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# 4B13 CRIB 2014

1(a) $R = R_0 e^{\beta'/T}$

$\therefore \; 200 = R_0 e^{3200/273} \qquad \therefore R_0 = 1.623 \times 10^{-3}$

Hence at 60° $\quad R = 1.623 \times 10^{-3} e^{3200/333} = 24.19\,\Omega$ nominal

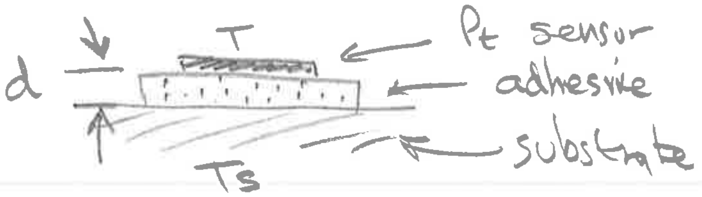
so output signal $= 24.19 \times 5 \times 10^{-3} = \underline{0.121\text{ V}}$

But with $\beta' \pm 100$ in $3200 = \pm 3.1\%$, we would expect the temperature for a given resistance (& signal) to vary by the same amount.

$\pm 3.1\%$ on $(273+60)\text{K} \simeq \pm 10.3\text{K}$ or $\underline{\pm 10.3\,°\text{C}}$

[25%]

(b)




Heat flux to $P_t$ $= \dfrac{kA(T_s - T)}{d} = m c_p \, dT/dt$

where: $d = 10^{-4}$ m $\qquad m = 0.05 \times 10^{-3}$ kg

$A = 10 \times 10^{-6}$ m2 $\qquad c_p = 1.2 \times 10^{3}$ J kg$^{-1}$K$^{-1}$

$k = 0.25$ W K$^{-1}$m$^{-1}$

$$\therefore \frac{dT}{dt} = \frac{-kAT}{d\,m\,c_p} + C = \frac{-T}{\gamma} + C$$

This has a sol$\underline{\text{n}}$ of the form $T = Xe^{-t/\gamma} + Y$

where $\gamma = \dfrac{m c_p d}{kA}$, $C = T_s/\gamma$ and $X$ and $Y$ depend on the initial and final temperatures.

1(b) contd

(i) Hence $\tau = \dfrac{5\times10^{-5} . 1{\cdot}2\times10^{3} . 10^{-4}}{0{\cdot}25 . 10^{-5}} = \underline{2{\cdot}4\ s}$

and $f_{-3dB} \simeq \dfrac{1}{2\pi\tau} = 0{\cdot}066$ Hz (15 sec. period) [30%]

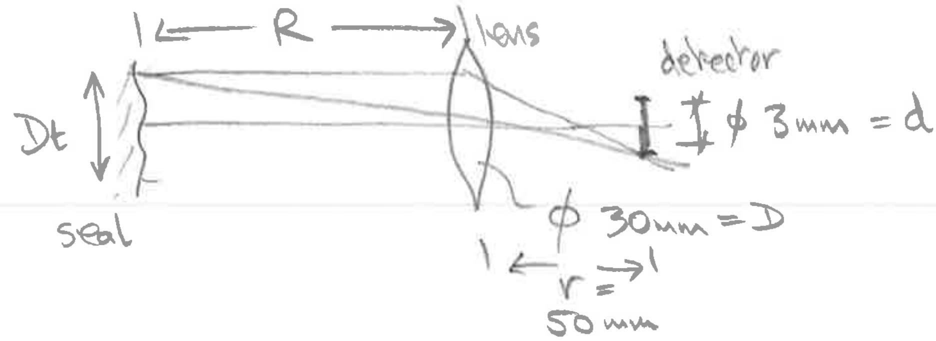
(ii) with 10 mA current and $R = 100 + (0{\cdot}385\times75) = 128{\cdot}9\,\Omega$
(nominal @ 100°C)

$P = i^2R = 0{\cdot}013$ W

steady state heat flow to substrate $= 0{\cdot}013 = \dfrac{kA\,\Delta T}{d}$

$\therefore \Delta T = 0{\cdot}013 . 10^{-4} / 0{\cdot}25 . 10^{-5} = \underline{0{\cdot}52\ °C}$ [15%]

(c)




Lambert's Law : $\delta W = \dfrac{W\cos\theta}{\pi} A\,\delta\omega = \dfrac{W}{\pi} . \dfrac{\pi D_t^2}{4} . \dfrac{\frac{\pi D^2}{4}\cancel{\times 4\pi}}{\cancel{4\pi}R^2}$

Stephan's Law : $W = \epsilon\,\sigma_{SB}\,T^4$

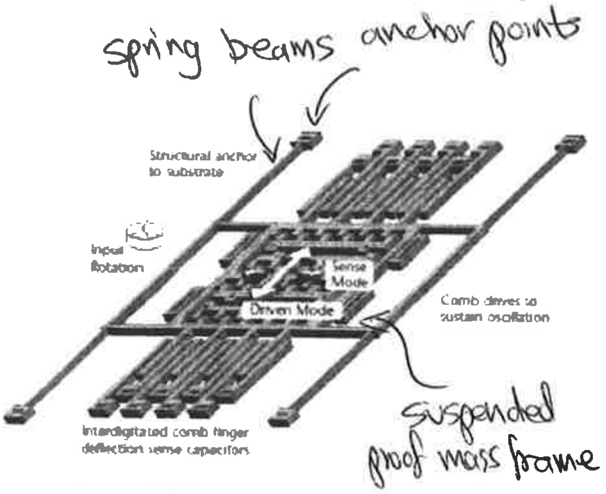
and here $\dfrac{D_t}{R} = \dfrac{d}{r}$ $\therefore$ $\delta W = \epsilon\,\sigma_{SB}\,T^4 \dfrac{\pi}{16}\dfrac{d^2D^2}{r^2}$

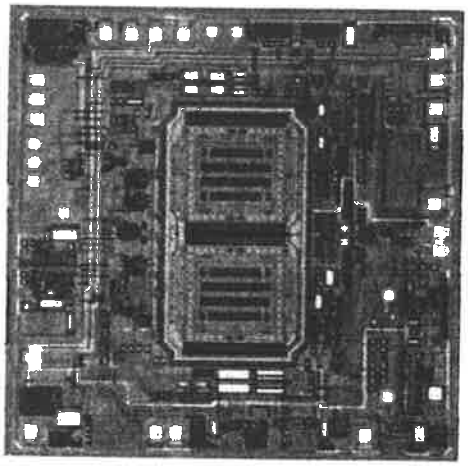
$\therefore \delta W = 0{\cdot}9 . 5{\cdot}6\times10^{-8} . (273+100)^4 . \dfrac{\pi}{16} . \dfrac{3^2 . 30^2}{50^2}\times10^{-6}$

$\therefore \delta W = 0{\cdot}62$ mW

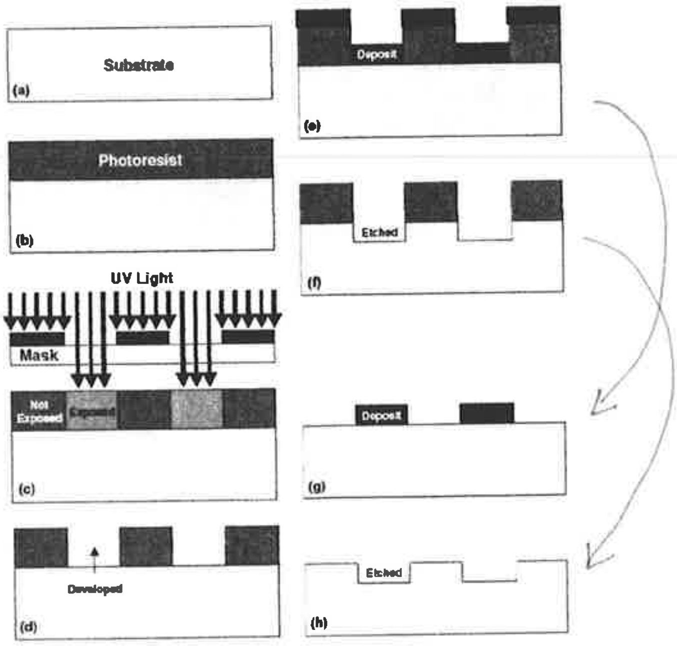
$\therefore \Delta T = 0{\cdot}124$ K $\therefore$ $V_{sig} = 0{\cdot}124\times50\times10^{-3}$
$= \underline{6{\cdot}2\ mV}$ [30%]

2(a) MEMs gyros use a vibrating proof mass, free to oscillate in 2 orthogonal axes. The proof mass is fabricated by surface micro-machining techniques i.e. a poly-Si layer is deposited onto the silicon crystal substrate and patterned by photolithography to define a proof mass, spring suspension, anchor points and capacitive drive & sensing fingers (plates).



A key process in IC and MEMs fabrication is photolithography. This process enables the replication of 2-d patterns in layers of materials which may be deposited and etched onto a substrate (a).




Photoresist is applied by dispensing a few $cm^3$ of liquid polymer onto a wafer, which is then spun at high speed to thin out the layer. It is then baked to form a solid film (b).

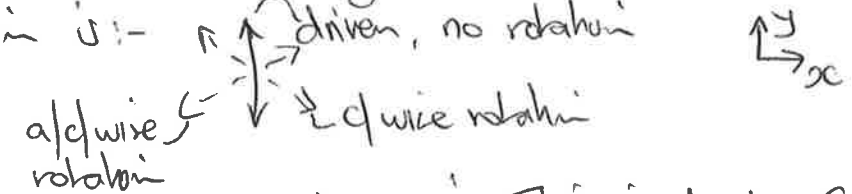
To transfer the pattern, the photoresist is exposed to UV light through a mask, which carries the desired pattern (usually chrome on glass). The UV light alters the properties of the polymer (c), allowing the exposed regions to be dissolved in dilute alkali (NaOH soln.) leaving the mask pattern replicated in resist (d).

Then, either a deposition process eg. Chemical Vapour Deposition (CVD), evaporation or sputtering deposits a metal, ceramic or Si layer on the wafer (e) or the underlying substrate / layer is etched (f). The photoresist layer is then usually removed by dissolving in solvent (eg. acetone) or plasma etched to leave the processed substrate (g) or (h).

2(a) contd.

The gyro proof mass is made from a layer of poly-Si deposited onto $SiO_2$. The poly-Si is typically around 5 μm thick, and the sacrificial $SiO_2$, which is etched away to leave clearance gaps is ~1 μm. Chemical isotropic etching with HF removes the $SiO_2$ layer and vertical planes are defined by plasma etching (reactive ion etching, RIE) ($CF_4 + O_2$) to form capacitor plates for driving resonance motion and sensing displacement.

For the gyro, one axis is driven into vibration by the capacitance drive and the orthogonal axis is used for sensing. Proof mass motion is :-



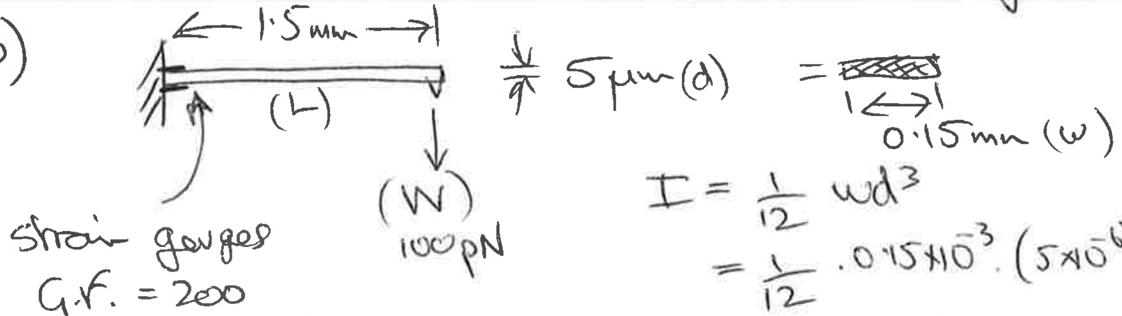

about vertical (z) axis. This is due to Coriolis forces.

$$a_{\perp} = 2s\Omega$$

$\Omega$ = angular rotation rate
$a_{\perp}$ = orthogonal accel.
$s$ = drive axis velocity.

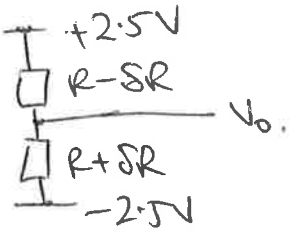
[35%]

(b)




$$I = \frac{1}{12} wd^3$$
$$= \frac{1}{12} \cdot 0.15\times10^{-3} \cdot (5\times10^{-6})^3$$
$$= 1.56\times10^{-21} \text{ m}^4$$

(i)




$$\frac{\delta R}{R} = \epsilon\, G.F. = 200\,\epsilon$$

$\rho_{Si} = 2.30\times10^{3}$ kgm$^{-3}$
$E_{Si} = 150$ GPa

$$M = WL = \frac{\sigma I}{y}, \quad \sigma = \epsilon E$$

$$y = d/2$$

$$\therefore \epsilon \frac{150\times10^{9} \cdot 1.56\times10^{-21}}{2.5\times10^{-6}} = 100\times10^{-12} \cdot 1.5\times10^{-3}$$

$$\therefore \epsilon = 1.6\times10^{-9} \qquad \therefore \frac{\delta R}{R} = 3.2\times10^{-7}$$

2(b)(i) contd.   $\Delta V = (V_+ - V_-)\,\frac{\delta R}{2R}$

$$\Delta V = \frac{2 \times 2.5 \times 3.2 \times 10^{-7}}{2} = 800\,nV = 0.8\,\mu V$$ [25%]

(ii) Beam mass, $m = \rho L d w = 2.59 \times 10^{-9}\,kg$

Guesstimate centre of vibrating mass as 2/3 along beam $\approx 1\,mm$, then spring constant, $S_1 = \frac{F}{\delta} = \frac{3EI}{L^3}$ with $L = 1\,mm$

$\therefore S = 0.702\ Nm^{-1}$

and $f_{res} = \frac{1}{2\pi}\sqrt{\frac{S}{m}} = 2.62\ kHz$

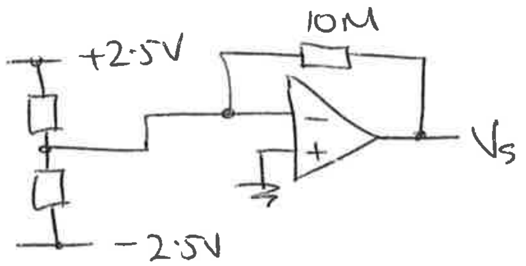
Q

Tip movement @ resonance $\simeq 80 \times$ static case

$$\delta = \frac{WL^3}{3EI} \times 80 = \frac{100 \times 10^{-12} \cdot (1.5 \times 10^{-3})^3}{3 \cdot 150 \times 10^9 \cdot 1.56 \times 10^{-21}} \times 80$$

$$= 3.85 \times 10^{-8}\,m = 38.5\,nm$$ [20%]

(iii)




Ignoring shot + flicker noise

$V_n = \sqrt{4kTRB}$, $B = 2620\ Hz$

$G = \frac{10 \times 10^6}{10 \times 10^3 / 2} = 2000$

$T = 300K$

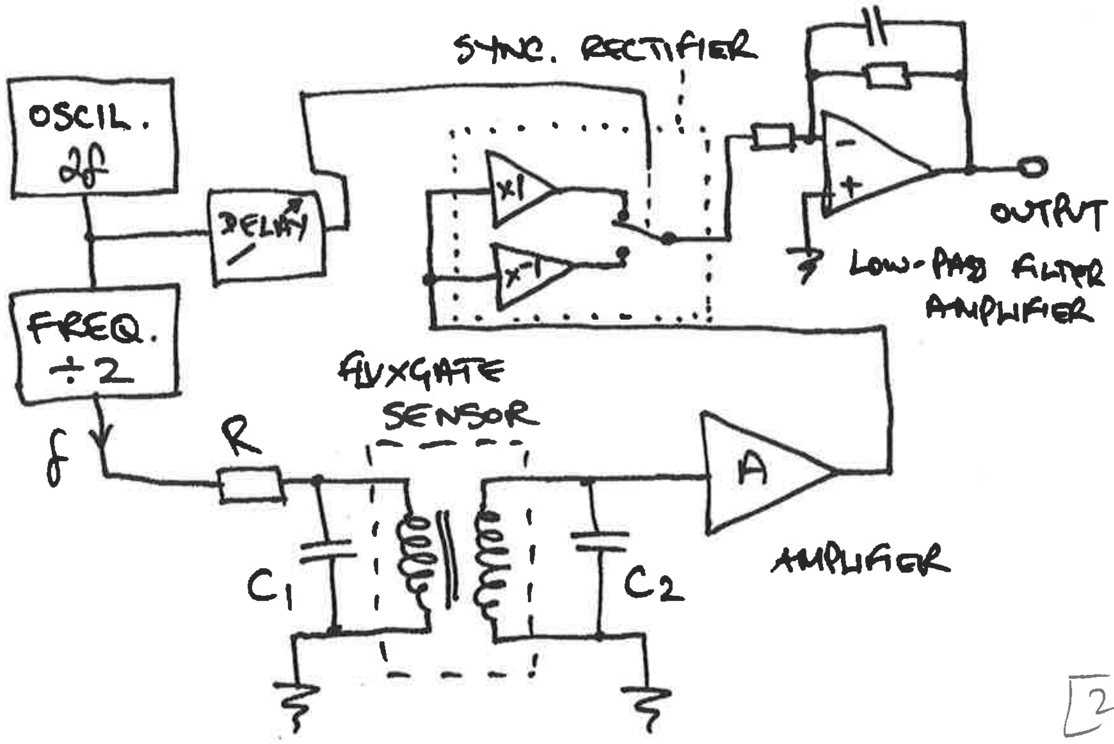
$k = 1.38 \times 10^{-23}$

| Source | $V_n$ ($\mu V_{rms}$) | GAIN | $V_n$ ($\mu V_{rms}$) |
|---|---|---|---|
| S/gauges, $R_s$ | 0.466 | 2000 | 931 |
| op-amp, $V_n$ | 0.256 | 2000 | 512 |
| Fback, R | 20.83 | 1 | 20.8 |
| op-amp, $i_n \times R_s$ | 0.05 | 2000 | 100 |

$\sqrt{\Sigma V_n^2}$  1067 $\mu V_{rms}$ $= 1.07\ mV_{rms}$

(about 0.5 $\mu V$ referred to input) [20%]

3 (a)




[25%]

(b) The flux in the core is given by $B_{core} \times A_{core} = \phi$

with a demagnetising factor of $D$, $B_{core} = B/D$

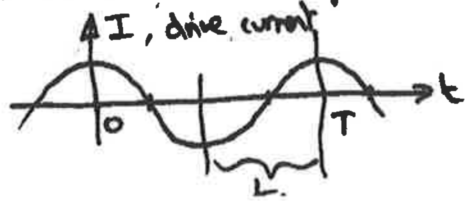
(effective relative permeability $\approx 1/D$)

$\therefore \ \phi = \frac{\pi d^2}{4} \cdot \frac{B}{D}$ and $D = \left(d/l\right)^2 \left[\ln \frac{2l}{d} - 1\right]$

for core of diameter $d$ and length $l$.

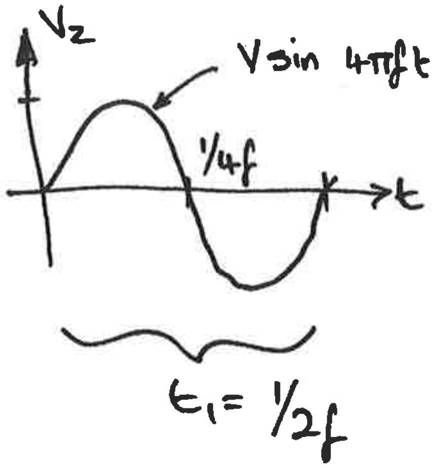
When driven by a gating current at frequency $f$, the core is saturated twice per cycle (once +ve and once −ve), hence the external field is switched in or out of the core 4 times.

Now $V_2 = N \frac{d\phi}{dt}$ for $N$ turns around the core, so if we take the time to de-saturate the core from −ve to saturation +ve to be $t_1$:



3(b) cont'd.

$t_1 = \frac{1}{2f}$ and during this time, the external field is switched into and back out of the core - causing a +ve and -ve voltage signal :-




If we assume the waveform is a sine wave and integrate a ½ cycle,

$$\int V_2 \, dt = N\phi$$

$$\therefore \quad N\phi = \int_0^{1/4f} V \sin 4\pi f t \, dt = \left[ -\frac{V \cos 4\pi f t}{4\pi f} \right]_0^{1/4f} = \frac{V}{2\pi f}$$

$$\therefore \quad V_2 = 2\pi f N\phi = 2\pi f N \frac{\pi d^2}{4} \frac{B}{D} = \frac{\pi^2 f N B L^2}{2\left(\ln \frac{2L}{d} - 1\right)}$$

Then $\times \frac{2}{\pi}$ to convert to d.c. with PSD.

$$V_0 = \frac{\pi f N B L^2}{\left(\ln \frac{2L}{d} - 1\right)}$$

[25%]

(c) The self-inductance is given by $L = \frac{N\phi}{I}$, for a current $I$ in the coil. The $H$ field in the coil is given by $H = \frac{NI}{L}$ and as $B = \mu H$ with $\mu = \mu_0 / D$ for the core,

$$\phi = \frac{\pi d^2}{4} . B = \frac{\pi d^2}{4} \frac{NI}{L} \frac{\mu_0}{D} \quad \therefore \quad L = \frac{\pi d^2}{4} \frac{N^2}{L} \frac{\mu_0}{D}$$

$$\Rightarrow L = \frac{\pi L N^2 \mu_0}{4\left(\ln \frac{2L}{d} - 1\right)}$$

3(c) contd.

$$L = \frac{\pi \cdot 0.03 \cdot 600^2 \cdot 4\pi \times 10^{-7}}{4\left(\ln 300 - 1\right)} = \underline{2.27\,mH}$$

If the drive is 30kHz, we want resonance at 60kHz

$$\therefore\ 60 \times 10^3 = \frac{1}{2\pi\sqrt{LC}}$$

$$\therefore\ C = \frac{1}{4\pi^2 \cdot (60\times10^3)^2 \cdot 2.27\times10^{-3}} = \underline{3.10\,nF}$$

[35%]

(d) $$V_2 = \frac{\pi^2 f N B L^2}{2\left(\ln\frac{2L}{d} - 1\right)} \times Q \qquad \text{where } Q = \frac{\omega L}{r} = 53.5$$

Hence d.c. voltage is $V_2 \times \frac{2}{\pi}$ for response of sync. detector included. So, with $B = 500\,nT$,

$$V_S = \frac{2}{\pi} \times V_2 = \frac{2}{\pi} \times \frac{\pi^2 \cdot 30\times10^3 \cdot 600 \cdot 500\times10^{-9} \cdot (30\times10^{-3})^2 \cdot 53.5}{2\left(\ln 300 - 1\right)}$$

$$= \underline{0.289\ V}$$ without op-amp. gain in low-pass filter @ end.

[15%]

4.(a) $t = \frac{(10+10)\text{ m}}{1550 \text{ m/s}} = \underbrace{12.90\text{ ms}}$ round trip pulse-echo time

$Z_{PZT} = 7500 \times 4000 = 30$ M Rayls.
$Z_{sea} = 1050 \times 1550 = 1.63$ M Rayls.

Power coupling PZT $\rightleftarrows$ sea water $= \frac{4 Z_{PZT} Z_{sea}}{(Z_{PZT} + Z_{sea})^2}$

$= 0.196$

$P_{electrical} = \frac{V^2}{R} = \frac{120^2}{250} = 57.6$ W

e-m conversion $\times 0.1 = 5.76$ W
couple to water $\times 0.196 = 1.13$ W
isotropic back scattered power ($\frac{1}{2}$ sphere) $\times 0.2 = 0.226$ W
$20 \times 1.5$ dB attenuation (both ways) $\times -30$dB $= 0.226$ mW
fraction collected by transducer
$\frac{\pi D^2/4}{2\pi R^2} = \times 3.125 \times 10^{-6} = 7.06 \times 10^{-10}$ W

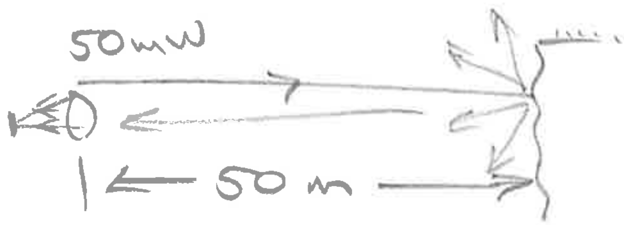
e-m conversion $\times 0.1 = 7.06 \times 10^{-11}$ W
couple back from water $\times 0.196 = 1.38 \times 10^{-11}$ W

$= \frac{V_r^2}{R} = \frac{V_r^2}{250} \quad \therefore V_r = 58.8\ \mu$V

(loaded)

or $2 \times 58.8\ \mu V = \underbrace{0.118\text{ mV}}$ [40%]

(b)




(i) $i_s = P_{laser} \times \frac{\frac{\pi D_{lens}^2}{4}}{2\pi \text{Range}^2} \times 2 \times 0.7 \times 0.15 = \underbrace{1.31 \text{ nA}}$

A/W, reflectivity, Lambertian

isotropic scatter in hemisphere [15%]

4(b) contd.

(ii) $E = \frac{hc}{\lambda}$ ← photon energy @ 780nm $\Rightarrow E = \frac{6.626\times10^{-34} \cdot 3\times10^{8}}{780\times10^{-9} \cdot 1.6\times10^{-19}}$ ← for eV

$= 1.59$ eV

$\therefore$ quantum efficiency for 1 photon = 1 electron $\Rightarrow \frac{1}{1.59} = 0.628$ A/W

$\therefore$ 0.7 A/W has q.e. of $\frac{0.7}{0.628}\times100\% = 111\%$

(ie: the photodiode has gain)

For a minimum detectable photon power of $4\times10^{-14}$ W/$\sqrt{Hz}$:

with 100 readings/sec $\Rightarrow$ 100 Hz say.

$4\times10^{-13}$ W is the min. power detectable.

$$\therefore \underset{\text{min power}}{4\times10^{-13}} = \underset{\text{laser power}}{50\times10^{-3}} \cdot \underset{\text{lens area / hemisphere area}}{\frac{\frac{\pi(50\times10^{-3})^2}{4}}{2\pi R^2}} \cdot \underset{\text{reflectivity}}{0.15} \cdot \overset{\text{Lambertian over isotropic}}{2} \cdot \underset{\text{optical atten.}}{10^{-\frac{3}{10000}\cdot 2R}}$$

re-arrange: $R^2 = 11.72\times10^{6}\ \underbrace{10^{-6R/10000}}_{\text{if applic.}}$

solve by iteration or directly on a clear day:

ie: $R = \sqrt{11.72\times10^{6}} = 3.42$ km clear day

try: R = 3000 in RHS. LHS: R = 431 : too small

2000 in RHS LHS: R = 860 : — " —

1000 in RHS. LHS: R = 1715 : too large

iterate to solve!

1500 in RHS ← 1215

1215 ← 1479 take avg. + iterate

1232

1347 1350 ← ✓

$\therefore$ range $\triangleq$ 1350 m

[45%]