frac{B\mu V_s w}{L} = V_h$

For GaAs: $\mu = 0.90$ V/s

$V_s = I_s R$ where $R = \frac{\rho L}{wd}$ $\therefore V_h = \frac{B\mu I_s \rho}{d}$

$\therefore \Delta V_h = \frac{-0.083 . 0.90 . 0.005 . 0.05}{10\times10^{-6}} = 1.867$ mV

Assuming linear excess carrier conc. as per lecture notes, with Fick's Law of diffusion with $D = \mu kT/q$, $f = -D\frac{dn}{dx} = -D\frac{2n_0}{w}$

$\frac{dN}{dt} = fLd = -D\frac{2n_0}{w}Ld = \frac{-8DN}{w^2}$ with soln. $N = N_0 e^{-t/\tau}$

with $\tau = \frac{w^2}{8D}$ and estimate response time $= 2.2\tau = t_{rise}$

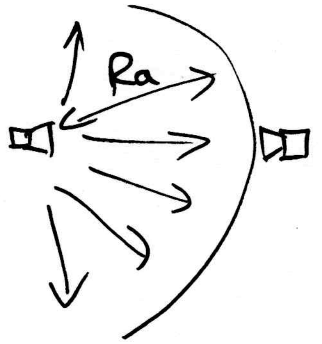
$\Rightarrow t_{rise} \simeq 2.2.\frac{(0.25\times10^{-3})^2}{8\times0.0233} = 0.73\ \mu s$

2(a) flight time = $\dfrac{\text{range}}{\text{velocity}}$ light : $\dfrac{5}{3\times10^{8}} = 0.017\ \mu s$

u/sonic : $\dfrac{5}{340} = 14.7\ ms$

∴ time difference = 14.7 ms

(b) $P_{\text{trans. density}} = \dfrac{V^2}{R} \cdot \eta \cdot \dfrac{4 Z_t Z_a}{(Z_t + Z_a)^2} \cdot \dfrac{1}{2\pi R_a^2}$



$Z_t = 7500 \times 4000 / 500 = 6\times10^{4}$

$Z_a = 1.22 \times 340 = 414.8$

∴ $P_{\text{trans. density}} = \dfrac{15^2}{500} . 0.2 . \dfrac{4 . 6\times10^{4} . 414.8}{60414.8^2} . \dfrac{1}{2\pi . 5^2} = 1.56\times10^{-5}$ W/m²

Air atten. = $-1.2 \times 5 = -6dB = \times 1/4$

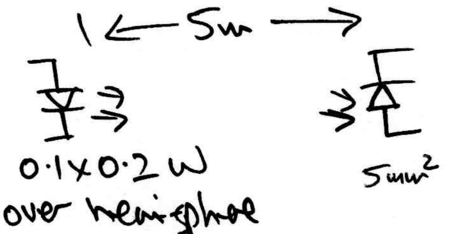
∴ Power recd. by transducer = $P_{t/d} \cdot \dfrac{\pi d^2}{4} . \dfrac{4 Z_t Z_a}{(Z_t + Z_a)^2} . \dfrac{1}{4}$ u/sonic

$= P_{u/s} = 1.88\times10^{-11}\ W \Rightarrow P_{elec} = \eta \times P_{u/s} = \dfrac{V^2}{R}$

$\Rightarrow V = (500 . 0.2 . 1.88\times10^{-11})^{1/2} = 43.4\ \mu V$ loaded

which is doubled into an open-cct $\Rightarrow 0.087\ mV$ (87μV)

(c)



Power @ p/diode = $\dfrac{0.02}{2\pi R_a^2} . 5(10^{-3})^2$

$= 0.637\ nW$

Photon energy @ 780nm : $\dfrac{hc}{\lambda} = \dfrac{6.626\times10^{-34} . 3\times10^{8}}{780\times10^{-9}} = 2.55\times10^{-19}$

∴ photo-current = $[2.55\times10^{-19}]^{-1} . 0.65 . 1.6\times10^{-19} . 0.637\times10^{-9} = 0.26\ nA$

(d) optical signal changes with inverse square law :

$\times\left(\dfrac{5}{2}\right)^2 \Rightarrow \times 6.25 \rightarrow 1.63\ nA,\ +1.36\ nA$ increase

2(d) contd.

ultrasonic signal has decreased attenuation, inverse square-law and $\sqrt{p}$ to v relationship:

Atten. $3 \times 1.2 = 3.6\,dB \qquad \times 2.29$

Sq. Law: $\quad \times 6.25$

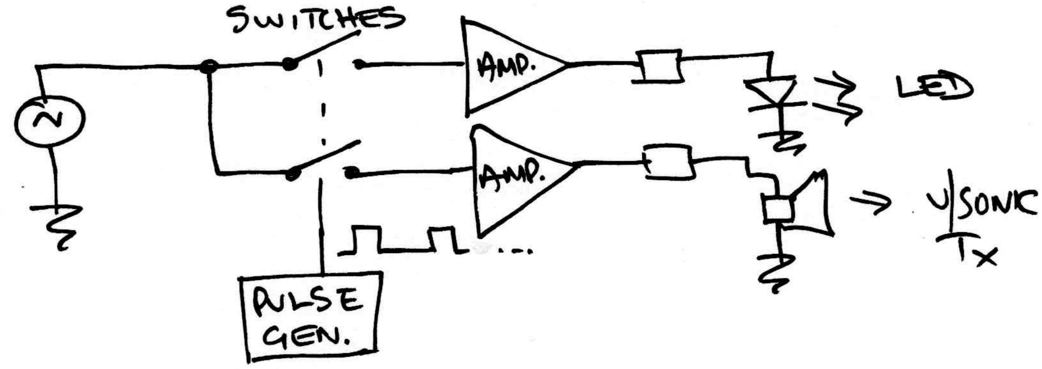
$\Rightarrow$ signal increases by $\times \sqrt{2.29 \times 6.25} = \times 3.78$

$\rightarrow 0.33\,mV$ signal

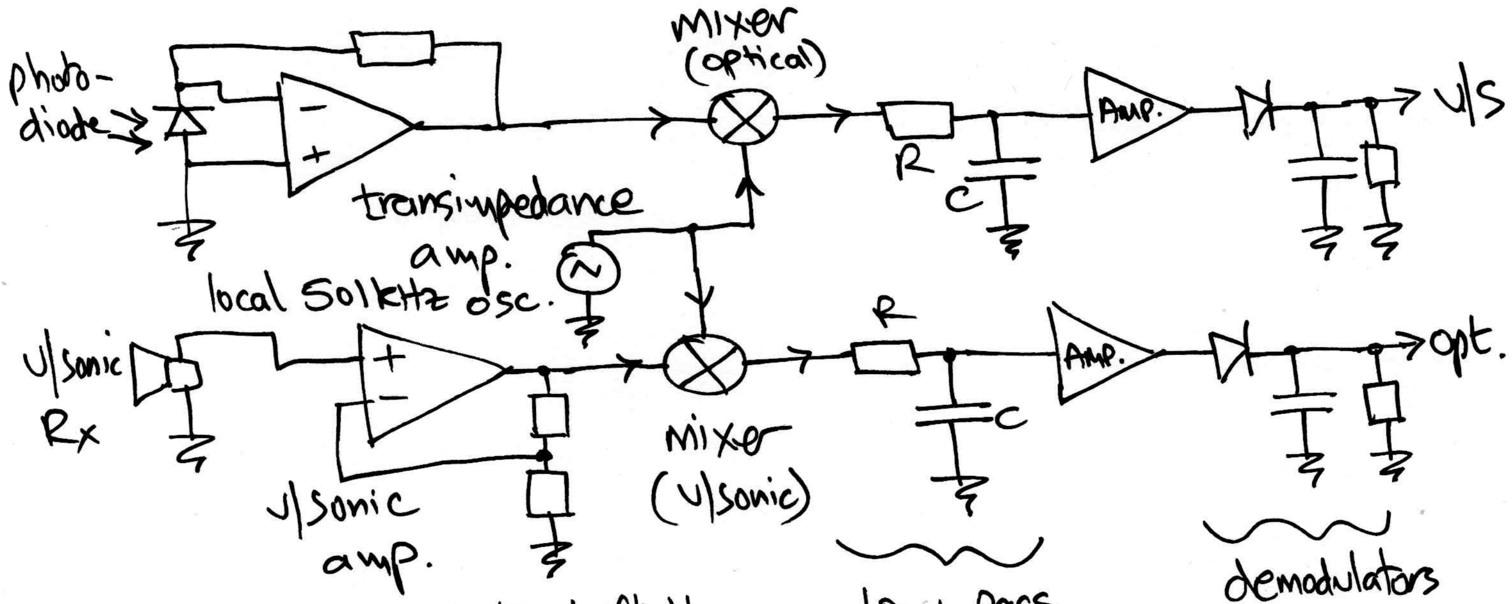
increase of $0.242\,mV$

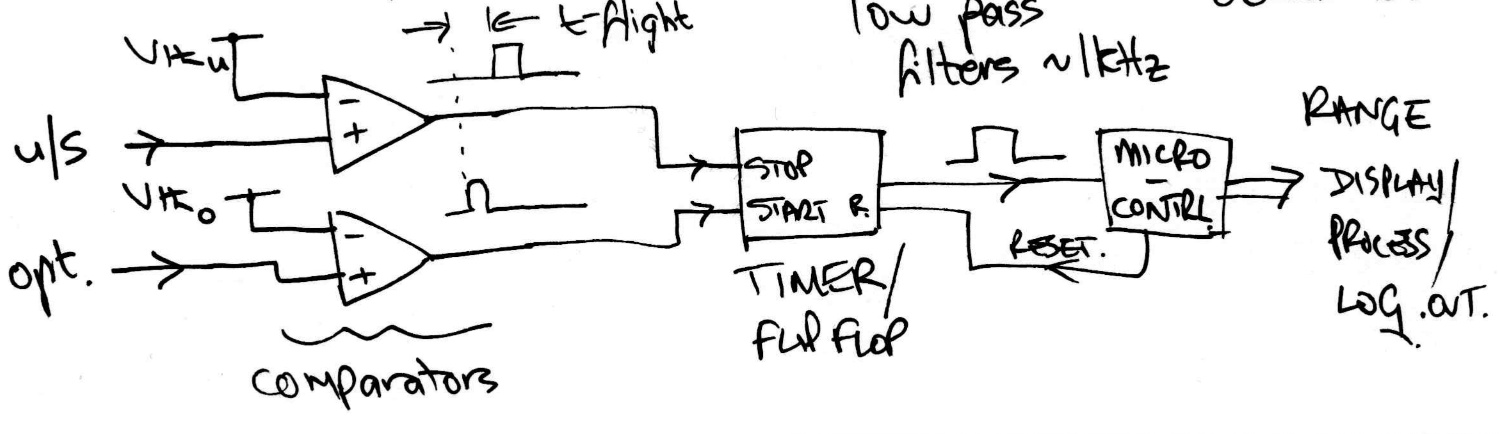
(e) Transmitter:

500kHz osc.

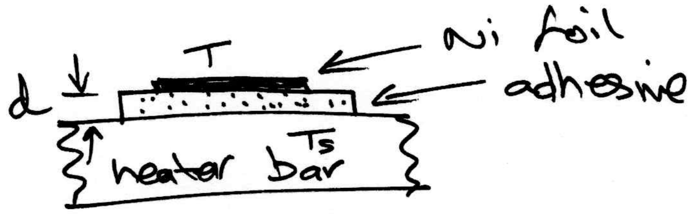


Receiver:





3(a)(i)



$$\text{Heat flux to Ni} = \frac{kA(T_s - T)}{d} = m c_p \, dT/dt$$

where $d = 50 \times 10^{-6}\,m$, $m = 0.025 \times 10^{-3}\,kg$

$A = 5 \times 10^{-6}\,m^2$, $k = 0.30\ W K^{-1} m^{-1}$

$c_p = 1.2 \times 10^{3}\ J\,kg^{-1} K^{-1}$

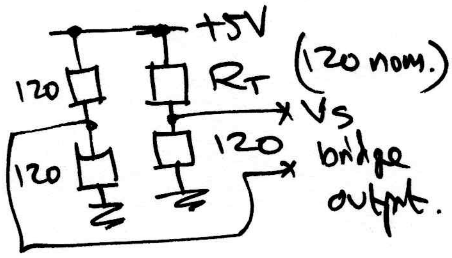
$$\therefore \frac{dT}{dt} = -\frac{kAT}{dmc_p} + C, \quad \text{soln. } T = Xe^{-t/\tau} + Y$$

where $\tau = \dfrac{m c_p d}{kA}$, $C = T_s/\tau$ and X and Y depend on initial and final temperatures. 10%–90% rise-time

$$\underline{t_r = 2.2\tau} = \frac{0.025 \times \cancel{10^{-3}} \cdot 1.2 \times \cancel{10^{3}} \cdot 50 \times \cancel{10^{-6}} \cdot 2.2}{0.30 \cdot 5 \times \cancel{10^{-6}}} = \underline{2.2\,s}$$

(a)(ii) Power dissipated $= \tfrac{1}{2} \cdot 5^2/(120+120)$

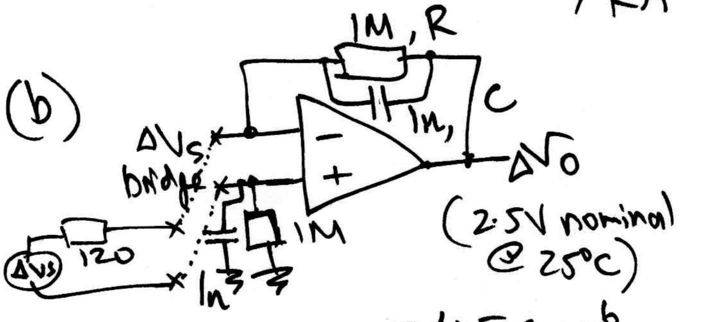
$P_d = 0.052\ W$



$$\text{Heat flux} = \frac{kA(\Delta T)}{d} = P_d$$

$$\therefore \Delta T = P_d \cdot d/kA = \underline{1.73\ °C}$$

(b)



for 1 element changing in 4 resistor bridge:

$$\Delta V_s = \frac{-\Delta R_T}{R_T} \cdot \frac{V_s}{4}$$

$$\Delta V_o = \frac{-\Delta V_s\, 2 \times 10^{6}}{120} = \frac{0.9}{120} \cdot \frac{5}{4} \cdot \frac{2 \times 10^{6}}{120} = \underline{156\ V K^{-1}}$$

for noise value at output, draw up noise/gain table:
where B, bandwidth $= \dfrac{1}{2\pi RC} = 159\,Hz$

3(b) cont'd.

$v_n = \sqrt{4kTRB}$ $\quad B = 159\ Hz$

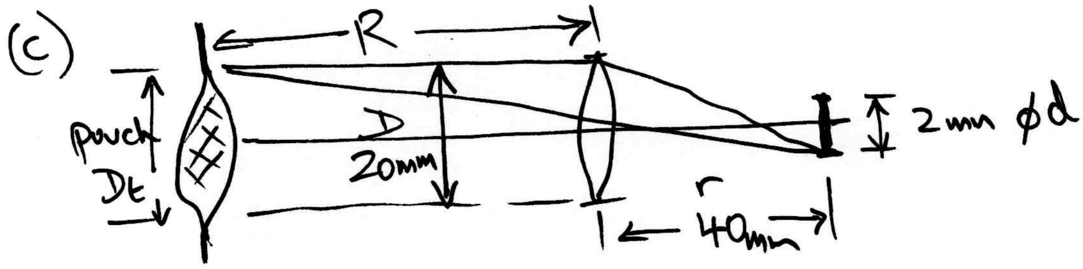
$k = 1.38\times10^{-23}$
$T = 300\ K$

$G = 2\times10^6/120 = 16667$

Source	v_n (μV rms)	GAIN	@ output V_n (μVrms)
bridge resistors (120 Ω), R_s	0.018	16667	300
op-amp, v_n	$3\times10^{-3}.\sqrt{159}$	16667	630
feedback, R (1MΩ)	1.62	1	2
— " —	1.62	1	2
op-amp, $i_n \times R_s$	0.008	16667	120

TOTAL NOISE $= \sqrt{\Sigma V_n^2} = \underline{708\ \mu V\text{rms}}$.

(c)



Lambert's Law: $\delta W = \frac{W\cos\theta}{\pi} . A . \delta\omega = \frac{W}{\pi} . \frac{\pi D_t^2}{4} . \frac{\frac{\pi D^2}{4}}{\cancel{4\pi} R^2} \cancel{.4\pi}$

Stephan's Law : $W = \epsilon\, \sigma_{SB} T^4$

and here by similar triangles : $\frac{D_t}{R} = \frac{d}{r}$ $\therefore \delta W = \frac{\epsilon\, \sigma_{SB} T^4 \pi d^2 D^2}{16 r^2}$

$$\therefore \delta W = \frac{0.95 . 5.67\times10^{-8} . (x+273)^4 . \pi . 2^2 . 20^2 \times10^{-6}}{16 . 40^2}$$

$= 8.91\times10^{-5}$ with $x = 30°C$ $\rightarrow$ (1.11 mV)
$= 1.84\times10^{-4}$ with $x = 90°C$ $\rightarrow$ (2.29 mV)

$\therefore \Delta\delta W = 9.45\times10^{-5}$ W with change from 30°C to 90°C

$\therefore \underline{V_{sig.}} = 9.45\times10^{-5} . 250 . 0.05 = \underline{1.18\ mV}$ $\quad 30\rightarrow90°C$

thermal rating $\quad$ pyro-elec. responsivity

4(a) Surface micro-machined devices use sequentially deposited layers of Si, metal, Si oxide/nitrides and photo-resist layers patterned by photolithography and etching (chemical, plasma). For photolith: spin on photoresist polymer, soft-bake, expose to UV thro' mask, develop in alkali or solvent, bake and then etch underlying layer. Etch can be wet chemical (fairly isotropic) or plasma reactive ion (directional). To remove underneath sacrifical layers, arrays of holes are reqd in large structures. Silane + N_2O, O_2, NH_3 plasma deposition is used to deposit ceramic SiO_xN_y layers – $SiH_4 + H_2$ for Si layers. Metals deposited by sputtering. Etching Si with hot KOH or CF_4 plasma for directional structure edges.
Sacrificial layer can form clearances under layers once removed eg: cantilevers. Si_3N_4 layers can provide patterns for doping Si with P or B to create semi-conductor regions eg: for strain gauges. Al metal contact layers are usually deposited by thermal evaporation and patterned with Ce or acid/alkali solutions – or lift-off with photoresist layer.

(b) (i)

$L = 500\,\mu m$

$F = \frac{1}{2}\rho v^2 A$, $A = \frac{\pi (50\times10^{-6})^2}{4}$

Assume load point on cantilever is 475µm from end

Deflection $\delta_1 = \frac{FL^3}{3EI} = 5\times10^{-6}$ approx. for end to touchdown.

$E = 148\ GN\,m^{-2}$ [data book 140-155], $I = \frac{1}{12}bd^3$

4(b)(i) contd. $I = 5\times10^{-23}\ m^4$

$$\delta = 2\times10^{-6} = \frac{F.(475\times10^{-6})^3}{3.148\times10^{9}.5\times10^{-23}} \Rightarrow F = 4.14\times10^{-7}\,N$$
$$= \tfrac{1}{2}\rho v^2 A$$

with $A = 1.96\times10^{-9}\,m^2$, $\rho = 1.22\ kg\,m^{-3}$ $\Rightarrow v = 18.6\,ms^{-1}$

$\Rightarrow V = 3.65\times10^{-8}\ m^3s^{-1}$ $= \underline{36.5\ mm^3s^{-1}}$ $(= vA)$

(ii) Half air flow gives $\frac{1}{4}$ force $\therefore F = 1.04\times10^{-7}\,N$

$$\therefore \frac{M}{I} = \frac{1.04\times10^{-7}.\ 475\times10^{-6}}{5\times10^{-23}} = \frac{\sigma}{y} = \frac{\epsilon E}{y} \quad \text{with } y = 10^{-6}\,m$$

$\therefore \epsilon = 6.67\times10^{-6}$ strain $\therefore \frac{\Delta R}{R}$ for strain gauges $= \epsilon\,G.f.$
$= 1.34\times10^{-3}$

Bridge output voltage $= 5\times1.34\times10^{-3} = \underline{6.67\,mV}$ @ half max. flow

(iii) Beam mass, $m = \rho_{Si}\times L\times b\times d = 1.74\times10^{-10}\ kg$ with $\rho_{Si} = 2.32\ kg\,m^{-3}$. Assume centre of vibrating mass is $\frac{2}{3}$ along beam length, $L = 333\,\mu m$ then spring constant, $k = \frac{F}{\delta}$
$= \frac{3EI}{L^3} = 0.60\ Nm^{-1}$ and $f_{res} = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = 9.35\,kHz$

Hence, assuming Q-cycles for beam to settle, the response time, $\underline{t_r} \approx 60/9.35\times10^3 = \underline{6.40\ ms}$

(iv) Force on capacitor plates: $C = \frac{A\epsilon_0}{x}$ $\therefore \frac{dC}{dx} = \frac{-C}{x}$

virtual work: $E = \frac{1}{2}CV^2$ $\therefore \delta E = \frac{1}{2}V^2\delta C = F\delta x$

$\therefore$ Force, $F = \frac{1}{2}V^2\frac{dC}{dx} = \frac{1}{2}V^2\frac{C}{x}$ with $x = 2\times10^{-6}\,m$ air gap

$$C = \frac{(75\times10^{-6})^2\ 8.854\times10^{-12}}{2\times10^{-6}} = 2.49\times10^{-14}\,F$$

$\therefore$ Force, $F = 1.04\times10^{-7} = \frac{1}{2}V^2\ 2.49\times10^{-14}/2\times10^{-6}$ $\Rightarrow \underline{V = 4.09\,V}$

(or 3.54V for 1.5μm air gap)

Examiner's comments:

Q1 Electromagnetic load cell and Hall effect sensor

A popular and fairly straightforward question, well-answered by most candidates. The operation of Hall effect devices was understood by most and the calculation of flux density vs. current was correct in most cases. Many remembered the Hall sensor bandwidth equation rather than deriving it.

Q2 Ultrasonic & optical range measurement

This question was attempted by nearly all candidates and was answered very well overall. Some candidates incorrectly assumed that the system was a pulse-echo arrangement, resulting in doubling of the flight times. The final section on the system schematic drew a range of responses – but often omitting a mixer element, which would be ideal in such a system.

Q3 Temperature measurement, noise and pyrometer

A very popular question which attracted good quality attempts. The thermal time-constant was correctly stated by most candidates, as was the thermal offset due to self-heating. The noise section was a bit more variable, and the final pyrometer section was generally well attempted although a number of candidates had apparently memorized the pyrometer response equation, without showing any derivation.

Q4 MEMs fabrication and device physics

A less popular question. The fabrication processes were reasonably well described in most cases although process details such as etchants and gases were sometimes omitted. The estimate of resonant frequency and capacitance force were often less well answered.

P. Robertson (Principal Assessor)