Version TJF/3

EGT3
ENGINEERING TRIPOS PART IIB

Wednesday 7 May 2025 2.00 to 3.40

Module 4B19

RENEWABLE ELECTRICAL POWER

*Answer not more than **three** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

*Write your candidate number **not** your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
Attachment: 4B19 data sheet (2 pages)
Engineering Data Book

10 minutes reading time is allowed for this paper at the start of the exam.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

You may not remove any stationery from the Examination Room.

1 (a) Give one advantage and one disadvantage of variable speed wind turbines. Define the terms 'tip-speed ratio' and 'power coefficient' in relation to wind turbines. [10%]

(b) A wind turbine has a power coefficient (C_p) vs tip-speed ratio (λ) characteristic given by:

$$C_p = 0.15\lambda \exp\left(-\frac{\lambda}{8}\right)$$

It operates as a variable speed turbine at its optimum tip-speed ratio from wind speeds of 6 m s^{-1} up to its rated wind speed of 12 m s^{-1}. The cut-in and stall wind speeds are 3 m s^{-1} and 20 m s^{-1}, respectively. For wind speeds between cut-in and 6 m s^{-1} the turbine's angular speed is fixed at the value corresponding to the wind speed of 6 m s^{-1}. At wind speeds between rated and stall its angular speed is held fixed at the value corresponding to rated wind speed. Its rated power is to be 5 MW. Wind speed data at the site of the wind turbine is provided in the table below.

Wind speed (m s^{-1})	2	5	8	11	14	17	21
Number of days	15	40	120	90	60	35	5

(i) Show that the optimum tip-speed ratio is 8, at which the corresponding power coefficient is 0.441. [10%]

(ii) Determine the turbine radius. [5%]

(iii) Find the output power at all the wind speeds given in the table and hence find the total annual energy produced in MWhr and the capacity factor. [30%]

You may quote without proof: $P = 0.5C_p\rho Av^3$, $\lambda = \frac{\omega R}{v}$, and take ρ to be 1.23 kg m^{-3}.

(c) The turbine of part (b) is coupled to a 16-pole doubly-fed induction generator (DFIG) via a gearbox. At the wind speed of 10 m s^{-1} the slip of the DFIG is -0.01. The stator winding of the DFIG is star-connected to the 11 kV, 50 Hz grid.

(i) Find the generator and turbine angular speeds at the wind speed of 10 m s^{-1}. Hence find the gearbox ratio (to three significant figures). [15%]

(ii) Draw a system diagram and use it to explain how the slip energy converter facilitates variable speed operation. [10%]

(iii) Find the range of output frequencies required of the slip energy converter. [20%]

2 (a) Show that the output power of a hydroelectric scheme is given by $P = \eta \rho g H Q$ and define all the terms in this expression. Also explain how pumped storage hydroelectric schemes work, and give three benefits of them as the grid moves towards net zero operation. [20%]

(b) Cruachan power station is an example of a pumped storage hydroelectric scheme. A high level resevoir holds 10 million m^3 of water when full, providing a net head of water of 363 m to the 4 turbine-generator sets. When empty the net head of water is 340 m. The system is 73% efficient when generating and is capable of operating at its maximum volumetric flow rate for 22 hours. Assume that the 4 turbine-generator sets are identical, and contribute equally when generating, and that the net head of water changes linearly with time over the 22 hours.

(i) Determine the maximum volumetric flow rate, and sketch a graph showing the net head of water vs time over the 22 hour generating period. [10%]

(ii) Find the peak and minimum power generated and hence find the total energy supplied to the grid in GWhr over the 22 hour generating period. [10%]

(c) The generators of part (b) are all 10-pole salient-pole synchronous machines star-connected to the 50 Hz, 3-phase, 16 kV grid. They are all rated at 100 MW and can deliver up to 30 MVAr of leading or lagging reactive power whilst delivering rated power. Their equivalent circuit parameters are $X_d = 2.5\ \Omega$ and $X_q = 1.8\ \Omega$.

(i) Find the VA rating of the generators and their angular speed. [10%]

(ii) Draw a phasor diagram for the case when a generator is operating at rated power and delivering 30 MVAr of lagging reactive power, and find the phase current, load angle and line-line excitation voltage. [20%]

(d) Recently the Energy System Operator has contracted Cruachan to provide grid support by providing additional inertia and reactive power. It does this by having one of its generators permanently synchronised to the grid on no-load.

(i) Explain how rotational kinetic energy helps to stabilise the grid frequency against sudden changes in demand or generated power. Why is grid-connected battery energy storage increasingly required? [10%]

(ii) When the frequency drops from 50 Hz to 49.8 Hz the generator supplies 3.2 MJ of energy to the grid from its rotational stored kinetic energy. Use this information to find its moment of inertia. [10%]

(iii) Find the line-line excitation voltage when the generator supplies 50 MVAr of lagging reactive power, and draw a phasor diagram to illustrate this situation. [10%]

3 A pn^+ junction Si diode has the parameters given below at a temperature of 300 K.

N_A	acceptor density on p-type side	5.0×10^{22} m^{-3}
N_D	donor density on n-type side	5.0×10^{24} m^{-3}
n_i	intrinsic carrier density	1.0×10^{16} m^{-3}
D_e	minority carrier electron diffusion constant	0.01 m^2 s^{-1}

The junction area is 10^{-2} m^2. The p-type region is 300 μm thick and uniformly doped. The n-type region is 1 μm thick and uniformly doped.

(a) Consider the excess minority electron concentration, Δn, in the p-type region. The continuity equation is

$$\frac{\partial(\Delta n)}{\partial t} = -\frac{\Delta n}{\tau_e} + \mu_e E \frac{\partial(\Delta n)}{\partial x} + D_e \frac{\partial^2(\Delta n)}{\partial x^2},$$

where μ_e is the hole mobility, E is the electric field, τ_e is the minority carrier electron lifetime and D_e is the minority carrier electron diffusion constant.

(i) Show that under steady-state conditions, Δn is given by

$$\Delta n = A\left[\exp\left(\frac{eV}{kT}\right) - 1\right]\exp(-Bx),$$

where A and B are constants, V is the applied bias, e the electronic charge, T the temperature and x the distance from the edge of the depletion region. State all assumptions. Find expressions for the constants A and B. [30%]

(ii) The current density, J_e, due to electrons is given by

$$J_e = -eD_e\frac{dn}{dx}.$$

Hence, estimate the reverse saturation current, I_s, given that $A \times B = 2 \times 10^{13}$ m^{-4}. State all assumptions. [20%]

(b) Assume the electrodes contacting the p-type and n-type sides are totally transparent, so that the junction can be illuminated from either side. For optimum solar cell operation, should the sunlight enter from the p-type side or from the n-type side of the diode? Explain your reasoning. [20%]

(c) When the diode is exposed to AM1.5 sunlight, it develops an open-circuit voltage of 0.6 V.

(i) What is the ideal short-circuit current which could be expected from the cell? You may assume the reverse saturation current is I_s = 0.3 nA. [15%]

(ii) Estimate the minority carrier diffusion length, L_e, of electrons if the optical generation rate is 2×10^{25} m^{-3} s^{-1}. State all assumptions. [15%]

4 (a) Sketch current–voltage characteristics (I–V curves) to describe how the following conditions affect the fill factor (FF_0), short-circuit current (I_{sc}) and open-circuit voltage (V_{oc}) of a solar cell.

(i) An increase in optical generation rate. [10%]

(ii) An increase in Shockley–Read–Hall recombination in the depletion region. [10%]

(iii) The presence of a shunt pathway for current at the edges of the solar cell. [10%]

(b) Describe the structure of a multijunction solar cell and explain how multijunction solar cells can achieve higher efficiencies than single-junction solar cells. [30%]

(c) Describe two strategies for increasing the optical path length within a solar cell. [10%]

(d) A multi-crystalline Si solar cell has experimentally measured V_{oc} and I_{sc} values of 0.6 V and 3 A respectively when exposed to AM 1.5 (1 kW m^{-2}) solar illumination. Under conditions of strong forward bias in the dark, the current is observed to be resistively limited above 2 A. The limiting resistance is 20 mΩ. The area of the cell is 100 cm^2 and the temperature during the measurement was 300 K.

(i) Estimate the fill factor for the p-n junction in the cell. [20%]

(ii) Give an estimate for the efficiency of the cell. [10%]

END OF PAPER

4B19 RENEWABLE ENERGY: SOLAR PHOTOVOLTAIC POWER

Formulae and Constants – Revised 2025

Built-in potential for a solar cell

$$V_{bi} = \frac{kT}{q} \ln\left(\frac{N_D N_A}{n_i^2}\right)$$

Reverse saturation current of a p-n junction diode

$$\boldsymbol{I_s = eA\left[\frac{n_i^2 D_e}{N_A L_e} + \frac{n_i^2 D_h}{N_D L_h}\right]}$$

where

$$\boldsymbol{L_{e,h} = \sqrt{D_{e,h}\tau_{e,h}}}$$

Optical generation rate in a semiconductor at distance x from the light-entering surface

$$g_0(x) = \int_{\lambda_1}^{\lambda_2} \phi_0(\lambda)\alpha(\lambda)\exp(-\alpha(\lambda)x)\, d\lambda$$

where $\phi_0(\lambda) = \big(1 - R(\lambda)\big)\phi(\lambda)$ and $\phi(\lambda)$ is the incident photon flux at wavelength λ of the solar spectrum.

Reflection co-efficient at normal incidence from the third layer of a 3 layer system comprising of a thin film sandwiched between two bulk materials extending away fromtheir interfaces with the thin film is given as:

$$R = \frac{n_2^2(n_1 - n_3)^2\cos^2\vartheta + (n_1 n_3 - n_2^2)^2\sin^2\vartheta}{n_2^2(n_1 + n_3)^2\cos^2\vartheta + (n_1 n_3 + n_2^2)^2\sin^2\vartheta}$$

where

$$\vartheta = \frac{2\pi n_2 d}{\lambda}$$

and n_1, n_2 and n_3 are the refractive indices of 3 materials m_1, m_2 and m_3 respectively: d is the thickness of the thin film m_2 sandwiched between materials m_1 and m_3. Direction of light flow is taken to be from m_1 to m_3.

Fill Factor for a Si solar cell

$$FF_0 = \frac{\frac{eV_{oc}}{kT} - \ln\left(\frac{eV_{oc}}{kT} + 0.72\right)}{\frac{eV_{oc}}{kT} + 1}$$

where V_{oc} is the open-circuit voltage for the cell.

Constants

Electronic charge unit **e**: 1.602×10^{-19} C

Boltzmann's constant **k**: 1.381×10^{-23} J K^{-1}

Speed of light **c**: 2.998×10^{8} m s^{-1}

Planck's constant **h**: 6.626×10^{-34} J s

Dielectric permittivity free space $\boldsymbol{\varepsilon_0}$: 8.854×10^{-12} F m^{-1}

Relative permittivity of Si $\boldsymbol{\varepsilon_r}$:11.9

Refractive index (for weakly and non-absorbing wavelengths) $n = \sqrt{\boldsymbol{\varepsilon_r}}$

Band-gap energy of Si: $\boldsymbol{E_g}$:1.12 eV