EGT3
ENGINEERING TRIPOS PART IIB

Tuesday 2 May 2023 14.00 to 15.40

Module 4B23

OPTICAL FIBRE COMMUNICATION

Answer not more than ***two*** *questions.*

All questions carry the same number of marks.

The ***approximate*** *percentage of marks allocated to each part of a question is indicated in the right margin.*

Write your candidate number ***not*** *your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
Engineering Data Book
Attachment: 4B23 Optical Fibre Communication formula sheet (2 pages)

10 minutes reading time is allowed for this paper at the start of the exam.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

You may not remove any stationery from the Examination Room.

1 SMF-28 ULL optical fibre is an ultra-low loss optical fibre that is used for submarine systems. 6500 km of SMF-28 ULL optical fibre is used in an optically amplified transatlantic link. Over this link, 88 WDM channels ranging from 1530 nm to 1610 nm are transmitted. Each WDM channel transmits 400 Gbit/s with a symbol rate of 100.5 GBd. The transceiver employs digital signal processing to create a rectangular Nyquist spectrum, with the chromatic dispersion compensated digitally at the receiver.

(a) The wavelength dependence of the refractive index, n, of an optical fibre can be modelled using the following equation

$$n = A + \left(\frac{B}{\lambda}\right)^2 - \left(\frac{\lambda}{b}\right)^2$$

where λ is the wavelength of light (λ always refers to the wavelength in vacuo), A, B and b are constants to be determined. At a wavelength of 1550 nm, SMF-28 ULL optical fibre has a group refractive index $n_g = 1.4620$ and a chromatic dispersion coefficient D =17 ps/nm/km, with the zero dispersion wavelength being 1312 nm. Determine the values of the constants A, B and b for the SMF-28 ULL optical fibre. [30%]

(b) Hence or otherwise determine the minimum latency experienced by the data transmitted using a WDM channel due to the 6500 km optical fibre link. [10%]

(c) The digital coherent receiver uses an oversampling rate of 16/15. Calculate the minimum number of taps N_{CD} that should be included in the transceiver to compensate the chromatic dispersion for any WDM channel when the 100.5 GBd signal is transmitted over the 6500 km link. [30%]

(d) Assuming appropriate frequency domain implementation, calculate the optimum FFT size and hence estimate the minimum power consumption required to realise digital chromatic dispersion compensation for the 400 Gbit/s 100.5 GBd signal given the energy required to perform a complex multiply is 0.5 pJ. [30%]

2 The EDFA underpins modern optical communication systems. If N_1 and N_2 represent the population densities for the number of atoms at level 1 and 2 respectively then the number of photons n in the erbium doped fibre will increase with distance z according to following differential equation

$$\frac{dn(z)}{dz} = \eta N_2 n(z) - \eta N_1 n(z) + \eta N_2$$

where η is a constant representing the efficiency of the process.

(a) By solving the differential equation for a fibre of length L show that the number of photons exiting n_{out} will be

$$n_{out} = n_{in} G + n_{sp}(G - 1)$$

where $n_{in} = n(0)$, $G = \exp[\eta(N_2 - N_1)L]$ is the gain and $n_{sp} = N_2/(N_2 - N_1)$ is the population-inversion factor. [20%]

(b) A 64GBd PDM-16QAM signal is transmitted over a 480 km optical fibre link. At the operating wavelength of 1550 nm, the optical fibre has an attenuation of 0.2 dB/km, dispersion coefficient of 17 ps/nm/km, a PMD coefficient of 0.1 ps/$\sqrt{\text{km}}$ and nonlinear coefficient of 1.3 $\text{W}^{-1}\text{km}^{-1}$. For the EDFA $N_2 = 2N_1$ and the gain of the EDFA can be tailored to exactly match the span loss.

(i) For the initial design, nonlinear impairments are neglected and the system is assumed to operate in the linear regime with an initial launch power of 0 dBm. Calculate the received signal to noise ratio for EDFA amplifier spacings of 80 km. [20%]

(ii) For the more detailed design, nonlinear impairments are included with an assumed total modulated optical bandwidth of up to 5 THz with WDM channels on a 75 GHz grid. Assuming the optimal launch power is used, calculate the received signal to noise ratio for EDFA amplifier spacings of 80 km. [20%]

(iii) Calculate the change in the SNR for both the initial design and the more detailed design if the amplifier spacing is reduced to 60 km. [20%]

(c) Finally for a certain EFDA, the power consumption is dominated by the pump laser power, which in turn is proportional to the gain of the amplifier. What amplifier spacing L_s minimises the overall power consumption for a long-haul link of length L with fibre attenuation of α if the gain G exactly matches the span loss such that $G = \exp(\alpha L_s)$. [20%]

3 A step index optical fibre with a core diameter of 10 μm, has core and cladding refractive index n_{co}=1.45 and n_{cl}=1.446 respectively.

(a) For this step index optical fibre determine the minimum wavelength for which only the LP_{01} mode propagates. [10%]

(b) The propagation constant β associated with the LP_{01} mode is given by

$$\beta^2 = \frac{k_0^2 \int_0^\infty n^2 E^2 r dr - \int_0^\infty \left(\frac{dE}{dr}\right)^2 r dr}{\int_0^\infty E^2 r dr}$$

where E and n are the electric field and refractive index which are both functions of radial distance r. The radial dependence of the electric field $E(r)$ of the LP_{01} mode can be approximated by a Gaussian such that:

$$E(r) = E_0 \exp\left(-\frac{r^2}{r_0^2}\right)$$

where E_0 corresponds to the peak electric field.

(i) Show, using the Gaussian approximation for the LP_{01} mode, that the propagation constant β is given by

$$\beta^2 = \left[1 - \exp\left(-2\frac{a^2}{r_0^2}\right)\right] n_{co}^2 k_0^2 + \exp\left(-2\frac{a^2}{r_0^2}\right) n_{cl}^2 k_0^2 - \frac{2}{r_0^2}$$

[20%]

(ii) By considering the stationary value of β^2 with respect to r_0 show that

$$r_0 = \frac{a}{\sqrt{\ln V}}$$

where V is the normalised wavenumber. [10%]

(c) The step index optical fibre is used to transmit an EDFA pump having a wavelength of 1480 nm. Estimate the maximum pump power if the electric field in the optical fibre should be kept below 6 MV/m to avoid damage to the fibre. [30%]

(d) The scattering of the light at the core cladding interface is proportional to the magnitude of the electric field $E(a)$ at the interface. Using the Gaussian approximation, for a fixed power level in the fibre, determine the value of V that gives the maximum scattering. [30%]

END OF PAPER

Formula	Notes
$\langle S \rangle = \frac{1}{2}\frac{n}{\eta_0}\lvert E_0 \rvert^2$	$\langle S \rangle$ - time averaged Poynting Vector, E_0 - complex electric field, $\eta_0 = \sqrt{\mu_0/\varepsilon_0} = 377\ \Omega$, n is refractive index.
$P(z) = P(0)\exp(-\alpha z)$	α in nepers/km, related to loss in terms of α_{dB}, the loss in dB/km via $\alpha \approx 0.23\alpha_{dB}$
$L_{eff} = [1 - \exp(-\alpha L)]/\alpha$	Effective length L_{eff} associated with L and loss α
$v_p = \frac{\omega_0}{\beta} = \frac{c}{n}$	Phase velocity v_p, for electric field $E = E_0 e^{j(\omega_0 t - \beta z)}$, where $\beta = k_0 n$ and n is refractive index and $k_0 = 2\pi/\lambda$ (note λ always refers to the wavelength in vacuo). $c = 3 \times 10^8$ m/s
$v_g = \frac{d\omega}{d\beta}$	Group velocity v_g of a modulated wave $e^{j(\omega t - \beta z)}$
$n_g = c/v_g$	Group refractive index $n_g = n + \omega\frac{dn}{d\omega} = n - \lambda\frac{dn}{d\lambda}$
$D = -\frac{2\pi c}{\lambda^2}\beta_2 = -\frac{\lambda}{c}\frac{d^2 n}{d\lambda^2}$	Dispersion coefficient D: $\beta_2 = \frac{d^2\beta}{d\omega^2}$. For λ =1550 nm $D\ (ps/nm/km) = -0.78 \times \beta_2 (ps^2/km)$
$\Delta t = D\Delta\lambda L$	Dispersion Δt, with dispersion coefficient D, spectral width $\Delta\lambda$, over a distance L. For 1550 nm 100 GHz spectrum corresponds to 0.8 nm
$V = k_0 a\sqrt{n_{co}^2 - n_{cl}^2}$	Normalised wavenumber for step index fibre. Core radius a, core refractive index n_{co}, cladding refractive index n_{cl}
$J_{m-1}(V) = 0$	Cutoff criterion for the modes. LP_{mn} is n^{th} solution of $J_{m-1}(V) = 0$ where J_m is the m^{th} order Bessel function of the first kind
$F(r) = \exp\left(-\frac{r^2}{r_0^2}\right)$	Gaussian approximation for fundamental mode with mode field radius: $r_0^2 = \frac{a^2}{\ln V}$
$\eta = \frac{2\sigma_1\sigma_2}{\sigma_1^2 + \sigma_2^2}\exp\left(-\frac{\Delta^2}{\sigma_1^2 + \sigma_2^2}\right)$	Overlap integral between two normalised Gaussian fields separated by Δ with the mode field radii σ_1 and σ_2

First n zeros for $J_k(x)$

k \ n	1	2	3	4	5
0	2.405	5.520	8.654	11.792	14.931
1	3.832	7.016	10.173	13.324	16.471
2	5.136	8.417	11.620	14.796	17.960
3	6.380	9.761	13.015	16.223	19.409
4	7.588	11.065	14.373	17.616	20.827
5	8.771	12.339	15.700	18.980	22.218

Formula	**Notes**
$S_{3dB} = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & j \\ j & 1 \end{bmatrix}$	Scattering matrix of a 3 dB coupler
$i = RP = R\|A\|^2$	Current in a photodiode, responsivity R, electric field amplitude A, optical power $P = \|A\|^2$
$\phi = \gamma P$	Kerr nonlinear phase shift: $\gamma = n_2 k_0 / A_{eff}$ where for optical fibres it can be assumed $n_2 = 2.6 \times 10^{-20}$ m^2/W
$\frac{\partial A}{\partial z} = j\frac{\beta_2}{2}\frac{\partial^2 A}{\partial t^2}$	Effect of dispersion in retarded frame of reference. With loss and nonlinearity becomes the NLSE $\frac{\partial A}{\partial z} = -\frac{\alpha}{2}A + j\frac{\beta_2}{2}\frac{\partial^2 A}{\partial t^2} - j\gamma \|A\|^2 A$
$P(\Delta\tau > x) \approx \frac{4}{\pi}\frac{x}{\langle\Delta\tau\rangle}\exp\left(-\frac{4x^2}{\pi\langle\Delta\tau\rangle^2}\right)$	Probability that with mean DGD $\langle\Delta\tau\rangle$ the instantaneous DGD $\Delta\tau$ exceeds x
$N_{NLI} = C_{NLI} G_{TX}^3$	Nonlinear noise power density for input PSD of G_{TX} with $C_{NLI} = \frac{8\gamma^2 L_{eff}^2 \alpha}{27\pi\|\beta_2\|}\ln\left(\frac{\|\beta_2\|}{\alpha}\pi^2 B^2\right)$
$N_{ASE} = 10^{NF/10} h\nu (G-1)$	Power spectral density for ASE amplifier with gain G and noise figure NF. $h = 6.634 \times 10^{-34}$ Js and for $\lambda \approx$1550 nm, $h\nu \approx 1.3 \times 10^{-19}$ J = 0.8 eV.
$G_{opt} = \sqrt[3]{\frac{N_{ASE}}{2C_{NLI}}}$	Optimum power spectral density
$N_q = h\nu P$	PSD for quantum noise
$N_{cm} = \frac{N\log_2(N) + N}{N - N_f + 1}$	Number of complex multiplications per sample for overlap and save implementation of a filter of length N_f using N point FFT (that in turn requires $0.5N\log_2 N$ complex multiplications)
$\boldsymbol{h} \coloneqq \boldsymbol{h} - \mu\frac{\partial\|\epsilon\|^2}{\partial \boldsymbol{h}^*}$	Stochastic gradient update for taps $\boldsymbol{h}$ with error ϵ and convergence parameter μ
$C = 1 - H_2(p_b)$	Capacity of binary symmetric channel where $H_2(p_b) = -(1-p_b)\log_2(1-p_b) - p_b\log_2 p_b$
$C = B\log_2(1 + SNR)$	Shannon capacity (for one polarisation), with bandwidth B and signal to noise ratio SNR