4B19 2022 Crib Dr Tim Flack, Prof Gehan Amaratunga

Q1 (a) Diversity of supply refers to being able to supply electrical power from as many different sources as possible. This helps to average out the variability of renewable energy sources that cannot be controlled, such as wind and solar. It also enables biomass and hydro to be integrated, which are highly controllable renewable energy sources. Doing this enables greater reliance on renewable energy sources and reduces the need for non-renewable sources.

[5%]

(b) The HVDC interconnector will allow the UK to import power from the Nordic countries, which have around 60 GW of hydroelectricity, which acts as a large energy store. Equally, if the UK integrates more wind energy, and there is a surplus, then it can export energy which will avoid depleting the Nordic hydro, so that it can be saved for times when other renewable sources are at low output. This is an example of diversity of supply at work. HVDC is used because of the large distance, and the problems associated with capacitive reactive power when ac is used. Furthermore, it avoids the need to synchronise the UK and Nordic grids.

[10%]

(c) Pumped storage schemes utilise cheap off peak electricity to store water at a high level reservoir (motor/pump), and then the system operates in reverse during periods of high demand to generate electricity (tubine/generator). Because they act as a large energy store, more renewables can be integrated by effectively shifting the time of production to match demand.

[5%]

(d) (i) Using $P = \eta\rho gHQ$ and putting in the numbers:

$400\times10^6 = 0.75\times1000\times9.81\times400\times Q$ giving $Q = 136\ m^3s^{-1}$

To maintain peak power output for T = 24 hours requires the resevoir to hold a volume of water given by:

$V = QT = 136\times24\times60\times60 = 11.7\times10^6\ m^3$

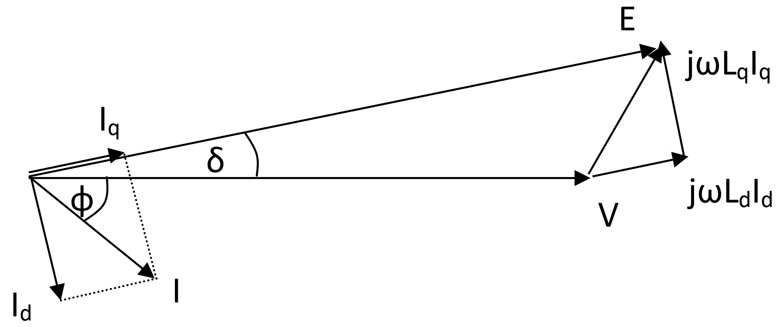
[10%]

(ii) Because the generators are salient-pole synchronous machines, and are connected to a fixed frequency supply, the entire system will be constrained to rotate at the synchronous speed of the generators, 60f/p rpm.

Therefore 60×50/p = 200 giving p=15 and so they are 30-pole generators.

[10%]

(iii) Phasor diagram is constructed remembering that for generating action E must lead V, I must lag V (lagging power factor specified), I_q is the component of the generator current resolved along E, and I_d is at right-angles to I_q such that $I_d + I_q = I$.



$\omega_s = 2\pi\times200/60 = 100\pi/12 = 20.9$ rads^{-1}.

Operating at peak power = 400/4 = 100 MW (4 identical turbine-generators)

Phase voltage is 11/√3 (star-connected) = 6350 V

$P = 3VI\cos\phi$ so $100\times10^6 = 3\times6350\times I\times0.8$ giving I = 6.56 kA

$I_q = I\cos(\phi+\delta) = I(\cos\phi\cos\delta-\sin\phi\sin\delta)$

$V\sin\delta = X_qI_q = X_qI(\cos\phi\cos\delta-\sin\phi\sin\delta)$

Divide through by cosδ and make tanδ the subject of the equation:

$\tan\delta = X_qI\cos\phi/(V+X_qI\sin\phi)= (0.75\times6560\times0.8)/(6350+0.75\times6560\times0.6) = 0.423$ giving $\delta = 22.9^0$

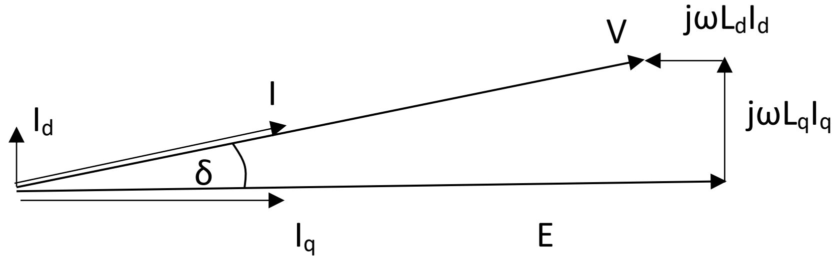
$I_d = I\sin(\phi+\delta) = 6560\sin(36.9+22.9) = 5668$ A

$E = V\cos\delta + I_dX_d = 6350\cos(22.9)+5668\times1.1= 12084$ V = 20.9 kV line

[35%]

(iv) Use motoring convention, and note that E always lags V for operation as a motor. Otherwise the phasor diagram is constructed as for a generator, bearing in mind that the governing equation is altered to:

$V = E + j\omega L_d I_d + j\omega L_q I_q$



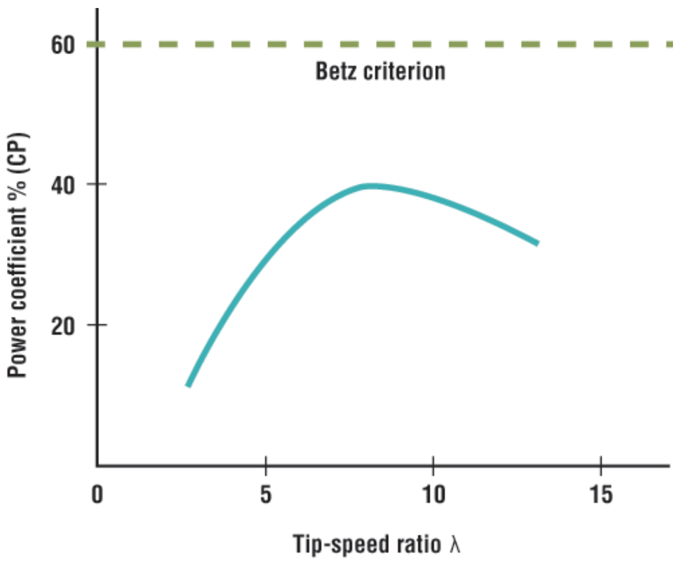
[15%]

(v) Pumping at rated output, the nett available power is 0.75×400=300 MW

This may be equated with ρgHQ giving Q = 76.5 m^3s^{-1}.

This is much less than the flow rate when generating because the losses act to reduce the power available to move the water upwards. When generating, the available gross power is 400/0.75 MW, giving 400 MW of power at the output of the turbines and hence input to the generators.

Q2 (a) Tip-speed ratio is the ratio of the speed of the tip of the turbine blade (ωR) to the wind speed, v giving λ= ωR/v. Power coefficient is the factor by which the power that can be extracted by the turbine differs from the peak theoretical power obtained by assuming that all the power is extracted.



By allowing ω to vary with v such that λ remains at its optimal value, Cp is also maximised, thereby extracting the maximum power available.

[10%]

(b) (i) Using $P=0.5C_p\rho Av3$ and putting in $C_p=0.4$, $\rho=1.23$ and $A=\pi\times94^2/4$ gives

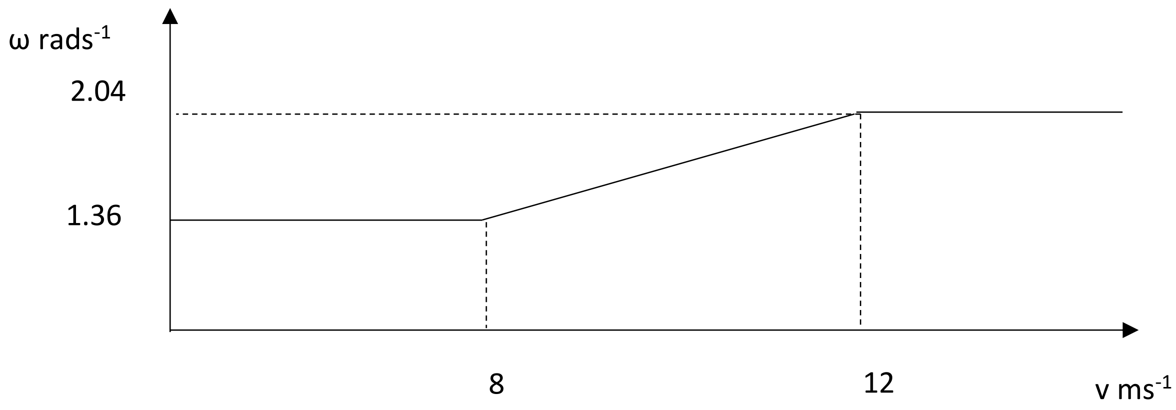
$P=1707v^3$

Rearranging $\lambda= \omega R/v$ to make ω the subject, and putting in $\lambda=8$ and $R=47$ gives $\omega=(8/47)\times v$

Finally, $T=P/\omega$

Putting in $v=8\ ms^{-1}$ gives P=874 kW, $\omega=1.36\ rads^{-1}$ and T=642 kNm

Putting in $v=12\ ms^{-1}$ gives P=2.95 MW, $\omega=2.04\ rads^{-1}$ and T=1.45 MNm



[20%]

(ii) For $v>12ms^{-1}$ the reduction in C_p that will occur because the system tip-speed ratio is no longer optimal helps to reduce the turbine power, which needs to be capped anyway to the rated power of the system. This is usually achieved by furling the blades to reduce their efficiency. For v<6 ms-1, the increased converter rating needed to continue to operate as a variable speed system is not justified by the small amount of additional power that could be extracted by allowing the turbine speed to reduce further. However, this might mean increasing the cut-in speed slightly.

[10%]

(c) (i) A gearbox increases the DFIG rotor speed which results in greater power density, hence cheaper, lighter, smaller volume and more efficient DFIG.

With a gearbox ratio of 32 the DFIG rotor speed will be 32×2.04=65.28 rads^{-1}

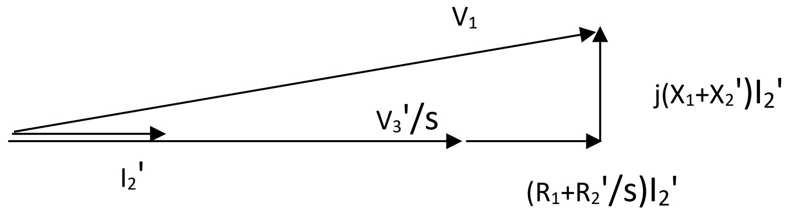
The input torque will be reduced by the gearbox ratio: 1450/32 = 45.3 kNm.

The synchronous speed is 2πf/p with f=50 Hz and p=6 giving 52.4 rads^{-1}

$s=(\omega_s - \omega_r)/\omega_s = (52.4-65.28)/52.4 = -0.247$

[10%]

(ii) Start from the given torque equation: $3\, I_2'(I_2'R_2'+V_3')/s\omega_s$



From phasor diagram: $V_1^2 = ((V_3'/s + (R_1+ R_2'/s)I_2')^2 + X^2\, I_2'^2$ where $X= X_1+X_2'$

Extract $V_3'/s + (R_2'/s)I_2'$ from this as $(V_1^2-I_2'^2X^2)^{1/2}- I_2'R_1$

Substitute into the given torque equation gives the result:

$T = 3I_2'((V_1^2-I_2'^2X^2)^{1/2}- I_2'R_1)/\omega_s$

[25%]

(iii) We have T=-45.3 kNm, V_1=11kV/√3, ω_s=52.4 rads^{-1}, X=0.6 Ω and R_1=0.15 Ω

Rearrange the torque equation to: $\omega_sT/3+I_2'^2R_1= I_2'(V_1^2-I_2'^2X^2)^{1/2}$

Square both sides and rearrange into a quadratic in I'^2_2. Note that there are some approximations that can be made: $I_2'^2R_1 \ll \omega_sT/3$ and can be ignored. Then, in the quadratic formula, '$b^2 \gg 4ac$' and so the binomial theorem can be used to approximate the solution to '-c/b'. Doing all of this leads to I_2'=-124 A.

To find V_3', rearrange $T=3I_2'(I_2'R_2'+V_3')/s\omega_s$

$V_3'=s\omega_sT/(3I_2')- I_2'R_2'$

Putting in the numbers gives V_3'=-1552 V

[25%]

Q3 (a)

i) $V_{bi} = \frac{KT}{q}\, ln\left(\frac{N_D N_A}{n_i^2}\right) = 0.026 ln\left(\frac{10^{25}.10^{23}}{25.10^{32}}\right) = 0.874\,V$

ii) The current – voltage equation for a diode

$I = I_S\left(exp\left(\frac{qV}{kT}\right) - 1\right)$

Hence

$$V = \frac{KT}{e}\ln\left(\frac{I}{I_S} + 1\right)$$

From data sheet

$$I_S = qA\left[\left(\frac{D_e}{L_e}\right)\frac{n_i^2}{N_A} + \left(\frac{D_h}{L_h}\right)\frac{n_i^2}{N_D}\right]\ and\ L_{e,h} = \sqrt{D_{e,h}\tau_{e,h}}$$

Substituting values and noting $N_D >> N_A$

$$I_S \approx q.10^{-2}\left[\frac{25.10^{32}}{10^{23}}\right] = 40\,pA$$

Substituting for I_s and I = 10mA in eqn. for V above

$$V = 0.026\ln\left(\frac{10^{-2}}{40.10^{-12}} + 1\right) = 0.503\,V$$

Is the turn-on voltage V_{ON}

iii) The minority peaks are given as:

In – n doped side

$p(0) = p_{no}\exp\left(\frac{qV_{ON}}{KT}\right) = \frac{n_i^2}{N_D}\exp\left(\frac{0.503}{0.026}\right) = 6.308.10^{16}\,m^{-3}$

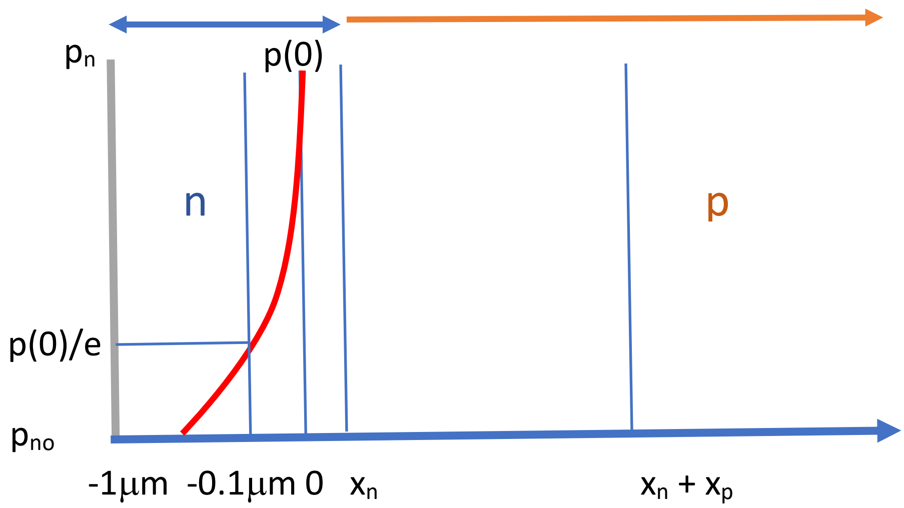
$$or\ p(o) = N_A\exp\left(\frac{-q(V_{bi} - V_{ON})}{KT}\right) = 6.352.10^{16}\,m^{-3}$$

There is a small difference in the answers due to rounding in exponential numbers.

In – p doped side

As V_{bi} and V_{ON} act in the same relative sense for electrons and holes

$$n(0) = p(0)\,\frac{N_D}{N_A} = 6.352.10^{18}\,m^{-3}$$



Where x_n and x_p are the widths of the depletion regions in n and p respectively.

(b)

i) Under AM1.5 solar illumination the diode current equation becomes:

$$I = I_S\left(exp\left(\frac{qV}{kT}\right) - 1\right) - I_{OP}$$

At short circuit V=0 and $I = I_{SC} = -I_{OP}$

And $$I_{OP} = qAg_{OP}(L_e + L_h)$$

$$|I_{SC}| = 1.602.10^{-19}.10^{-2}.2.10^{25}.100.10^{-6} \approx 3.2\ A$$

ii) Under open circuit I = 0

$$0 = I_S\left(exp\left(\frac{qV_{OC}}{kT}\right) - 1\right) - I_{OP}$$

$$V_{OC} = \frac{KT}{q}\ ln\left(\frac{I_{OP}}{I_s} + 1\right) = 0.026\ ln\left(\frac{3.2}{40.10^{-12}} + 1\right) = 0.653V$$

iii)

$$Efficiency = \frac{V_{OC}I_{SC}FF_0}{P_{in}}$$

FF_0 obtained from data sheet using $\frac{qV_{OC}}{KT} = 25.12$ This gives FF_0= 0.837

P_{in}= 10^3. 10^{-2} = 10 Watts. Substituting values, Efficiency = 17.5 %

The efficiency is likely to be lower because FF_0 is fill factor for the p-n junction without taking into account the series and parallel resistances associated with the entire solar cell. This will lower the fill factor and hence the efficiency.

Q4

a) The band gap of Si (data sheet) is 1.12 eV.
   Energy of 900nm photon:
   $hc/\lambda = 6.626.10^{-34}x3.10^{8}/900.10^{-9} = 2.21.10^{-19}$ J
   or $2.21.10^{-19}/1.602.10^{-19} = 1.38$ eV
   For the 600nm photo it would be 1.38x9/6 = 2.07 eV

   Both photons have energy greater than the band -gap of Si and can generate an electron-hole pair. Therefore, the same photon flux (Φ s^{-1}) the expected short circuit current would be the same for the 900nm and 600nm light. But power requires a current at a given voltage. The maximum theoretical voltage is V_{bi} at which current can be provided by an p-n junction. This is smaller than the band-gap in a p-n junction. In practice at any temperature above 0 K the V_{oc} of the cell will be lower than V_{bi}. Hence considering the term $V_{oc}I_{sc}$ term in the efficiency calculation the nominal power output will be the same with 600nm and 900nm photon fluxes. But the 600nm photon flux has greater power (2.07 Φ eV S^{-1}) than the 900nm photon flux (1.38 Φ eV $^{S-1}$). **Therefore, the power conversion efficiency will be higher with the 900nm photon flux**.

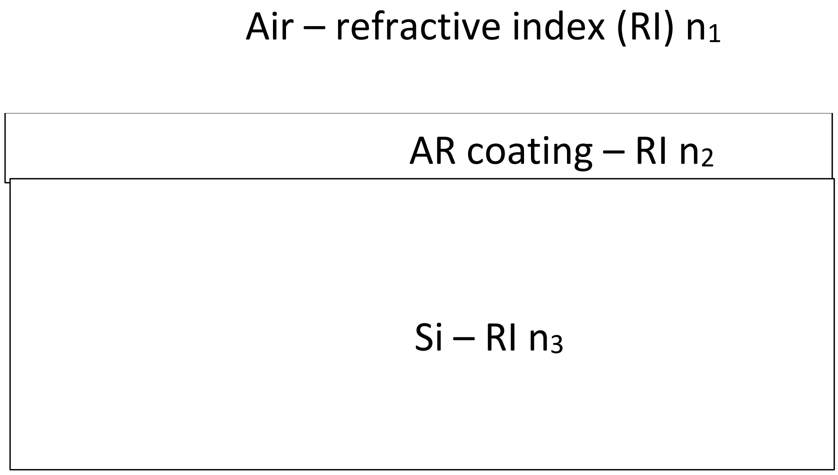
   The excess energy of the photon fluxes above V_{oc} are dissipated as heat within the solar cell. The 900nm photon flux leads to less loss to heat and hence higher efficiency.

b) Light absorption in Si (or any other semiconductor) varies exponentially with distance when the photon energy of the light is greater than the band-gap energy. From Fig. 1 the absorption coefficient for Si it is seen that at the peak power intensity of 600nm the absorption co-efficient is between 10^4 (500nm) – 10^3 cm^{-1} (700nm). Therefore, the peak power in the solar spectrum drops to

1/e within 10^{-4} to 10^{-3} cm (1 – 10 μm). A key feature of a solar cell is the built in electric field at the p-n junction which provides the force field to separate electron and holes generated by absorbing photons (light). The probability of a photo-generated electron-hole pair being separated is highest (~ 1) if it is absorbed in the depletion region of the junction. Therefore, having a depletion region for absorption of the solar peak requires the p-n junction to be placed within 1 – 10 μm of the light entering surface. This determines the p-n junction to have a highly doped n-region within 1 μm of the surface from which light enters the solar cell, an n^+-p junction.

The n-region is highly doped to minimise its resistance to the lateral current flow parallel to the surface which is required to reach the front electrical contacts (typically interspersed line wires printed on the surface allowing light to enter the cell between the lines).

c) Anti-reflection coating on Si



From data sheet:

$$R = \frac{n_2^2\,(n_1 - n_3)^2\,Cos^2\vartheta + \left(n_1 n_3 - n_2^2\right)^2 Sin^2\vartheta}{n_2^2\,(n_1 + n_3)^2\,Cos^2\vartheta + \left(n_1 n_3 + n_2^2\right)^2 Sin^2\vartheta} \quad \text{Where} \quad \theta = \frac{2\pi n_2 d}{\lambda}$$

For $\theta = \pi/2$ $d = \frac{\lambda}{4n_2}$ where d is the thickness of the AR coating.

Also R = 0 under this condition if $n_1 n_3 - n_2^2 = 0$

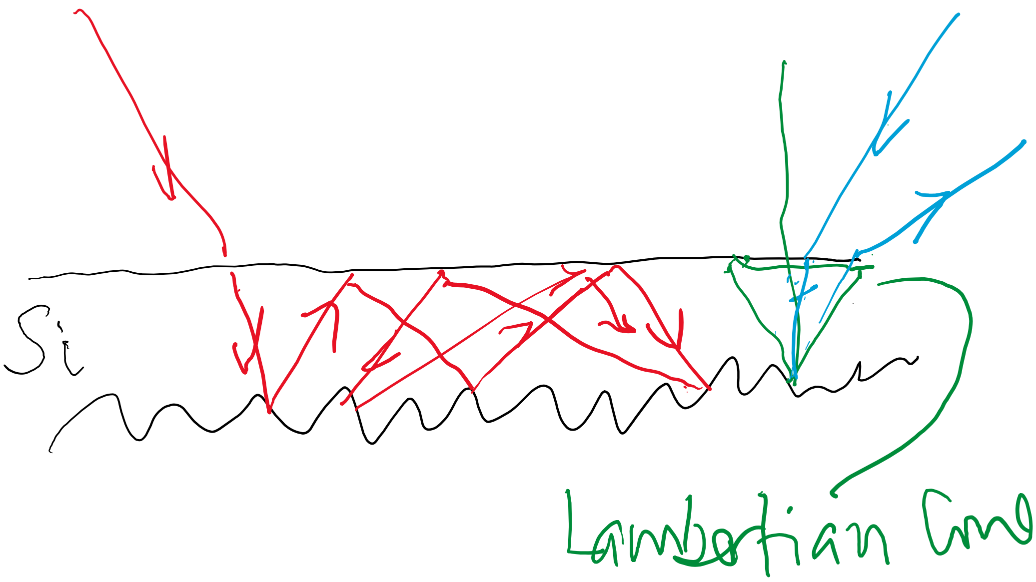
$n_1 = 1$ $n_3 = (11.9)^{1/2}$ which gives n_2 – 1.857 , rounded to 1.9 refractive index for the AR coating.

The coating is to be optimised for 600nm light. This gives

$$d = \frac{600}{4x1.9} = 78.95\, nm$$

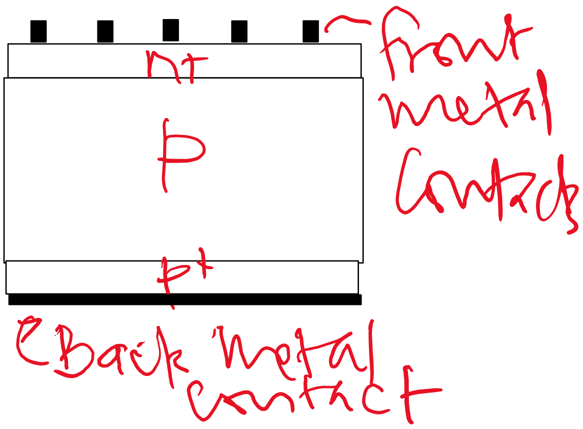
Which can be rounded to 80nm. Therefore the parameters for the AR coating are : Refractive index 1.9 and thickness 80nm.

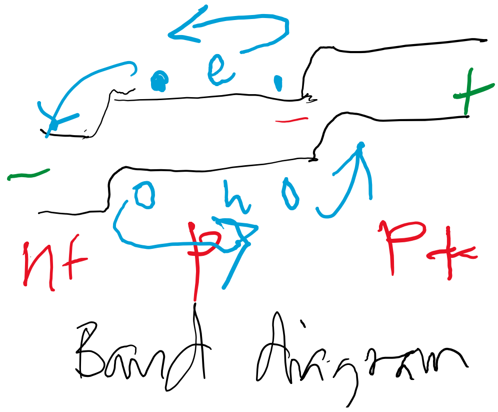
d) As seen from the optical absorption co-efficient data for Si, to absorb the longer wave length components in the solar spectrum the wafer thickness would need to reach almost 1mm (10 cm^{-1}absorption co-efficient). But such an excessive thickness increases the cost of a solar cell as more Si has to be used. One way of solving this problem is to divert the light path once it enters the Si to be parallel (rather than normal) to the surface. This can result in an effective increase in the optical path required for absorption. Such a diversion of the light path can be achieved by randomising the direction of reflection from the back surface of light which reaches it without being absorbed. Roughening the back surface can achieve such randomised reflection. With such randomised angles of reflections, all light which reaches the front surface with angle greater than that of the Lambertian cone for Si will undergo total internal reflection and be trapped in the wafer, as in an optical wave guide.



Roughening the back surface allows for randomised reflection and light trapping through total internal reflection.

e) A back surface field refers to an additional p+ doping region placed at the back of the p-doped wafer before the contact. It acts to form a p^+-p junction. This provides an additional built-in potential at the back and hence a separating field for electron hole pairs created deep in the wafer far more than a diffusion length away from the main p-n junction at the front. Makes the contribution of longer wavelength light to the photocurrent greater.





Examiners' comments

Q1 Hydroelectric and pumped storage power: 21 IIB attempts, mean 12.6/20

A very popular question which received many good attempts, especially on the more discursive parts. Most errors concerned the salient pole generators in section (d)(iii), although most candidates demonstrated a good understanding of the phasor diagram. Very few candidates were able to draw a phasor diagram for the machine in motoring mode.

Q2 Wind power and DFIG: 17 IIB attempts, mean 13.5/20

Most candidates did very well on parts (a) and (b). Part (c) is very challenging, especially parts (ii) and (iii), which require good analytical skills. Nevertheless, a number of candidates achieved perfect or near-perfect scores. The main errors concerned algebraic rather than conceptual failings.

Q3 Solar PV: 19 IIB attempts, mean 16.3/20

A question on the semiconductor fundamentals and operating principles of a Si p-n junction and its function as a photovoltaic cell. It was very pleasing to see that the vast majority of the candidates attempting the question understood the operations very well and were able to do the numerical derivation of the fundamental operating parameters of open circuit voltage, short circuit current and conversion efficiency of a PV-solar cell correctly.

Q4 Solar PV: 9 IIB attempts, mean 15.2/20

A question on the optoelectronic design principles of a Si solar cell. Attempted by a minority of candidates, but those attempting it were competent in understanding the required design details. There was one perfect answer.