

4B19 2024

Crib

Qs 1 and 2 : T. Flack

Qs 3 and 4 : T. Coombs

1. a) As the sea level rises, water is retained in a basin behind a barrage in which turbine-generator sets are mounted. As the tide recedes, generation commences at the minimum viable head of water, typically around 2m, by allowing the trapped water to flow out via the turbines. Thus, the gravitational potential energy is converted into electricity. This continues until the once again the minimum viable head is reached on the next rising tide. At this point, generation ceases and the basin allowed to refill.

These schemes resemble low-head hydro, so typically high flow rate propeller turbines are used, directly coupled to salient-pole generators. These allow low speeds of rotation at high torque, as required.

$$b) i) W_s = \frac{1}{2}(R + R \cos \omega t) \quad \text{where } R = 16$$

$$T = 12 \text{ hrs and } 25 \text{ minutes} = (12 \times 60 + 25) \times 60 \text{ secs}$$

$$\text{so } \omega = \frac{2\pi}{T} = 1.41 \text{ rad s}^{-1} \times 10^{-4} \text{ rad s}^{-1}$$

$$\text{Generation starts when } W_B - W_s = 2 \text{ so } 16 - \left(\frac{1}{2}(16 + 16 \cos \omega t) \right) = 2$$

$$\text{giving } \cos \omega t = 3/4 \text{ so } \omega t = 0.723 \text{ and } t = 5126 \text{ s} = 85.4 \text{ mins.}$$

$$\text{Generation ends when } W_B = 0.4R = 6.4 \text{ m}$$

$$W_B = 16 \text{ m for } 0 < t < 85.4 \text{ min}$$

$$= 16 - \frac{Q_{\text{out}} \Delta t}{A} \text{ for } t > 85.4 \text{ min and } t < t_{\text{end}} \text{ where } Q_{\text{out}} = \text{total} \\ \text{and } \Delta t = t - 85.4 \times 60$$

$$\text{flow rate in the basin} = 125 \times 250 = 31250 \text{ m}^3 \text{ s}^{-1} \text{ and } A = \text{basin area}$$

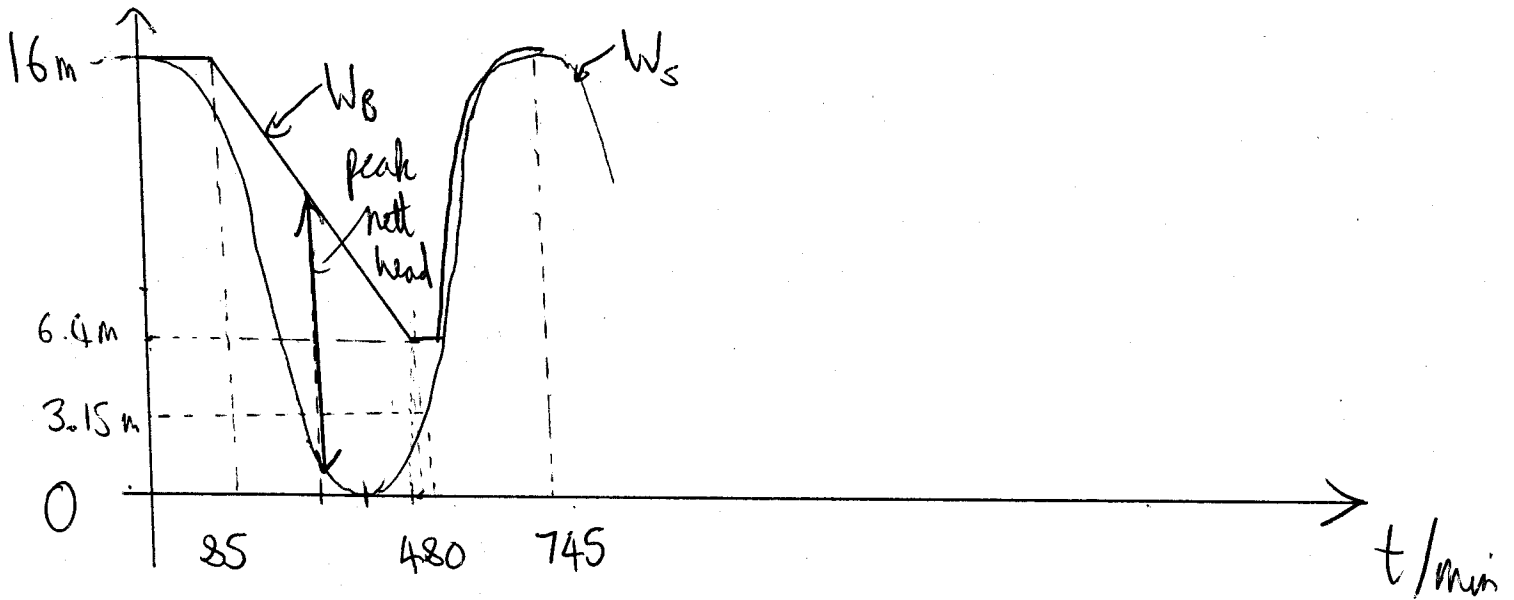
$$= \frac{\pi \times 7000^2}{2} = 77.0 \times 10^6 \text{ m}^2$$

$$\therefore 6.4 = 16 - \frac{31250 \Delta t}{77 \times 10^6} \text{ so } \Delta t = 23654 \text{ s} = 394 \text{ mins}$$

$$\therefore t = 85.4 + 394 = 480 \text{ mins } \approx 8 \text{ hours.}$$

$$W_s = 8(1 + \cos \omega t) = 8(1 + \cos(1.41 \times 10^{-4} \times 480 \times 60)) = 3.15 \text{ m}$$

$$\therefore \text{Nett head} = W_B - W_s = 6.4 - 3.15 = 3.25 \text{ m.}$$



iii) From sketch, peak nett head is $\sim 10 \text{ m}$ at $\sim 320 \text{ mins}$

$$\text{Alt Nett head } H = R - \frac{Q}{A}(t - 85 \times 60) - \left(\frac{R + R \cos \omega t}{2} \right)$$

$$\frac{dH}{dt} = -\frac{Q}{A} + \frac{R \omega \sin \omega t}{2} = 0 \quad \text{giving } \sin \omega t = 0.36$$

$$\omega t = \sin^{-1} 0.36 = 0.368 \text{ or } \pi - 0.368$$

$$t = 43 \text{ mins or } t = 328 \text{ mins}$$

Ignore

At $t = 328 \text{ min}$ $W_s = 0.54 \text{ m}$, $W_B = 10.1 \text{ m}$ so $H = 9.54 \text{ m}$

Peak power = $0.85 \times 1000 \times 9.81 \times 9.54 \times 250 = \underline{19.9 \text{ MW}}$ per generator

so $125 \times 19.9 = \underline{2.49 \text{ GW}}$ in total.

ii) Times of high ~~low~~ tides will shift forwards by ~~20~~⁵⁰ minutes each day so there will be days where electricity production does not coincide with demand. Mitigations include: energy storage (can even do this with the scheme itself by running as pumped storage, which is included in the proposal); interconnecting to the grid and using it as supply base load as scheduling is possible as times of generation completely predictable.

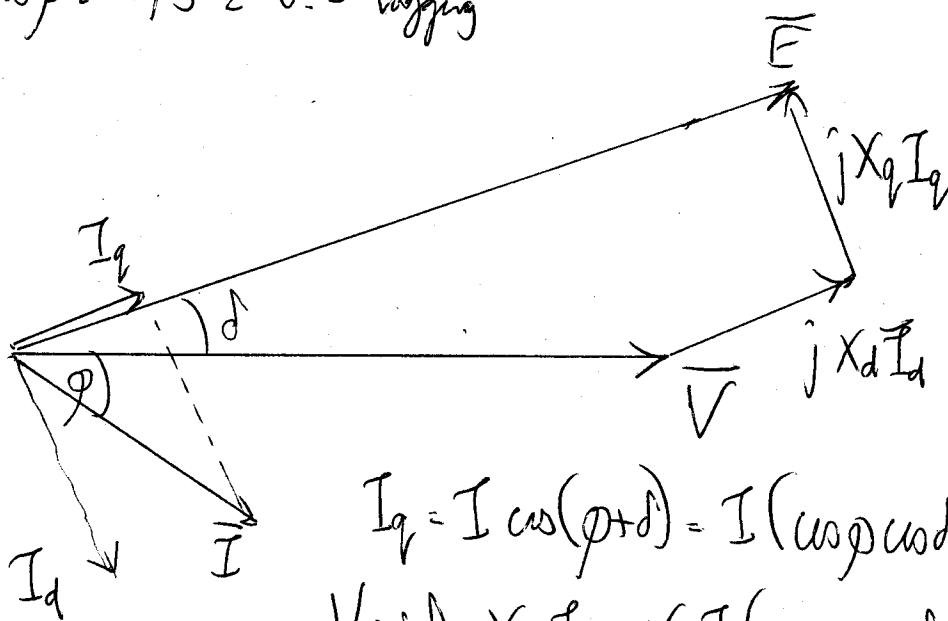
c) ~~$\omega_s = 2\pi \times 50 / 20 = 15.7 \text{ rad s}^{-1} = 150 \text{ rpm}$~~

$\omega_s = 2\pi \times 50 / 30 = 10.47 \text{ rad s}^{-1} = 100 \text{ rpm}$

$T\omega_s = 20 \times 10^6 \text{ s} \Rightarrow T = 1.91 \text{ MNm}$

$S = \sqrt{3} V_c I_c \Rightarrow 25 \times 10^6 = \sqrt{3} \times 11000 \times I \quad I = 1.31 \text{ kA}$

$\cos \phi = P/S = 0.8 \text{ lagging}$



$I_q = I \cos(\phi + \delta) = I (\cos \phi \cos \delta - \sin \phi \sin \delta)$

$V \sin \delta = X_q I_q = X_q I (\cos \phi \cos \delta - \sin \phi \sin \delta)$

$\tan \delta = \frac{X_q I \cos \phi}{V + X_q I \sin \phi} = \frac{1.2 \times 1310 \times 0.8}{11000/\sqrt{3} + 1.2 \times 1310 \times 0.6}$

so $\delta = 17.0^\circ$

2 (a) Rated power is the maximum power that a wind turbine can sustain indefinitely, without damage. Rated wind speed is the wind speed at which rated power is developed.

Power coefficient represents the factor by which the output power differs from the total kinetic energy per second that would pass through the swept area unimpeded. Tip speed ratio is the ratio of the speed of the tip of a blade to the wind speed. Maximum power for a given wind speed is only developed at the optimum tip-speed ratio i.e. $\lambda_{opt} = \omega R / v$, meaning that ω should always be proportional to v for maximum power extraction, hence use of variable speed devices. [15%]

b) i) Rated power $\times 24 = 384 \therefore P_{rated} = \underline{16 \text{ MW}}$. We deduce this from the information that $v > v_{rated}$ over the 24 hours the record was set, so the turbine operated at 16 MW the entire time.

Also, turbine always operated at λ_{opt} , and we know that $\omega R = 0.7 \times 343$ when $v = 24 \text{ m s}^{-1}$.

$$\therefore \lambda_{opt} = 0.7 \times 343 / 24 = \underline{10}$$

When operating at rated wind speed the turbine produces rated power of 16 MW.

So in one hour of operation it would produce $16 \text{ MWh} = 34.2 \text{ kWh} \times N$ where N is the number of revolutions in one hour.

$$\therefore N = 468 \text{ revs/hour} = 468/60 = 7.80 \text{ rpm} = 0.817 \text{ rad s}^{-1}$$

$$\text{Since } \lambda = \lambda_{opt} = 10 = \frac{\omega R}{v} = \frac{0.817 \times (252/2)}{v} \Rightarrow \underline{v = 10.3 \text{ m s}^{-1}}$$

$$\text{Using } P = \frac{1}{2} C_p \rho A v^3 \Rightarrow 16 \times 10^6 = \frac{1}{2} C_p \times 1.23 \times \frac{\pi \times 252^2}{4} \times 10.3^3$$

$$\text{giving } \underline{C_p = 0.477}$$

Note: At wind speeds $> v_{rated}$, ω is still $\propto v$, but C_p is reduced by real-time, rapid response blade feathering. This is how the GWA252 is still safe to operate in storm force winds. [30%]
[25%]

$$\text{ii) } v = 5 \text{ m s}^{-1} \text{ by } v^3 \text{ scaling } P = \left(\frac{5}{10.3}\right)^3 \times 16 = 1.83 \text{ MW}$$

$$\omega = 0.817 \text{ at } v = 10.3 \text{ so } \omega = \frac{5}{10.3} \times 0.817 = \underline{0.397 \text{ rad s}^{-1}}$$

$$\therefore T = 1.83 / 0.397 = \underline{4.6 \text{ MNm}}$$

$$v = 20 \text{ m s}^{-1} \text{ } P \text{ capped at } 16 \text{ MW and } \omega = \frac{20}{10.3} \times 0.817 = \underline{1.59 \text{ rad s}^{-1}}$$

$$\text{so } T = P/\omega = \underline{10.1 \text{ MNm}}$$

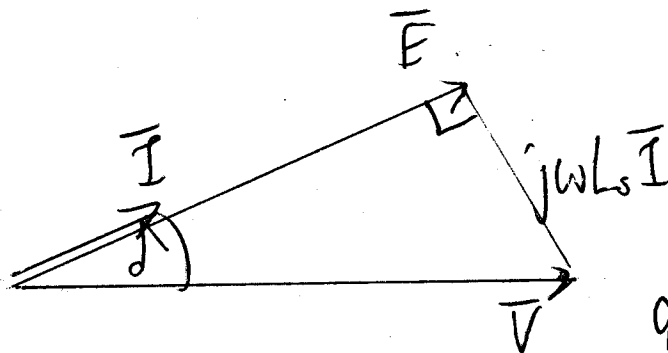
[10%]

c) At 5 ms^{-1} : $T = 3kI$ so $4.6 \times 10^6 = 3 \times 2500 \times I \Rightarrow I = \frac{613 \text{ A}}{6.13 \text{ kA}}$

$E_{ph} = kW_r = 2500 \times 0.397 = 993 \text{ V}$ (1720 V line)

$\omega_r = \frac{\omega}{p}$ so $\omega = 30\omega_r = 11.9 \text{ rad s}^{-1}$ and $\omega L_s I = 453 \text{ V}$

$\bar{E} = \bar{V} + j\omega L_s \bar{I}$, \bar{I} in phase with \bar{E} for 90° torque angle



$V^2 = E^2 + (\omega L_s I)^2 \Rightarrow V = 1091 \text{ V}$ (1890 V line)
 $\delta = \cos^{-1} E/V = 2.6^\circ$

[20%]

d) Capital cost = $25 \times 16 \times \pounds 2 \text{ million} = \pounds 800 \text{ million}$.

Annual O&M = $0.04 \times \pounds 800 = \pounds 32 \text{ million}$.

Cost of borrowing = $\frac{\pounds 800 \text{ million}}{\pounds 1000} \times \pounds 71 = \pounds 56.8 \text{ million}$

\therefore Total annual costs = $\pounds 32 + \pounds 56.8 = \pounds 88.8 \text{ million}$.

Scheme produces 1.6 million MWh / annum, so cost/MWh = $\frac{\pounds 88.8}{1.6}$
 = $\pounds 55.5 / \text{MWh}$

Capacity factor = $\frac{1.6 \times 10^6}{25 \times 16 \times 365 \times 24} = 0.457$ is typical of offshore wind, very healthy. [15%]

e) Scheme is not viable if electricity sold to grid only fetches £40/MWh, this is well below the £55/MWh it costs to produce. The government can subsidise the market rate by guaranteeing a minimum payment. This is how Contracts for difference work as guarantee £80/MWh. It could also levy a greater carbon tax on CCGT/Coal/Oil which would force the market price upwards.

[10%]

Q1 Tidal barrage scheme: 23 IIB attempts, mean 65.7%

There were many good attempts at this question. Many students answered (a) very well, (b) (i) caused problems in the calculation of the end of the generating period and the net head of water. (b) (ii) was well answered, but very few candidates were able to find the peak head of water in (b) (iii). Most candidates realised that the timing of the peak power generation would change every day but some only alluded to the generation being 'lumpy'. Part (c) received many excellent answers, although a few misunderstandings of the difference between MVA and MW were apparent.

Q2 Offshore wind power: 28 IIB attempts, mean 71.1%

This was a popular question and was mainly very well answered. (b) (ii) caused some problems, with some candidates taking 24 ms^{-1} as the rated wind speed and getting some obviously wrong answers. Most candidates showed a good understanding of permanent magnet generators in part (c), and parts (d) and (e) on the economics of the scheme received many excellent answers, the main errors stemming from incorrectly transcribing the information given in the question to their scripts.

Q3

(a)

(i)

$$V_{bi} = \frac{kT}{q} \ln \left(\frac{N_D N_A}{n_i^2} \right)$$

$n_i = 3.18 \times 10^{16}$ at $T = 300$ K from expression given. N_D (n-doping), N_A (p-doping) defined in Q and k , q from data book or formulae and constants sheet

hence $V_{bi} = 0.832$ V

(ii) D_e and D_h diffusion length for electrons (minority carriers in the p-region) and holes (minority carriers in the n-region) and τ_e and τ_h their respective lifetimes are data given in the table. As is A the junction area and N_D and N_A .

Data sheet tells us that. $I_s = qA \left[\frac{n_i^2 D_e}{N_A L_e} + \frac{n_i^2 D_h}{N_D L_h} \right]$

Where

$$L_{e,h} = \sqrt{D_{e,h} \tau_{e,h}}$$

Plugging in values on gets $I_s = 1.16 \times 10^{-10}$ A

(iii)

From lecture notes

$$V_{oc} = \frac{kT}{q} \log \left(\frac{I_{op}}{I_s} + 1 \right)$$

$I_{op} = 10^{-2}$, all other parameters known and calculated in (ii)

$$V_{turn\ on} = 0.472$$
 V

b) (i) The short circuit current is equal to the optically generated current.

$$I_{SC} = I_{OP} = qAg[L_e + L_h]$$

g is the generation rate defined in the Q, A junction area, $L_e = 40 \mu\text{m}$ and $L_h = 1 \mu\text{m}$ from the parameters for $D_{e,h}$ and $\tau_{e,h}$ given.

$$I_{OP} = 0.39$$
 A

(ii) As T has changed to 340K n_i changes and hence I_s also changes to:

$$n_i = 4.76 \times 10^{17} \text{ cm}^{-3} \text{ and } I_s = 2.68 \times 10^{-8}$$
 A

$$V_{oc} = 0.484$$
 V

- (iii) The built in voltage is the voltage which would be applied to make the band across the junction flat. But before the band becomes flat, with reduction of the potential barrier to minority current flow, current starts to flow as carriers can be thermally excited over the barrier (gain voltage KT/q). Hence the open circuit voltage is lower.

Comments: Probably because the answers were numerical, and the expressions were standard ones the students had little difficulty with question 3. Generally they only lost marks in part b iii)

Q4

- a) Internal losses due to bulk resistance, contact resistance, leakage current. Cell is essentially a black body and not all radiation is converted to heat. Solar cells are between 20% and 30% efficient some of that is reflected but some of it is simply converted into heat

The following factors effect heat generation in the cell

Back surface reflection

Surface shading by top contacts

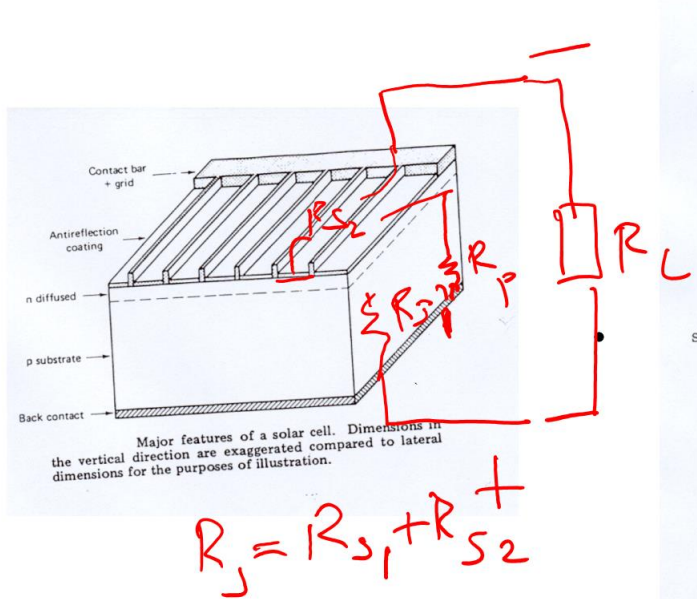
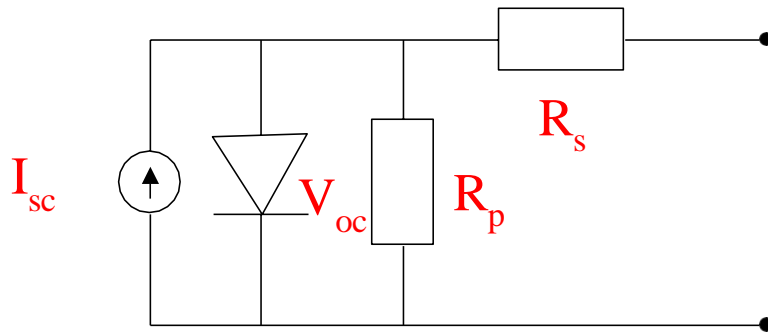
Recombination e-h pairs generated near surfaces.

Hotter cell equals lower V_{oc} and reduced power output

Comments: General impression was that this was a question about how much of the spectrum can be converted in cell. Hardly anyone mentioned the losses in the cell itself

- b) A practical solar cell is not a simple junction. There are series resistances with junction due to the bulk Si across which the photogenerated current flows to the terminals. Additionally there is always resistance between the Si and the metal contacts (contact resistance) which is also in series (generally both are lumped together in a single resistance). Additionally there can be a leakage resistance parallel to the diode, takes current out from the main current, due to damage at the perimeter edges of the cell. If the area/perimeter ratio is very high then this parallel resistance can be taken to approach infinity.

The modified equivalent circuit is:



Comments: Students had little difficulty with this section

c) *Shockley-Read-Hall recombination (SRH)*, also called *trap-assisted recombination*, the electron in transition between bands passes through a new energy state (localized state) created within the band gap by a dopant or a defect in the crystal lattice; such energy states are called *traps*. Non-radiative recombination occurs primarily at such sites. The energy is exchanged in the form of lattice vibration, a phonon exchanging thermal energy with the material.

In *Auger recombination*, the energy is given to a third carrier which is excited to a higher energy level without moving to another energy band. After the interaction, the third carrier normally loses its excess energy to thermal vibrations. Since this process is a three-particle interaction, it is normally only significant in non-equilibrium conditions when the carrier density is very high. The Auger effect process is not easily produced, because the third particle would have to begin the process in the unstable high-energy state.

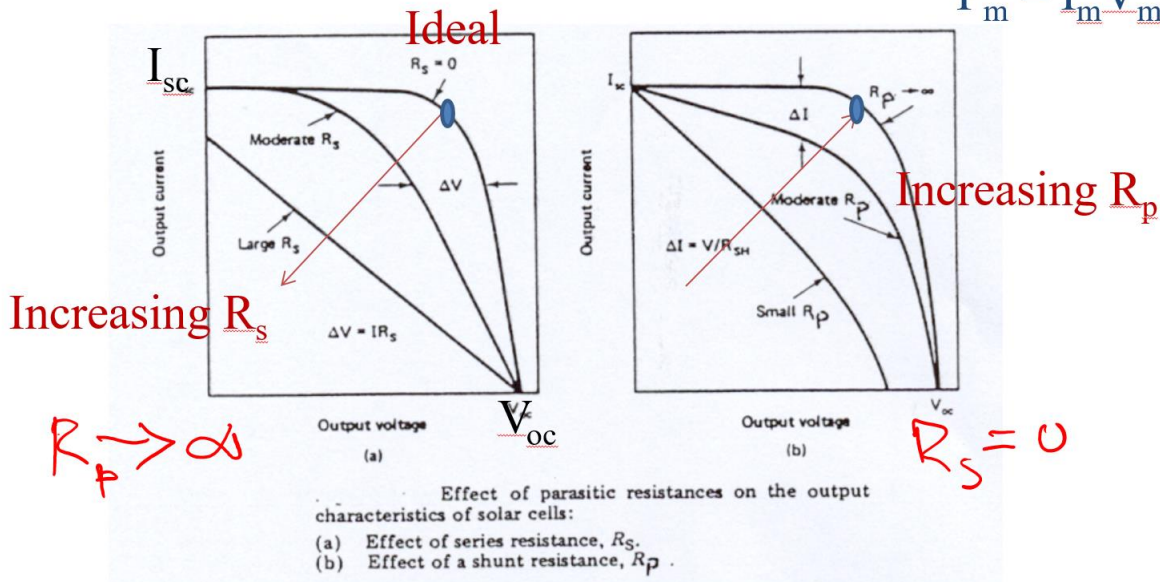
SRH is the dominant recombination process in silicon and other indirect band gap materials.

Comments: Lots of confusion over which is dominant mechanism. Otherwise fairly well answered

d)

The impact of R_s and R_p on the solar cell I-V characteristics:

$$P_m = I_m V_m$$



Reducing R_s brings the characteristic closer to the ideal. Increasing R_p also brings it closer to the ideal.

Comments: Generally well done although some confusion with how the intercepts on the graphs vary with R_s and R_p

Overall: There were 40 attempts for question 3 and 22 for question 4. The marks awarded for each question were similar with question 3 scoring an average of 16 and question 4 an average of 14.7