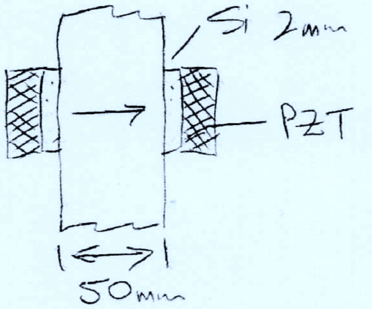
4B13 CRIB 2021

1(a)



$$t_{plset} = t_{Si} + \frac{50\times10^{-3}}{1300} = 38.5\,\mu s + t_{Si}$$

$$t_{full\,set} = t_{Si} + \frac{50\times10^{-3}}{1600} = 31.3\,\mu s + t_{Si}$$

$$\therefore \Delta t = 7.2\,\mu s$$

(b)

$Z_{plset} = \rho v = 900\times1300 = 1.17$ MRyls

$Z_{full\,set} = 900\times1600 = 1.44$ MRyls

$Z_{PZT} = 7500\times400 = 30$ MRyls

$Z_{Si} = 1200\times1800 = 2.16$ MRyls

Coupling coeff. $= \frac{4Z_1Z_2}{(Z_1+Z_2)^2}$ $\therefore$ $PZT \to Si = 0.251$

$Si \to$ Resin plset $= 0.91$

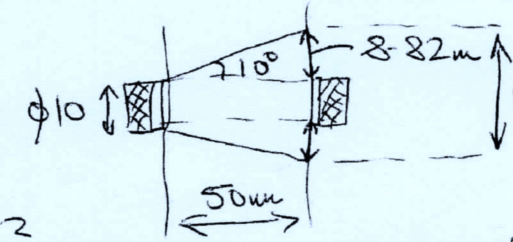
$Si \to$ Resin fullset $= 0.96$

Attenuation : Si : $\times 10^{-\frac{7\times2\times10^{-3}}{10}} = \times 0.997$

Resin plset : $\times 10^{-\frac{25\times50\times10^{-3}}{10}} = \times 0.75$

Resin full set : $\times 10^{-\frac{18\times50\times10^{-3}}{10}} = \times 0.81$

Geometry of beam:



$\phi = 10+2\times8.82$mm
$= 27.6$mm

$$\therefore \frac{A_{transducer}}{A_{beam}} = \left(\frac{10}{27.6}\right)^2 = 0.13$$ (proportion of beam captured) [or can use 52mm spacing]

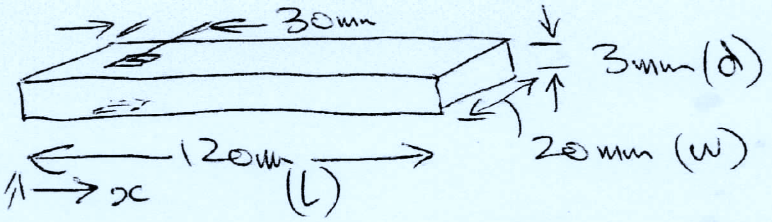
with 20V drive, $P_t = V^2/R \cdot \eta = 0.4W$

(i) $P_{recd,\,plset} = 0.4\times0.251\times0.997^2\times0.91^2\times0.75\times0.13 = 2.01\times10^{-3}\,W = \frac{V_r^2}{R}$

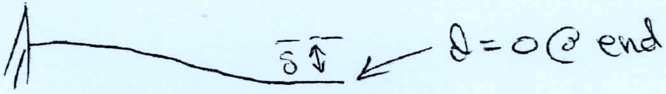
with $R = 250\,\Omega$, $V_r = 0.71V_{loaded} \Rightarrow 1.42V_{open\,cct.}$

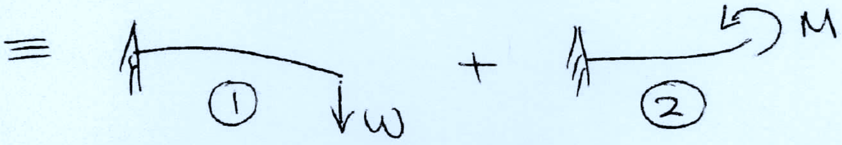
(ii) $P_{recd,\,full\,set} = 2.42\times10^{-3}W \Rightarrow V_r = 0.78V_{loaded} \Rightarrow 1.56V$ open cct.

1(c)



Beam ends are clamped for no rotation:



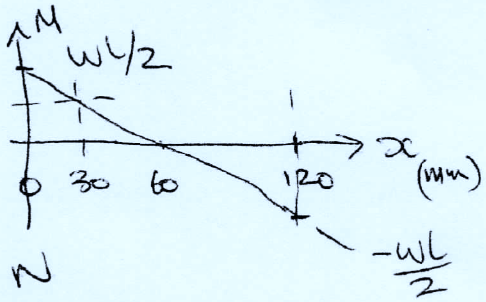


① $\theta = \dfrac{wL^2}{2EI}$, $\delta = \dfrac{wL^3}{3EI}$ $\qquad I = \dfrac{bd^3}{12} = 4{\cdot}5 \times 10^{-11}\ \mathrm{m}^4$

② $\theta = \dfrac{ML}{EI}$, $\delta = \dfrac{ML^2}{2EI}$

For zero end rotation $\dfrac{wL^3}{3EI} = \dfrac{ML}{EI}$ $\therefore$ $\underline{M = \dfrac{wL}{2}}$

∴ Bending moment along beam :



∴ @ strain gauges $M = \dfrac{wL}{4}$

$\dfrac{\sigma}{y} = \dfrac{\epsilon E}{y} = \dfrac{M}{I}$ with $w = 200$ N

E stainless steel ≃ 200 GN/m²

$y = d/2 = 1{\cdot}5 \times 10^{-3}$ m , $L = 0{\cdot}12$ m

$\therefore \dfrac{200 \times 10^{9}\,\epsilon}{1{\cdot}5 \times 10^{-3}} = \dfrac{200 \times 0{\cdot}12}{4 \times 4{\cdot}5 \times 10^{-11}} \Rightarrow \epsilon = 0{\cdot}001$ (0·1%)

∴ For full-bridge output: $\Delta V = V_s . \epsilon . G.F.$
(all 4 gauges under strain) $\underline{\Delta V = 10 . 0{\cdot}001 . 2 = 0{\cdot}02\,V}$ (20mV)

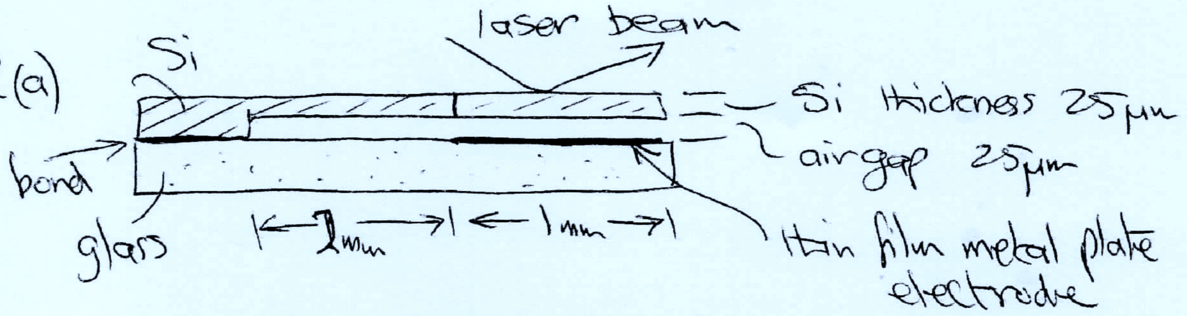
With $M = \dfrac{wL}{4}$, end deflection $\delta = \dfrac{wL^3}{3EI} - \dfrac{wL}{2} \cdot \dfrac{L^2}{2EI}$

$\delta = \dfrac{wL^3}{12EI} = \dfrac{200 . 0{\cdot}12^3}{12 . 200 \times 10^9 . 4{\cdot}5 \times 10^{-11}}$

$\underline{= 3{\cdot}2\ \mathrm{mm}}$

(note ε @ ends = 0·2%, just linear still for some materials)

2(a)

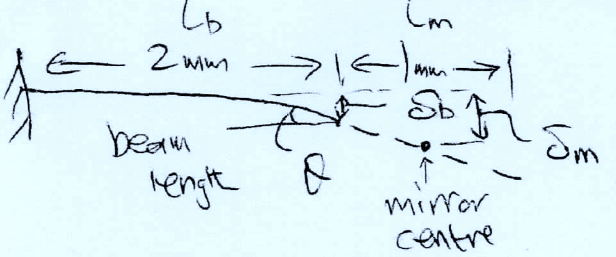


- photolithography :- deposit photoresist onto surfaces (+ve)
  - expose to UV light where it is to remain
  - develop photoresist to leave patterns
- etching : exposed areas can be etched chemically eg: metals to leave patterns
- deposition : to etch Si, SiN is first deposited by plasma enhanced CVD. Photoresist is applied and plasma etching with CF_4 etches patterns. These act as etchant barriers to wet chemical etch of crystalline Si in hot KOH solution. Boron doping into Si can also provide etch-stop to leave thin membrane eg: 25μm

To make structure above, thin Si wafer to 25μm by KOH etching with B-doped stopper. Then pattern beam and mirror profiles with plasma etching or wet etching with SiN patterns. Electrode is deposited in AL by evaporation or sputtering and patterned with photolith and chemical etching onto the glass. Si and glass anodically bonded by heat, pressure & applied DC voltage.

(b) $$C = \frac{A\epsilon_0}{d} = \frac{(10^{-3})^2\ 8.854\times10^{-12}}{25\times10^{-6}} = \underline{0.354\,\text{pf}}$$

(c)



$$\delta = \frac{WL_b^3}{3EI}, \quad \theta = \frac{WL_b^2}{2EI}$$

2(c) contd. Spring constant @ centre of mirror: need to include effects of mirror length. $L_b = 2\times10^{-3}$ m, $L_m = 10^{-3}$ m

$$\delta_m = \delta_b + \frac{\theta L_m}{2} = \frac{WL_b^3}{3EI} + \frac{WL_b^2}{2EI}\frac{L_m}{2}, \quad L_m = \frac{L_b}{2}$$

$$\therefore \delta_m = \frac{W}{EI}\left[\frac{L_b^3}{3} + \frac{L_b^3}{8}\right] \text{ and spring constant } k = \frac{W}{\delta_m}$$

$$\therefore k = \frac{EI}{L_b^3}\cdot\frac{24}{11} \qquad E_{Si} = 150\times10^9 \text{ N/m}^2$$

$$I = \frac{1}{12}\underset{\underset{0.1\times10^{-3}}{\uparrow}}{b}\underset{\underset{25\times10^{-6}}{\uparrow}}{d}^3 = 1.302\times10^{-19}\text{ m}^4$$

$$\therefore \underline{k = 5.33 \text{ N/m}}$$

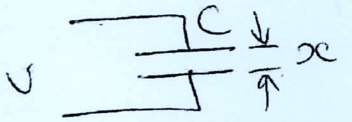
$$m = \rho A d = 2330 . 10^{-6} . 25\times10^{-6} = 5.825\times10^{-8}\text{ kg}$$

$$f_{res} = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = 1522 \text{ Hz}$$

(d) $$0.5° \text{ deflection} = \frac{0.5\times\pi}{180} \text{ rads} = \frac{WL_b^2}{2EI} = 8.73\times10^{-3} \text{ rads.}$$

$$\therefore W = 8.52\times10^{-5}\text{ N} \quad \therefore \delta_m = \frac{W}{k} = 16\,\mu\text{m} \therefore \text{airgap} = 25-16 = \underline{9\,\mu\text{m}}$$

$$C = \frac{A\epsilon_0}{\underset{\leftarrow 9\mu m}{x}} = \underline{0.98 \text{ pF}}$$



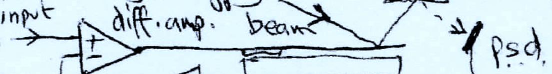
$$E = \tfrac{1}{2}CV^2 \qquad F\delta x = \delta E = \tfrac{1}{2}V^2\delta C$$

$$\therefore F = \frac{1}{2}V^2\frac{dC}{dx} = -\frac{1}{2}V^2\frac{A\epsilon_0}{x^2}$$

$$\therefore \text{with } x = 9\,\mu\text{m},\ F = 8.52\times10^{-5}\text{ N} \Rightarrow \underline{V = 40.2\text{ V}}$$

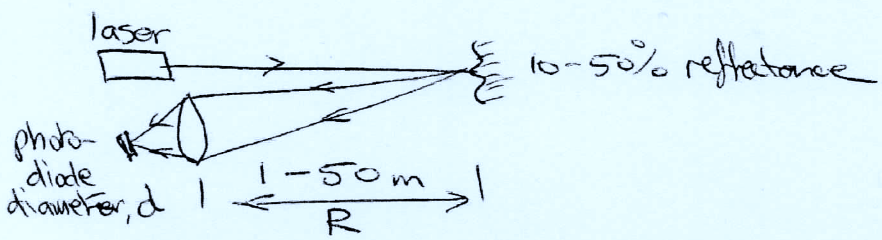
Note: for simplification, we have ignored additional beam end moment applied as a result of ½ mirror length and have assumed even force over mirror area (would need tilted metal plate to achieve).

e) For position feedback, we could split partial beam power with a semi-mirror and detect with a position-sensitive diode to give beam scan position. This can be fed back to control the drive voltage: can improve linearity, apply electronic damping and improve transient response.



Strain gauges diffused into beam flexure point could also be used.

3(a)



(i) $\text{Min. flight time} = \dfrac{2}{3\times10^{8}} = 6.67\,\text{ns}$

$\text{Max. flight time} = \dfrac{100}{3\times10^{8}} = 333\,\text{ns}$

(ii) Max. photo-current : range = 1m
reflectance = 50%

Min. photo-current : range = 50m
reflectance = 10%

$$i = \frac{\pi \overset{20\times10^{-3}}{d}^{2}}{4}\cdot\frac{1}{2\pi R^{2}}\cdot\overset{0.8}{\eta_{quant}}\cdot\overset{10^{-2}}{P_{laser}}\cdot\eta_{reflect}\cdot\frac{\overset{q}{1.6\times10^{-19}}}{hc/\lambda}$$

$h = 6.626\times10^{-34}$, $c = 3\times10^{8}$, $\lambda = 650\times10^{-9}$

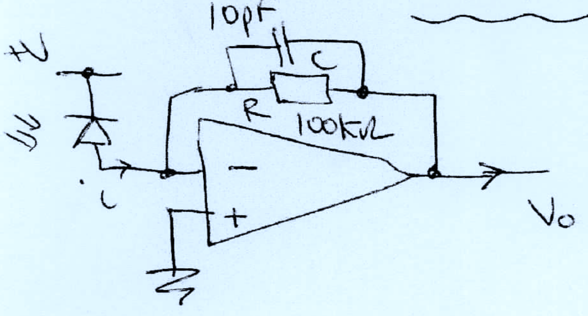
photon energy $3.06\times10^{-19}\,\text{J}$

$$\therefore\ i = \frac{\eta_{refl.}}{R^{2}}\times 2.09\times10^{-7}\,\text{A}$$

$\therefore\ \text{max.} = 1.04\times10^{-7}\,\text{A}$

$\text{min.} = 8.36\times10^{-12}\,\text{A}$

(iii)



$\text{Bandwidth}, f = \dfrac{1}{2\pi RC}$

$= 159\,\text{kHz}$

$\sqrt{f} = 399\,\sqrt{\text{Hz}}$

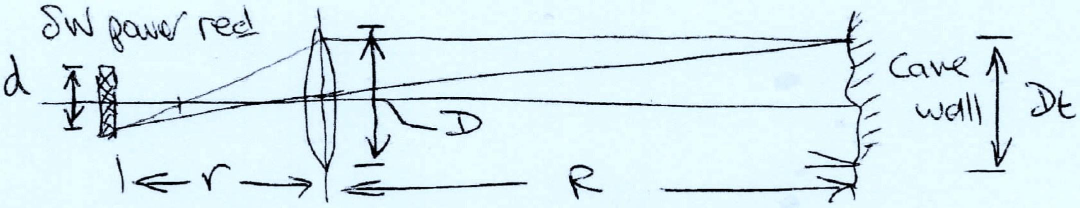
noise source	value	v_n, noise voltage (rms)
resistor, thermal	$\sqrt{4kTRf}$ =	$16.2\,\mu\text{V}$
op-amp, current	$1.4\times10^{-12}\times399\times10^{5}$ =	$55.9\,\mu\text{V}$
	$\left(\Sigma v_n^{2}\right)^{1/2}$	$58.2\,\mu\text{V} = N$

$\text{min. signal} = 8.36\times10^{-12}\times10^{5} = 0.836\,\mu\text{V} = S$

$\therefore\ S/N \text{ ratio} = 0.014\ (-37\text{dB})$

Hence, averaging reqd to reduce bandwidth and increase S/N.

3(b)(i)



Lambert's Law:

$$\delta W = \frac{W \overset{1}{\cos\theta}}{\pi} A\, \delta\omega = \frac{W}{\pi}\,\frac{\pi D_t^2}{4}\,\frac{\pi D^2}{4R^2} \quad \text{as } \delta\omega = \frac{\frac{\pi D^2}{4}\cdot\cancel{4\pi}}{\cancel{4\pi} R^2}$$

and with $\frac{D_t}{R} = \frac{d}{r}$ by similar triangles:

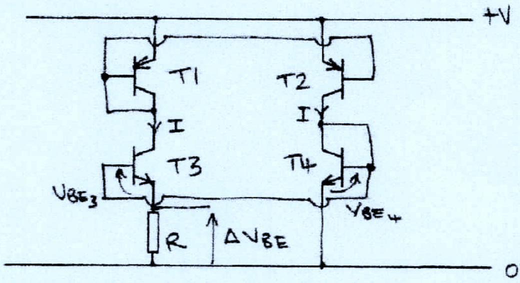
$$\delta W = \frac{W}{\pi}\,\frac{\pi^2}{16}\cdot\frac{d^2}{r^2}\cdot D^2 = \epsilon\sigma_{SB}T^4\,\frac{\pi}{16}\,\frac{d^2D^2}{r^2}$$

$$\text{So, } \delta W = 0.9 \times 5.67\times10^{-8} \times (273+30)^4 \times \frac{\pi}{16} \times \frac{(20\times10^{-3})^2(10^{-3})^2}{(50\times10^{-3})^2}$$

$$\delta W = 13.5\ \mu W \ @\ 0.15\ A/W \rightarrow i_{ph} = 2.03\ \mu A$$

(ii) Choices for long wavelength detection are bolometer (thermocouples in series) or pyroelectric detectors using a chopped source. Bolometers with Si thermocouples are often used. High thermal constant ~100's K/W are possible with small, thin film devices and with many hundred elements in series, the voltage signal can be quite reasonable. eg: with Seebeck coeff. of 1000μV/K and 250 K/W 13 μW incident IR power would give $V_{sig} \simeq 3\,\mu V$ per element ×100 say = 0.3 mV signal amplitude.

(c)



It is a pair of current mirrors connected together. T1 and T2 are matched transistors which source equal current I to T3 and T4. T3 is arranged to have a collector area of a fixed ratio larger than T4 such that its current density is lower (by a factor of r).

3(c) contd.

Then $J_{C_3} = J_{S_3} e^{V_{BE3}/V_t}$ and $J_{C_4} = J_{S_4} e^{V_{BE4}/V_t}$

J = current density, A/m^2

$I_{C3} = I_{C4}$ ∴ $r J_{C3} = J_{C4} = J$ and $J_{S_4} = J_{S_3} = J_S$

material property ↗
- not geometry dependant

∴ $r \cancel{J_s} e^{V_{BE3}/V_t} = \cancel{J_s} e^{V_{BE4}/V_t}$

∴ $\frac{kT}{q} \ln r + V_{BE3} = V_{BE4}$ as $V_t = \frac{kT}{q}$

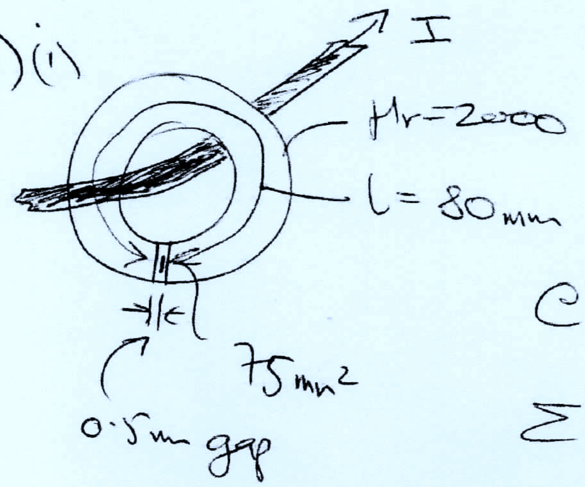
$\Rightarrow \Delta V_{BE} = \frac{kT}{q} \ln r$ voltage seen @ output across R

So for $\Delta V_{BE} = 0.1 \times 10^{-3} . T$ we need $\frac{k \ln r}{q} = 10^{-4}$

∴ $\ln r = 1.159$ and $r = 3.188$ (area ratio of transistors).

3

4(a)(i)



@ full power I = 300A

$$\Sigma H.dl = I$$

$$\therefore \; 80\times10^{-3} H_m + 0.5\times10^{-3} H_g = I$$

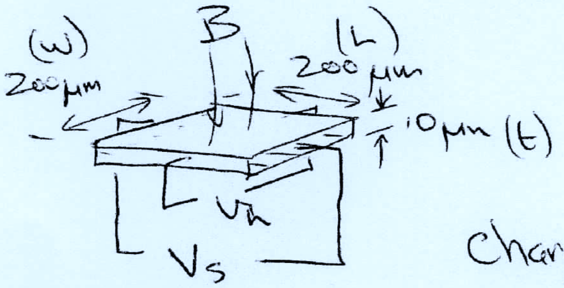
B = μH where $B_m = B_g$ and so:

$$80\times10^{-3}\frac{B}{\mu_0 . 2000} + \frac{0.5\times10^{-3}B}{\mu_0} = I \qquad \mu_0 = 4\pi\times10^{-7}\text{ H/m}$$

$$\therefore \; 0.54\times10^{-3} B = \mu_0 I$$ so with I = 300A

$$B = 0.698\text{ T}$$ (just ok for ferrite)

(ii)



Charge balanced forces on carriers with drift velocity v_d in flux density, B.

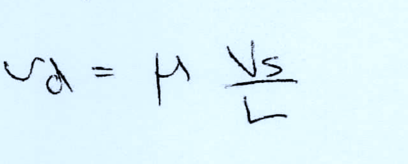
$$Bqv_d = qE = q\frac{V_h}{w}$$

$$v_d = \mu\frac{V_s}{L} \qquad \therefore \; \frac{V_h}{B} = \frac{\cancel{w}\,\overset{0.16}{\mu}\,\overset{5V}{V_s}}{\cancel{L}} = 0.8\text{ V/T}$$

responsivity

∴ output voltage @ max. power = 0.8×0.698 = 0.558V

(b)(i)



$$L = \frac{N\phi}{i}, \quad \phi = BA$$

$$B = \left(\frac{0.698}{300}\right)\times Ni$$ ← from part (a)

and $A = 75\times10^{-6}$ m²

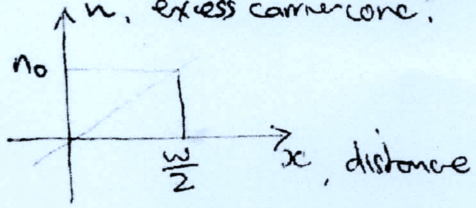
$$\therefore \; L = \underset{\uparrow\,800}{N}^2\,\frac{0.698}{300}\,75\times10^{-6} = 0.112\text{ H}$$

4(b)(ii) when voltage = 850V, $i = \frac{850}{10^5} = 8.5\,\text{mA}$ and with 800 turns $\Rightarrow$ 6.8 Amp-turns. ($\rightarrow$ 15.8 mT)

$$\therefore V_h = \left(\frac{0.698}{300}\right) \times 6.8 \times 0.8 = 12.7\ \text{mV}$$

(iii)

For response bandwidth consider carrier diffusion back across slice centre line - assuming linear excess carrier conc. for simplicity:



$n = \frac{2n_0}{w} x$

Fick's Law

$$F = -D\frac{dn}{dx}, \quad D = \frac{\mu kT}{q} = 4.14\times10^{-3}$$

@ 300k

$$\frac{dn}{dx} = \frac{2n_0}{w}$$

N = total excess carriers one side $= Ld\int_0^{w/2} \frac{2n_0}{w} x\,dx = \frac{n_0 w L d}{4}$

Consider 1 side: $\frac{dN}{dt} = FLd = -D\frac{2n_0}{w}Ld$ where $n_0 = \frac{4N}{wLd}$

$$\therefore \frac{dN}{dt} = -D\frac{2}{w}\frac{4N}{wLd}Ld = -\frac{8DN}{w^2}$$

Soln. of form $N = N_0 e^{-t/\tau}$ where $\tau = \frac{w^2}{8D} = 1.21\,\mu\text{s}$

$t_{rise} \simeq 2.2\tau = \frac{(200\times10^{-6})^2\, 2.2}{8\times4.14\times10^{-3}} = 2.66\,\mu\text{s}$ $\quad f_{-3dB} = \frac{1}{2\pi\tau} = 132$ kHz

Hall sensor

Hall sensor only

Compare this to L/R time constant of resistance and inductance

$L = 0.112\,\text{H}$, $R = 10^5\,\Omega$ $\therefore L/R = 1.12\,\mu\text{s}$

So, the time constants are very similar, with Hall sensor slightly dominant. Overall, the rise time will be $t_r \approx \sqrt{1.21^2 + 1.12^2} \times 2.2 \approx 3.6\,\mu\text{s}$

and overall $f_{-3dB} \approx 96$ kHz vs. 132 kHz for Hall only.

(actually 87 kHz by LTSPICE analysis) $\approx 3.9\,\mu\text{s}$

Examiner’s comments:

Q1 Ultrasonic testing and strain sensing

A popular and fairly straightforward question, well-answered by most candidates. Coupling coefficients and signal amplitudes were generally well attempted, although taking account of beam spread was more variable in standard. The final part on strain in a beam was also generally well attempted but the formula employed did not always take into account the non-tilting constraint of the beam ends.

Q2 MEMs fabrication and device physics

A rather unpopular question. The fabrication processes were well described in most cases although the estimate of resonant frequency and capacitance was less well answered. Deriving an accurate estimate of the cantilever spring constant defeated many.

Q3 LIDAR scanner and pyrometer

This question was attempted by nearly all candidates and was answered very well overall. The pyrometer section was quite straightforward although for the LIDAR, some candidates omitted the collection lens area and worked out the signal amplitude for direct detection by the photodiode.

Q4 Hall effect and induction current sensing

This question was quite popular and generally well done. The operation of Hall effect devices was understood by most and the calculation of flux density vs. current was correct in most cases. The effect of the L/R time constant was not often considered in determining the measurement bandwidth and the sensor inductance was sometimes also incorrectly calculated.