Answers 4B19 – 2021   Dr T J Flack, Prof G A J Amaratunga

Question 1

a) (i) $\lambda=\omega_r R/v$ so $8=\omega_r\times42.5/10$ giving $\omega_r=1.88$ rads^{-1} (=18.0 rpm, reasonable)

$P=0.5C_p\rho Av^3 = 0.5\times0.38\times1.23\times\pi\times42.5^2\times10^3$ giving P=1.33 MW

$T\omega_r=p$ giving T = 705 kNm   [10%]

(ii) $E=k\omega_r =280\times1.88=526$ V giving $\sqrt{3}\times526=912$ V as the line-line excitation emf.

T=3kI and solving for I gives 839 A

$\omega_r=\omega/p$ and with p=20 gives $\omega=37.6$ rads^{-1} and $\omega L_s=0.143\Omega$

$V^2=E^2+(\omega L_s I)^2$ giving V=539 V i.e. 934 V line-line.

$\cos\delta=E/V=526/539$ giving $\delta=12.6^0$ [15%]

b) Rated wind speed is 12 ms^{-1} and so the angular speed at this wind speed can be found from $\lambda=\omega_r R/v$ giving $\omega_r=2.26$ rads^{-1}. Rated power is the power at rated wind speed, and this can be found from the power equation, or more easily by using the fact that at fixed, optimum tip-speed ratio the power scales as v^3 giving $P=(12/10)^3\times1.33=2.30$ MW.

(i) At the wind speed of 14 ms^{-1}, because we are told that the system is controlled so that the turbine angular speed is the same as at rated wind speed i.e. 2.26 rads-1, then this will be the turbine angular speed. We are also told that the power is capped at rated power, so P=2.30 MW from which the torque may be found using $P=T\omega_r$ giving T=1000 kNm.   [10%]

(ii) T=3kI gives I=1191 A and $E=k\omega_r$ gives E=633 V which gives 1096 V as the line-line excitation emf. [15%]

c) Every wind turbine is rated at 2.3 MW and assuming that all the wind farm turbines are operating at rated output, and neglecting power losses leading up to the input to the platform converter means that the converter has a rating of 230 MVA.

(i) The input power to the converter is 230 MW and if it is 99% efficient then 1% of the input power is lost, so the power loss is 2.3 MW. [10%]

(ii) The input power to the dc link is 230 -2.3 = 227.7 MW and this is equated with $V_{DC}I_{DC}$ giving the dc link current as 759 A. The dc link power loss is I^2R, and with R = 5 Ω this gives 2.88 MW. [10%]

(iii) Thus, the power reaching the shore is 227.7 – 2.88 = 224.8 MW and the overall efficiency is 224.8/230 = 97.7%   [10%]

d) All of main powertrain components would need to be upgraded to operate at $(14/12)^3$ more power, including the blades and the main drive shaft. Then there are the electrical upgrades to the generator, local transformer, local converters, platform converter and the dc link. The benefit would be that at wind speeds greater than 12 ms^{-1}, the full power available can be exploited up to 14 ms^{-1}, and for wind speeds greater than the new rated wind speed, the output power will be 1.58 times what was previously available. Thus, more electrical energy is produced which can be sold to the grid operators. Whether this makes sense depends on how often these higher wind speeds occur in practice, and whether this additional capability is utilised regularly enough to offset the upgrade costs.

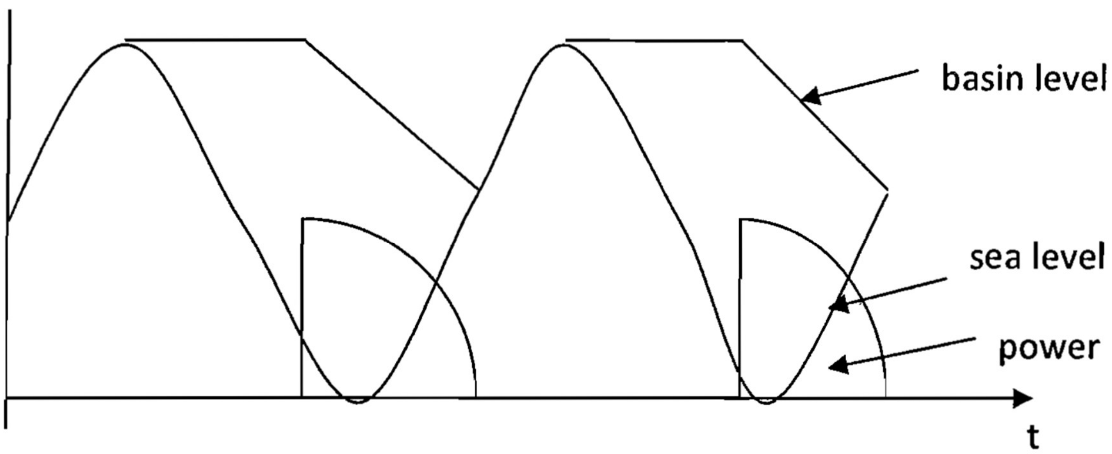
Discounted cash flow analysis enables an economic assessment of the proposal by rebasing all future incomes and costs to the start of the project. Annuitisation tables are used which enable the cost of borrowing the money over the lifetime of the project (often 25 years) at a given nett interest rate to be included in the calculation. [20%]

Question 2

(a) The potential energy of a body of water of volume V and at head H with respect to the power extraction point is ρVgH. If water is allowed to flow then the rated of change of this energy is given by P= ρgHdV/dt and replacing dV/dt with volumetric flowrate Q gives ρgHQ. This is an upper bound on the power available, in reality there are power losses due to things like turbulence of the flow, turbine and generator losses, and these are all accounted for by the efficiency, η, to give P=ηρgHQ. Taking the density of water to be 1000 kgm^{-3}, and g ~ 10 ms^{-2} gives P ~ 10000ηHQ. [10%]

(b) As the sea level rises water becomes trapped behind a barrage. Thus, potential energy is acquired. When the sea level has fallen to close to its low tide level, the water behind the barrage is released via turbines coupled to electrical generators, thereby converting the potential energy to electrical power.

Basin area is A, tidal range is R, at high tide centre of gravity of water is at R/2 and so potential energy = MgR/2 = ρARgR/2 = $\rho gAR^2/2$. If this is converted to power in period T between high tides then the average power is energy/T = $\rho gAR^2/2T$. This is an upper limit - in reality there will be power losses etc. The graphs below show the relationship between basin level, sea level and power ouput, illustrating these ideas.



The difficulty of integrating these schemes are the relatively short bursts of high power output may not be matched to the demand for electricity (the tidal period is 12 hours, 25 minutes and so the timing of the bursts of power production will keep shifting). Also, the best geographical locations may not be close to major load centres. Means of mitigation are plant scheduling (the times of generation are completely predictable) and grid upgrades in order to enable the grid to cope with the large power flows when the scheme is generating. [25%]

(c) (i) The total volume of water trapped is AR = $15\times140\times10^6$ m^3 = 2.1×10^9 m^3

The time taken for this volume of water to drain out of the barrage is 5 hours = 5×60×60 s = 18000 s.

The flow rate is then $2.1\times10^9/18000$ = 116.7×10^3 m^3s^{-1}. [5%]

(ii) The upper limit to the average power available is given by

$P = \rho g A R^2/2T = 1000\times9.81\times15^2\times140\times10^6/(2\times12\times60\times60) = 3.58$ GW. 60% of this is extracted giving 2.15 GW as the average output power.

Peak output power is when the scheme first starts generating and so the head of water is 15 m giving

$P_{max} = \eta\rho gHQ = 0.6\times1000\times9.81\times15\times116.7\times10^3 = 10.3$ GW

Total annual energy is $365\times24\times2.15 = 18.8$ TWhr.

Capacity factor = Average power/Peak power = 2.15/10.3 = 0.21 [20%]

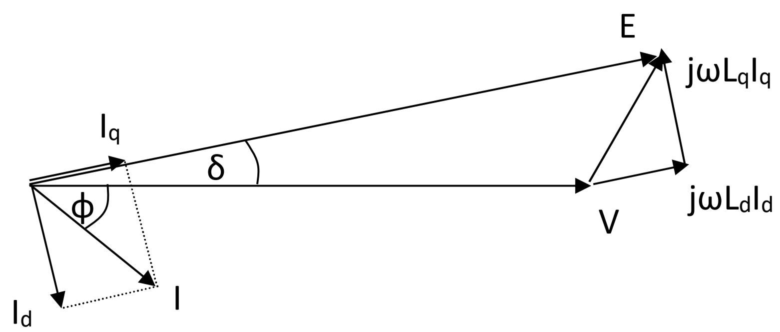
(d) (i) The turbines have to be rated for the peak power of 10.3 GW and so because there are 150 identical turbines, they are individually rated at 10.3/150=68.7 MW.

The turbines rotate at 150 rpm which is 15.7 rads^{-1} and so using $T\omega_r$=p gives the torque as 4.37 MNm. [10%]

(ii) In the worst-case scenario the generators will be operating at 0.6 lagging power factor at maximum power of 68.7 MW giving a VA rating of 68.7/0.6=114.5 MVA.

A 2 pole generator connected to a 50 Hz grid rotates at 3000 rpm, so for 150 rpm operation the generator needs 20 times as many poles i.e. 40 poles. [10%]

(e) The first step is to draw a generic phasor diagram:



The first step is to draw a generic phasor diagram:

V_{ph}=11 kV/√3 (star-connected)=6.35 kV and from $P=3VI\cos\phi$ with P=68.7 MW, V=6350 V and $\cos\phi$=0.8 we find I=4.51 kA.

$I_q = I\cos(\phi+\delta) = I(\cos\phi\cos\delta-\sin\phi\sin\delta)$

$V\sin\delta = X_qI_q = X_qI(\cos\phi\cos\delta-\sin\phi\sin\delta)$

Divide through by $\cos\delta$ and make $\tan\delta$ the subject of the equation:

$\tan\delta = X_qI\cos\phi/(V+X_qI\sin\phi) = 0.209$ and $\delta=11.1^0$

Putting in $X_q=0.75\Omega$, I=4510 A, $\cos\phi$=0.8, V=6351 V gives $\tan\delta = 0.323$ and so $\delta=17.9^0$

I_d = Isin(ϕ+δ) = 3684 A

E = Vcosδ + I_dX_d = 9727 V and so 16.8 kV line [20%]

Question 3

(i) Built in potential given by:

$$V_{bi} = \frac{KT}{q}\ln\left[\frac{N_D N_A}{n_i^2}\right]$$

$n_i = 3.18 \times 10^{16}$ at T = 300 K from expression given. N_D (n- doping), N_A (p-doping) defined in Q and K, q from data book or formulae and constants sheet

$$V_{bi} = 0.026\ln\left[\frac{25x10^{46}}{16.10^{32}}\right] = 0.85\,V$$

(ii)

$$n_p(0) = n_{p0}e^{Vfq/kT} = N_D e^{-q(Vbi-Vf)/kT} = 5x10^{24}e^{-0.35/0{,}026} = 7.12x10^{18}\,m^{-3}$$

n_{po} is the intrinsic minority (electron) carrier concentration on the p side of the junction and N_D is the doping concentration on the n – side.

(iii) The minority electron current density J_e on the p side is:

$$J_e = -qD_e\frac{dn_p}{dx} \;\; and - \frac{dJ_e}{dx} = \frac{n_p(x) - n_{po}}{\tau_e}$$

$$n_p(x) - n_{po} = Aexp\left(-\frac{x}{L_e}\right) where\ A = n_p(0) - n_{po}\ hence$$

$$n_p(x) = n_p(0)\exp\left(-\frac{x}{L_e}\right) - n_{po}\exp\left(-\frac{x}{L_e}\right) - n_{po}$$

$$substituting\ for\ n_p(0) from\ (ii) \quad J_e(0) = J_e = qD_e n_{po}\left\{\exp\left(\frac{qV_f}{kT}\right) - 1\right\}/L_e$$

Similarly for holes $J_h = qD_h p_{no}\{\exp\left(\frac{qV_f}{kT}\right) - 1\}/L_h$

Total current is A($J_e + J_h) = qA\left[\frac{n_i^2 D_e}{N_A L_e} + \frac{n_i^2 D_h}{N_D L_h}\right]\left\{\exp\left(\frac{qV_f}{kT}\right) - 1\right\}\ and\ L_{e,h} = \sqrt{D_{e,h}t_{e,h}}$

D_h diffusion length for electrons (minority carries in the p-region) and holes (minority carriers in the n-region) and t_e and t_h their respective lifetimes. A junction area.

Plugging in values one gets *$I_f = 6.67 \times 10^{-3}$* A , The pre exponent factor $I_s = 2.63\ x10^{-11}\ A$

(iv) The short circuit current per cell $I_{sc} = I_{op} = qA(L_e + L_h)G_{op} = 1.602x0.514x5 = 4.1\ A$

This is also the short circuit current for the panel as the cells are connected is series.

(v) The open circuit voltage per cell is:

$V_{oc\ cell} = \frac{KT}{q}\ln\left[\frac{I_{sc}}{I_s} + 1\right] = 0.026\ln[(4.1/1.28x10^{-10})$+1] = 0.63 V

Since there are 36 cells connected in series $V_{oc-panel} = 0.63\ x\ 36 = 22.6$

(vi) The nominal efficiency is

$$\eta = V_{oc} I_{sc} FF / P_{in}$$

$$P_{in} = 1.0\ x\ 0.4 = 0.4\ kW$$

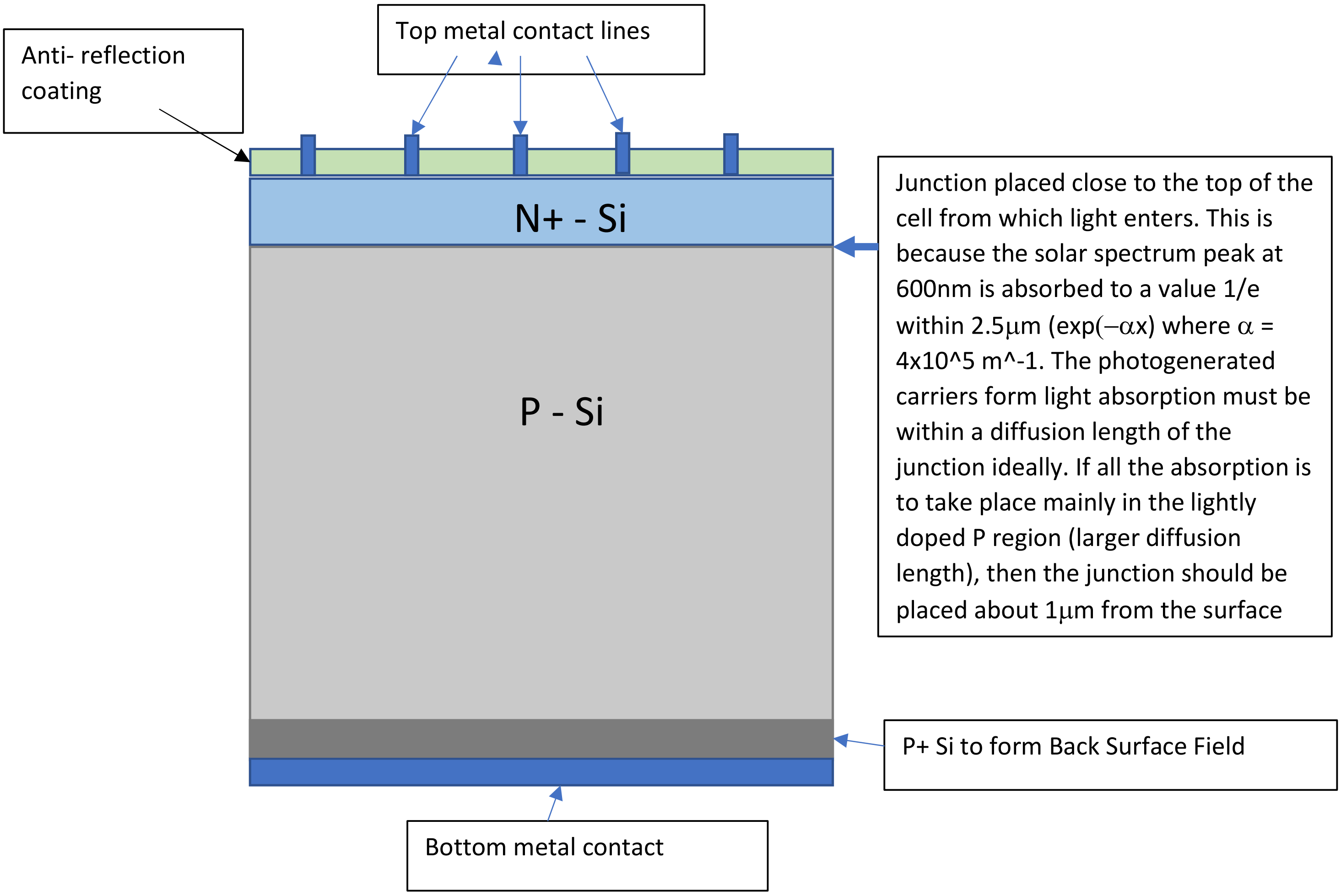
FF the fill factor has to be taken to be FFo assuming that the series resistance of each cell is negligible. From the data sheet

$$FF_o = \left(\frac{qV_{oc\,cell}}{KT} - \ln\left[\frac{qV_{oc\,cell}}{KT} + 0.72\right]\right) / \left(\frac{qV_{oc\,cell}}{KT} + 1\right) = 0.83$$

Hence $\eta = 22.6 x 4.1 x 0.83/400 = 0.19$ or 19%

Question 4

(a)



(b)

Si can only absorb light which has energy above its band gap energy of 1.12eV. It is stated that the wavelength of light which corresponds to be just above the band gap energy is at 1100nm and has an absorption coefficient of 10^3 m^{-1}. Given that absorption is given by

$I = I_0 e^{-\propto x}$ where α is the absorption coefficient, then the Si should be at least be 1mm thick. But more accurately light at 1100nm should have an optical path length in the Si of at least 1mm. Since the back metal acts as reflector, then normally incident light will have a nominal 1mm path length with **0.5mm thick Si**.

(c)

One way of reducing the cost of a solar cell is to use less Si. This is possible if the Si cell thickness is reduced. But then from (b) above the light absorbed is reduced. However, there is the possibility of reducing the thickness since not all light is normally incident and that the back surface contact has some roughness (not a perfectly smooth mirror) the reflected light (reflection angle larger than 0) and incident light (refraction on entering the Si) will have on average a path length longer than that of ideal normally incident light. Hence it is possible to reduce the Si thickness to about 0.25mm (250μm). It is infact possible to design in high roughness at the back metal contact so that light is reflected at large angles. In this case one can achieve large optical path length of light in Si layer approaching 50μm in thickness through confining light by total internal reflection in the Si – referred to as light trapping.

(d)

Considering normally incident light with an intensity I_0 at the Si surface where light enters, light absorption is given as the decay intensity as the light goes through the Si.

$I = I_0$ exp(-$\propto$x) ; at any given location x the light 'lost' contributes to electron-hole generation. This occurs on the basis of the photons absorbed. Hence light intensity needs to be converted to a photon flux at a given wavelength.

$I_0(\lambda)/(hc/\lambda) = \Phi_0(\lambda)$; but not all the photons enter the Si as there is some reflection at the surface.

$\Phi_0(\lambda) = (1-R(\lambda))\Phi_I(\lambda)$ therefore photon flux in the Si is given as

$\Phi(\lambda,x) = \Phi_0(\lambda)\exp(-\alpha(\lambda)x)$

The photons 'lost' in any given interval $\Delta x = -\Delta\Phi$. Therefore the electron hole pairs generated at any given X is:

$G(x,\lambda) = -d\Phi(\lambda,x)/dx = (1 - R(\lambda))\Phi_I(\lambda)\alpha(\lambda)\exp(-\alpha(\lambda)$

Therefor the total e-h pairs generated at 600nm is:

$G(\lambda) = \int_{x1}^{x2} (1 - R(\lambda))\Phi_I(\lambda) 4x10^5 \exp(-4X10^5 x)$ where λ = 600nm

(e) Not all the carriers G(λ) can contribute to the photogenerated current as some of them will recombine as minority carriers before they diffuse to the junction and get separated by the electric field across the junction. The recombination rate for minority carrier electrons in the p-region is given as :

$R_n = (n_p(x) - n_{po})/\tau_e$

and in the n-region the hole recombination rate is $R_p = (p_n(x) - p_{no})/\tau_h$

There can also be Schokley-Read-Hall type recombination in the depletion region of the junction as the carriers are swept across the junction.

(f) From the data sheet the condition for minimum reflectivity requires:

$\Theta = \frac{2\pi n d}{\lambda} = \frac{\pi}{2}$ where n is the refractive index of the coating, λ is the wave length and the thickness of the coating.

d= $\frac{\lambda}{4X1.8} = 150/1.8 = \mathbf{83nm}$.

With this thickness the reflectivity at normal incidence for 600 nm light is (from data sheet)

$$\frac{n_1 n_3 - n_2^2}{n_1 n_3 + n_2^2}$$

$n_1 = 1$ $n_2 = 1.8$ and $n_3 = \sqrt{11.9} = 3.45$ and **R(λ=600) = 0.2/6.69 = 0.03**

Examiners' comments

Q1 Offshore wind power: 21 IIB attempts, mean 14.3/20

A very popular question, with lots of good attempts. Part (b) caused the most difficulties, as it required a combination of understanding how the turbine is being controlled, and some generator calculations. A number of candidates confused terminal voltage and excitation voltage. Part (d) resulted in some incomplete answers, and candidates forgetting that the system comprises many different entities that would all need to be upgraded.

Q2 Hydroelectricity and tidal barrage scheme: 20 IIB attempts, mean 12.2/20

Another popular question, attracting lots of very good answers. Part (c) caused a number of difficulties, requiring the candidates to appreciate the difference between peak and average output power, and then realise that the system has to be rated for peak power. Part (d) also caused difficulties for similar reasons. There were many good attempts at part (e), with candidates showing a good understanding of salient-pole synchronous generators. However, there were few correct answers, and a lot of consequentially incorrect ones, which were not penalised.

Q3 – 16 attempts (out of 20 candidates), Average mark 71%

A question on the fundamental principles of a semiconductor p-n junction in Si and how they can be exploited to devise a photovoltaic solar cell. A large majority of those attempting the question showed a very good understanding of the underlying principles of the p-n junction solar cell and how they can be translated to the major solar cell output parameters for generating electric power. It is possible therefore to conclude that one of the main pedagogic aims of the course have been met.

Q4 – 6 attempts (out of 20 candidates), Average mark 56%

A question on the design details of a solar cell and how it is linked to optical absorption in Si and the resulting photon to electron – hole pair generation. This was the least popular question on the paper. Probably as it requires some detailed understanding of how a solar cell operates as an optoelectronic device. There were two answers at the first class standard.