

EGT3  
ENGINEERING TRIPOS PART IIB

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Wednesday 1 May 2024 2.00 to 3.40

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**Module 4B19**

**RENEWABLE ELECTRICAL POWER**

*Answer not more than **three** questions.*

*All questions carry the same number of marks.*

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

*Write your candidate number **not** your name on the cover sheet.*

**STATIONERY REQUIREMENTS**

Single-sided script paper

**SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM**

CUED approved calculator allowed

Attachment: 4B19 data sheet (2 pages)

Engineering Data Book

**10 minutes reading time is allowed for this paper at the start of the exam.**

**You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.**

**You may not remove any stationery from the Examination Room.**

1 (a) Explain the principles of electricity generation using ebb-flow tidal barrage schemes. Outline the turbine and generator technologies used, giving reasons for them. [10%]

(b) The West Somerset Lagoon scheme is a proposal to create a semi-circular tidal barrage, of diameter 14 km, which backs onto the straight coast line. The tidal range  $R$  is 16 m, and the scheme is to utilise 125 turbine-generators, with every turbine operating at a fixed flow rate of  $250 \text{ m}^3 \text{ s}^{-1}$  when generating. The combined efficiency of the turbine-generators is 85%. The scheme operates as follows. As high tide approaches, sluice gates are opened such that the water level in the basin,  $W_B$ , is the same as the sea level,  $W_S$ , which is given by  $W_S = R(1 + \cos(\omega t))/2$ . The time interval between high tides is 12 hours and 25 minutes. Taking  $t = 0$  to coincide with high tide, such that  $W_B = W_S = R$ , the sluice gates are closed. As the sea level falls, generation commences when the net head of water,  $H = W_B - W_S$  reaches 2 m. At this point the turbine gates are opened and subsequently controlled such that the flow rate through every turbine is held constant. This continues until the basin level reaches  $0.4R$  at which point generation ceases and the turbine gates are closed. When the sea level starts to exceed the basin level the sluice gates are opened and the basin allowed to refill.

(i) Determine the times at which generation commences and ends, to the nearest minute, and the net head of water at the end of the generating period. [15%]

(ii) Sketch graphs of sea level, basin level and net head of water vs time on the same axes, indicating where generation commences and finishes. [15%]

(iii) Using your sketch, or otherwise, estimate the peak net head of water and hence the maximum power output of the scheme. You may quote  $P = \eta\rho gHQ$  without proof, and take  $g$  to be  $9.81 \text{ m s}^{-2}$  and  $\rho$  to be  $1000 \text{ kg m}^{-3}$ . [20%]

(iv) Making reference to the tidal period, explain why tidal barrage schemes are not always well-matched to periods of high demand on the grid. Give two mitigations for this issue. [10%]

(c) The turbines are all directly coupled to 60 pole, 25 MVA, star-connected salient pole synchronous generators. The generators are connected to the 3-phase, 50 Hz, 11 kV grid and have direct and quadrature synchronous reactances of  $3.2 \Omega$  and  $2.4 \Omega$ , respectively. Assuming generation at 20 MW and rated MVA at a lagging power factor find:

(i) the generator angular speed in rpm, and the torque; [10%]

(ii) the generator phase current, power factor and load angle. [20%]

2 (a) Define the terms ‘rated wind speed’, ‘rated power’, ‘power coefficient’ and ‘tip-speed ratio’ in the context of wind turbines. Explain why it is best to operate wind turbines as variable speed devices. [15%]

(b) Currently the world’s largest wind turbine is the Goldwind GWH252. Unlike most other wind turbines it operates at optimum tip-speed ratio at all wind speeds from cut-in to hurricane wind speeds of up to  $24 \text{ m s}^{-1}$ , at which its blade tips reach 70% of the speed of sound. It achieved the record for the most energy generated by a wind turbine in a single 24 hour period of 384 MWhr. It did this during a storm, during which it may be assumed that wind speeds were always in excess of the rated wind speed. It has a turbine diameter of 252 m, and at its rated wind speed it generates 34.2 kWhr of energy per revolution of the turbine. Using this information find:

(i) the rated power of the turbine, its optimum tip-speed ratio, its rated wind speed and its power coefficient at optimum tip-speed ratio and for wind speeds at or below rated wind speed. You may quote without proof:  $P = 0.5C_p\rho Av^3$ ,  $\lambda = \frac{\omega R}{v}$ , and take  $\rho$  to be  $1.23 \text{ kg m}^{-3}$  and the speed of sound to be  $343 \text{ m s}^{-1}$ ; [30%]

(ii) the turbine torque and angular speed at wind speeds of  $5 \text{ m s}^{-1}$  and  $20 \text{ m s}^{-1}$ . [10%]

(c) The turbine of part (b) is mechanically coupled directly to a 3-phase, star-connected, 60 pole permanent magnet generator with a synchronous inductance of 6.2 mH and an emf constant of  $2500 \text{ V s rad}^{-1}$ . Assuming that the system is controlled so that the generator always operates with a torque angle of 90 degrees, find the line-line excitation emf and output voltage, the phase current and the load angle at the wind speed of  $5 \text{ m s}^{-1}$ . [20%]

(d) An offshore wind farm is proposed that utilises 25 of the wind turbine generators specified in parts (b) and (c). It is predicted to produce 1.6 million MWhr of electrical energy per year. The capital cost per rated MW of fully installed generating capacity is £2 million, annual operation and maintenance is 4% of the capital cost, and the initial capital investment is paid back at the rate of £71 per £1000 borrowed, per annum, over the 25 year lifetime of the scheme. Use discounted cash flow analysis to evaluate the economic viability of this proposition, and find the current cost per MWhr of electricity produced by the scheme. Also find its capacity factor. [15%]

(e) If the current market rate for selling electricity to the grid is £40 per MWhr, comment on the viability of the scheme. Give two ways that the government could intervene to make the scheme viable. [10%]

3 A Si p–n junction solar cell made of crystalline Si has the following parameters given in the table below.

n-region doping $N_D$	$1 \times 10^{24} \text{ m}^{-3}$
p-region doping $N_A$	$1 \times 10^{23} \text{ m}^{-3}$
minority carrier diffusion coefficient in the n-region $D_h$	$10^{-6} \text{ m}^2 \text{ s}^{-1}$
minority carrier diffusion coefficient in the p-region $D_e$	$16 \times 10^{-6} \text{ m}^2 \text{ s}^{-1}$
Minority carrier lifetimes for electrons and holes respectively $t_e, t_h$	$1 \mu\text{s}, 1 \text{ ns}$
Junction area	$10^{-2} \text{ m}^2$

The intrinsic carrier concentration  $n_i$  as a function of temperature  $T$  is given by the expression:

$$n_i = n_0 \exp \frac{-T_0}{T}$$

where  $n_0 = 3.1 \times 10^{26} \text{ m}^{-3}$  and  $T_0 = 6.9 \times 10^3 \text{ K}$ .

- (a) Assume that the junction temperature is 300 K under dark conditions.
- (i) Determine the built-in potential for the junction under dark conditions. [10%]
  - (ii) Find the reverse saturation current of the junction. [10%]
  - (iii) If the threshold for turn-on of the p-n junction under dark conditions is taken to be  $1 \text{ A m}^{-2}$  estimate the open circuit voltage of the junction at turn on. [10%]
- (b) The solar cell is assembled in a panel and exposed to AM 1.5 solar illumination of  $1 \text{ kW m}^{-2}$ . Under these conditions the temperature of the cell rises to 340 K. The average generation rate is then  $6 \times 10^{25} \text{ m}^{-3} \text{ s}^{-1}$ .
- (i) Find the corresponding short-circuit current of the cell. [20%]
  - (ii) Calculate the corresponding open-circuit voltage of the cell. [30%]
  - (iii) Why is the built-in potential of the junction different from the open-circuit voltage obtained under solar illumination? [20%]

- 4 (a) Explain why generation of heat is inevitable when converting light energy of the solar spectrum to electrical energy through the photovoltaic process using a semiconductor. What effect does a rise in temperature have on the open circuit voltage? [20%]
- (b) Draw the equivalent circuit for a solar cell labeling the various resistances. Illustrate your answer with a sketch of the cell itself showing the origin of the resistances. [20%]
- (c) Explain the difference between Shockley-Read-Hall and Auger recombination. Which one dominates in a solar cell? Explain your answer. [30%]
- (d) Draw I-V characteristics which illustrate the effects of  $R_s$  and  $R_p$  on the characteristics and the fill factor. How does fill factor affect the optimum efficiency? Give reasons for your answer. [30%]

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Short answers

1 (b) (i) 85 minutes, 480 minutes, 3.25 m (iii) 9.54 m, 2.49 GW

(c) (i) 100 rpm, 1.91 MN m (ii) 1.31 kA, 0.8 lagging,  $17^\circ$

2 (b) (i) 16 MW,  $\lambda=10$ ,  $v=10.3 \text{ m s}^{-1}$ ,  $C_p=0.477$

(ii) 4.6 MN m,  $0.397 \text{ rad s}^{-1}$ , 10.1 MN m,  $1.59 \text{ rad s}^{-1}$

(c)  $E=1720 \text{ V}$ ,  $V=1722 \text{ V}$ ,  $I=613 \text{ A}$ , load angle= $2.6^\circ$  (d) £55.50 per MWhr, 0.457

3 (a) (i) 0.832 V (ii) 0.116 nA (iii) 0.472 V (b) (i) 0.39 A (ii) 0.484 V

## 4B19 RENEWABLE ENERGY: SOLAR PHOTOVOLTAIC POWER

### Formulae and Constants – Revised 2024

#### Built-in potential for a solar cell

$$V_{bi} = \frac{kT}{q} \ln \left( \frac{N_D N_A}{n_i^2} \right)$$

#### Reverse Saturation Current of a p-n junction diode

$$I_s = qA \left[ \frac{n_i^2 D_e}{N_A L_e} + \frac{n_i^2 D_h}{N_D L_h} \right]$$

Where

$$L_{e,h} = \sqrt{D_{e,h} \tau_{e,h}}$$

#### Optical generation rate in a semiconductor at distance x from the light entering surface

$$g_o(x) = \int_{\lambda_1}^{\lambda_2} \phi_0(\lambda) \alpha(\lambda) \exp(-\alpha(\lambda)x) d\lambda$$

Where  $\phi_0(\lambda) = (1 - R(\lambda))\phi(\lambda)$  and  $\phi(\lambda)$  is the incident photon flux at wavelength  $\lambda$  of the solar spectrum.

**Reflection co-efficient** at normal incidence from the third layer of a 3 layer system comprising of a thin film sandwiched between two bulk materials extending away from their interfaces with the thin film is given as:

$$R = \frac{n_2^2(n_1 - n_3)^2 \cos^2 \vartheta + (n_1 n_3 - n_2^2)^2 \sin^2 \vartheta}{n_2^2(n_1 + n_3)^2 \cos^2 \vartheta + (n_1 n_3 + n_2^2)^2 \sin^2 \vartheta}$$

Where  $\mathcal{G} = \frac{2\pi n_2 d}{\lambda}$

$n_1$ ,  $n_2$  and  $n_3$  are the refractive indices of 3 materials  $m_1$ ,  $m_2$  and  $m_3$  respectively:  $d$  is the thickness of the thin film  $m_2$  sandwiched between materials  $m_1$  and  $m_3$ . Direction of light flow is taken to be from  $m_1$  to  $m_3$ .

#### Fill Factor for a Si solar cell

$$FF_0 = \frac{\frac{qV_{oc}}{kT} - \ln \left( \frac{qV_{oc}}{kT} + 0.72 \right)}{\frac{qV_{oc}}{kT} + 1}$$

where  $V_{oc}$  is the open circuit voltage for the cell.

## Constants

Electronic charge unit      **q:  $1.602 \times 10^{-19}$  C**

Boltzmann's Constant      **k:  $1.38 \times 10^{-23}$  J K<sup>-1</sup>**

Speed of light      **c:  $3 \times 10^8$  m s<sup>-1</sup>**

Planck's Constant      **h:  $6.626 \times 10^{-34}$  J s**

Dielectric permittivity free space  $\epsilon_0$ :  **$8.85 \times 10^{-12}$  F m<sup>-1</sup>**

Relative permittivity of **Si**  $\epsilon_r$ : **11.9**

Refractive index (for weakly and non-absorbing wavelengths) **n =  $\epsilon_r^{0.5}$**

Band-gap energy of **Si**: **1.12 eV**