## EGT3 ENGINEERING TRIPOS PART IIB

Wednesday 4 May 2022 2.00 to 3.40

## Module 4B19

## **RENEWABLE ELECTRICAL POWER**

Answer not more than three questions.

All questions carry the same number of marks.

The *approximate* percentage of marks allocated to each part of a question is indicated in the right margin.

Write your candidate number <u>not</u> your name on the cover sheet.

### STATIONERY REQUIREMENTS

Single-sided script paper

### SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM

CUED approved calculator allowed Attachment: 4B19 data sheet (2 pages) Engineering Data Book

10 minutes reading time is allowed for this paper at the start of the exam.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

You may not remove any stationery from the Examination Room.

1 (a) Explain the concept of 'diversity of supply', and how it enables greater integration of renewable electrical power sources into the grid. [5%]

(b) The North Sea link is a 1.4 GW high voltage direct current (HVDC) link that connects the UK to Norway. Illustrate your answer to part (a) by explaining the benefits of this link, referring to the availability of 60 GW of Nordic hydroelectric power. Also explain why HVDC is preferred to three-phase ac as the means of transmitting the power. [10%]

(c) Explain the principles of pumped storage schemes, and how they enable greater integration of renewable sources into the electrical supply system. [5%]

(d) A pumped storage scheme is capable of producing peak electrical output power of 400 MW. The resevoir is situated 400 m above the turbines, and the system is 75% efficient when both generating and pumping. The scheme uses four identical turbine-generators. The generators are salient-pole synchronous machines which are all star-connected directly to the 11 kV, 50 Hz three-phase grid. They have equivalent circuit parameters  $X_d = 1.1 \Omega$  and  $X_q = 0.75 \Omega$ .

(i) Determine the peak volumetric flow rate of the system, and the volume of water stored in the resevoir if peak output can be maintained for 24 hours. You may quote the result  $P = \eta \rho g H Q$  without proof. [10%]

(ii) The turbine-generators always rotate at 250 rpm. Explain why they are fixed speed, and determine the number of poles of the salient-pole generators. [10%]

(iii) Draw a phasor diagram of the salient-pole synchronous machine assuming generating operation at peak output power, and 0.8 lagging power factor. Hence find the phase current, excitation voltage and the load angle of the generators under these conditions. [35%]

(iv) Draw a phasor diagram of the salient-pole synchronous machine assuming pumping operation at peak output power and unity power factor. No calculations are necessary.

(v) Find the volumetric flow rate of the water being pumped up to the resevoir when the system is operating as a pump at rated power, and explain why this figure is lower than that obtained when generating at rated power. [10%]

2 (a) Define the terms 'tip-speed ratio' and 'power coefficient'. Sketch a typical power coefficient vs tip-speed ratio characteristic, and hence explain why variable speed wind turbines are preferred to fixed speed ones. [10%]

(b) A wind turbine generator system utilises a turbine of blade diameter 94 m. It is operated as a variable speed system between wind speeds of 8 m s<sup>-1</sup> and the rated wind speed of  $12 \text{ m s}^{-1}$ . Between these wind speeds the turbine is controlled to be at its optimum tip-speed ratio of 8, at which the power coefficient is 0.4. For wind speeds below 8 m s<sup>-1</sup> the system operates at the angular speed corresponding to the wind speed of 8 m s<sup>-1</sup>. For wind speeds above  $12 \text{ m s}^{-1}$  the system operates at the angular speed corresponding to the wind speed of 8 m s<sup>-1</sup>.

(i) Determine the turbine output power, torque and angular speeds corresponding to the wind speeds of 8 m s<sup>-1</sup> and 12 m s<sup>-1</sup>, and sketch a graph of turbine angular speed vs wind speed. The following may be quoted without proof:  $P = 0.5C_p \rho A v^3$ ,  $\lambda = \frac{\omega R}{v}$ , and take  $\rho$  to be 1.23 kg m<sup>-3</sup>. [20%]

(ii) Explain why the system is operated as a variable speed system over only a limited range of wind speeds. [10%]

(c) The turbine of part (b) is coupled to a doubly-fed induction generator (DFIG) via a gearbox of ratio 32. The stator winding of the DFIG is star-connected to the 11 kV, 50 Hz, three-phase grid. The parameters of the DFIG are:  $R_1 = 0.15 \Omega$ ,  $R'_2 = 0.12 \Omega$ ,  $X_1 = X'_2 = 0.3 \Omega$ , the iron loss resistance and magnetising reactance are large enough to be ignored. The DFIG has 12 poles.

(i) Explain why the system uses a gearbox, and for rated wind speed determine the input torque to the DFIG, and its rotational speed and hence slip. [10%]

(ii) The DFIG is controlled such that the rotor current and slip energy converter voltage are in phase. Show that the DFIG torque may be expressed as:

$$T = \frac{3I_2' \left( \left( V_1^2 - I_2'^2 X^2 \right)^{1/2} - I_2' R_1 \right)}{\omega_s}$$

where  $X = X_1 + X'_2$ . You may quote  $T = \frac{3I'_2(I'_2R'_2+V'_3)}{s\omega_s}$  without proof. [25%] (iii) Using the result in (ii) above, and for rated wind speed only, determine the phase current of the DFIG, and hence find the referred converter voltage,  $V'_3$ . [25%]

## Version TJF/2

| n-doping                                    | $10^{25} \text{ m}^{-3}$          |
|---|-----------------------------------|
| p-doping                                    | $10^{23} \text{ m}^{-3}$          |
| Electron minority carrier lifetime $\tau_e$ | 100 µs                            |
| Hole minority carrier lifetime $\tau_h$     | 100 ns                            |
| Electron diffusion length $L_e$             | 100 µm                            |
| Hole diffusion length $L_h$                 | 25 nm                             |
| Intrinsic carrier concentration $n_i$       | $5 \times 10^{16} \text{ m}^{-3}$ |

## 3 A Si p–n junction diode at 300 K has the following parameters given in Table 1:

### Table 1

(a) The n-doping extends 1  $\mu$ m from the front surface of the p-doped wafer which is 250  $\mu$ m thick. The front surface area of the diode is 10<sup>-2</sup> m<sup>2</sup>.

(i) Calculate the built-in potential  $V_{bi}$  of the junction. [10%]

(ii) Under dark conditions, the diode is considered to be turned ON when the current through it is 10 mA. What is the corresponding turn on voltage  $V_{on}$ ? [15%]

(iii) Derive the minority carrier concentrations at the edges of the depletion regions, and sketch the minority carrier distribution in the n-doped region at turn on. [20%]

(b) When the diode operates as a photovoltaic cell under AM 1.5 (1 kW m<sup>-2</sup>) solar illumination, the overall optical generation rate can be taken as  $2 \times 10^{25}$  m<sup>-3</sup> s<sup>-1</sup>.

- (i) What is the short-circuit current  $I_{SC}$  under AM 1.5 conditions? [15%]
- (ii) Calculate the associated open-circuit voltage  $V_{OC}$ . [15%]
- (iii) Estimate the solar power conversion efficiency of the diode when operating under the above conditions, and explain why it would be expected to be lower. [25%]

4 (a) For the same photon flux, would the power conversion efficiency of a Si solar cell be higher for 900 nm or 600 nm light? Give reasons for your answer. [15%]

(b) Referring to Fig. 1, explain why the n-doping region of a solar cell is typically placed within 1  $\mu$ m of the light entry surface, and is consequently highly doped. [20%]

(c) Considering the peak power intensity of the solar spectrum is at 600 nm, determine the parameters for an anti-reflection coating on a Si solar cell placed directly in the atmosphere.

(d) Explain why the thickness of a Si wafer used in a solar cell can be reduced by roughening its back surface. [20%]

(e) Sketch the cross section of a solar cell which includes a back surface field and explain its function. [25%]

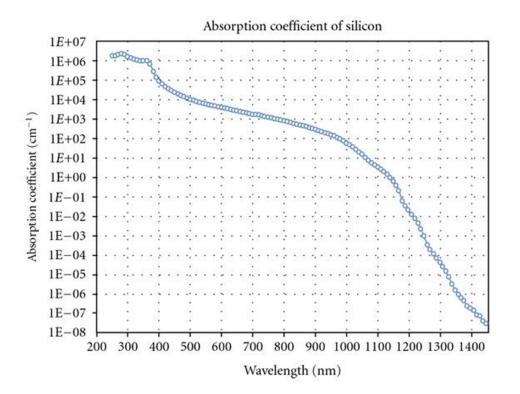


Fig. 1 The optical absorption coefficient of Si (in  $\text{cm}^{-1}$ ) as a function of the light wavelength (in nm). The intensity of the solar spectrum can be considered to be significant at wavelengths greater than 300 nm and peaks at 600 nm.

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Version TJF/2

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Short answers

1 (d) (i) 136 m<sup>3</sup> s<sup>-1</sup>, 11.7 × 10<sup>6</sup> m<sup>3</sup> (ii) 6.56 kA, 20.9 kV (line), 22.9° (v) 76.5 m<sup>3</sup> s<sup>-1</sup>

2 (b) (i) 874 kW, 642 kN m, 1.36 rad s<sup>-1</sup>, 2.95 MW, 1.45 MN m, 2.04 rad s<sup>-1</sup>

(c) (i) 45.3 kN m, 65.3 rad s<sup>-1</sup>, -0.247 (iii) -124 A, -1552 V

3 (a) (i) 0.874 V (ii) 0.503 V (iii) 6.352  $\times 10^{16}$  m^-3, 6.352  $\times 10^{18}$  m^-3

(b) (i) 3.2 A (ii) 0.653 V (iii) 17.5%

4 (c) 1.9 refractive index, 80 nm thickness

#### 4B19 RENEWABLE ENERGY: SOLAR PHOTOVOLTAIC POWER

#### Formulae and Constants – Revised 2021

**Reverse Saturation Current of a p-n junction diode** 

$$I_{S} = qA\left[\left(\frac{D_{e}}{L_{e}}\right)\frac{n_{i}^{2}}{N_{A}} + \left(\frac{D_{h}}{L_{h}}\right)\frac{n_{i}^{2}}{N_{D}}\right]$$

Where  $L_{e,h} = \sqrt{D_{e,h}\tau_{e,h}}$ 

<u>Optical generation rate in a semiconductor at distance x from the light entering</u> <u>surface</u>

$$g_{opt}(x) = \int_{\lambda_1}^{\lambda_2} \phi_0(\lambda) \alpha(\lambda) \exp(-\alpha(\lambda)x) \, d\lambda$$

Where  $\phi_0(\lambda) = (1 - R(\lambda))\phi(\lambda)$  and  $\phi(\lambda)$  is the incident photon flux at wavelength  $\lambda$  of the solar spectrum.

**<u>Reflection co-efficient</u>** at normal incidence from the third layer of a 3 layer system comprising of a thin film sandwiched between two bulk materials extending away from their interfaces with the thin film is given as:

$$R = \frac{n_2^2 (n_1 - n_3)^2 \cos^2 \vartheta + (n_1 n_3 - n_2^2)^2 \sin^2 \vartheta}{n_2^2 (n_1 + n_3)^2 \cos^2 \vartheta + (n_1 n_3 + n_2^2)^2 \sin^2 \vartheta}$$

Where  $\vartheta = \frac{2\pi n_2 d}{\lambda}$ 

 $n_1$ ,  $n_2$  and  $n_3$  are the refractive indices of 3 materials  $m_1$ ,  $m_2$  and  $m_3$  respectively: d is the thickness of the thin film  $m_2$  sandwiched between materials  $m_1$  and  $m_3$ . Direction of light flow is taken to be from  $m_1$  to  $m_3$ .

#### Fill Factor for a Si solar cell

$$FF_{o} = \frac{\frac{qV_{oc}}{kT} - \ln\left(\frac{qV_{oc}}{kT} + 0.72\right)}{\frac{qV_{oc}}{kT} + 1}$$

where  $V_{oc}$  is the open circuit voltage for the cell.

# **Constants**

| Electronic charge unit   | q: $1.602 \times 10^{-19}$ C                  |  |
|--|---|--|
| Boltzmann's Constant   | k: 1.38 × 10 <sup>-23</sup> J K <sup>-1</sup> |  |
| Speed of light   | c: $3 \times 10^8$ m s <sup>-1</sup>          |  |
| Planck's Constant  | h: 6.626 × 10 <sup>-34</sup> J s              |  |
| Dielectric permittivity free space $\epsilon_0: 8.85 \times 10^{-12} \text{ F m}^{-1}$ |   |  |
| Relative permittivity of Si ɛr: 11.9   |   |  |

Refractive index (for weakly and non-absorbing wavelengths)  $\mathbf{n} = \boldsymbol{\epsilon}_{r}^{0.5}$ 

Band-gap energy of Si: 1.12 qV