EGT3
ENGINEERING TRIPOS PART IIB

Tuesday 30 April 2024 2.00 to 3.40

Module 4B23

OPTICAL FIBRE COMMUNICATION

Answer not more than ***two*** *questions.*

All questions carry the same number of marks.

The ***approximate*** *percentage of marks allocated to each part of a question is indicated in the right margin.*

Write your candidate number ***not*** *your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
Engineering Data Book
Attachment: 4B23 Optical Fibre Communication formula sheet (2 pages)

10 minutes reading time is allowed for this paper at the start of the exam.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

You may not remove any stationery from the Examination Room.

1 In a 3 x 3 fused fibre coupler the three electric field amplitudes $A_1(z)$, $A_2(z)$ and $A_3(z)$ are governed by the following matrix differential equation

$$\frac{d}{dz}\mathbf{A}(z) = -j\kappa J_3 \mathbf{A}(z)$$

where κ is the coupling coefficient, $j = \sqrt{-1}$, $\mathbf{A}(z) = [A_1(z), A_2(z), A_3(z)]^T$ and J_3 is the 3 x 3 all-ones matrix (where all elements are 1).

(a) Obtain a differential equation for $A_s(z) = A_1(z) + A_2(z) + A_3(z)$ and hence obtain an expression for $A_s(z)$ in terms of κz and the initial value $A_s(0)$. [20%]

(b) Hence or otherwise show that for a coupling length L the output fields are given by

$$A_m(L) = \frac{e^{-3j\kappa L} - 1}{3} A_s(0) + A_m(0)$$

where $m \in \{1, 2, 3\}$. [20%]

(c) The 3 x 3 coupler is to be used for coherent detection with the initial fields being $\mathbf{A}(0) = [A_{sig}, 0, A_{lo}]^T$. Each of the output fields $\mathbf{A}_m(L)$ are detected by a photodiode with the corresponding photocurrents given by

$$i_m = R|A_m(L)|^2 = a_m|A_{sig}|^2 + b_m|A_{lo}|^2 + c_m A_{sig}A_{lo}^* + d_m A_{sig}^* A_{lo}$$

where $m \in \{1, 2, 3\}$, R is the responsivity of the photodiode, A^* denotes the conjugate of A and a_m, b_m, c_m and d_m are constants to be determined. Show that

$$c_1 = \frac{R}{9}(\cos 3\kappa L - 1 + 3j \sin 3\kappa L)$$

$$c_3 = \frac{R}{9}(\cos 3\kappa L - 1 - 3j \sin 3\kappa L)$$

[30%]

(d) The power splitting ratio gives the proportion of power in the three outputs when only the first port is illuminated e.g. 1:1:1 for a symmetric coupler. Determine the power splitting ratio for which the phase between the coherent products c_1 and c_3 is 90°. [30%]

2 An erbium doped fibre amplifier (EDFA) can be modelled as a two level process where N_1 and N_2 represent the population densities for the number of atoms at level 1 and level 2 respectively. The corresponding number of photons n in the erbium doped fibre (EDF) will increase with distance z according to the following differential equation

$$\frac{dn(z)}{dz} = \eta N_2 n(z) - \eta N_1 n(z) + \eta N_2 \tag{1}$$

where η is a constant representing the efficiency of the process.

(a) State the processes modelled by:

(i) $\eta N_2 n(z)$ [2%]

(ii) $-\eta N_1 n(z)$ [2%]

(iii) ηN_2 [2%]

(b) By solving Eq. (1) for an amplifier of bandwidth B, employing an EDF of length L, show for a polarisation division multiplexed (PDM) system, that the output power P_{out} is given by

$$P_{out} = P_{in}G + 2h\nu n_{sp}(G-1)B$$

where P_{in} is the input power, $G = \exp[\eta(N_2 - N_1)L]$ is the gain and $n_{sp} = N_2/(N_2 - N_1)$ is the population-inversion factor. [24%]

(c) A submarine link uses two identical EDFAs (both with $n_{sp} = 1$), one at the midpoint and one at the receiver, with the gain tailored to match the loss of the fibre which is 0.15 dB km^{-1} in the C-band (1530 nm to 1565 nm) and 0.18 dB km^{-1} at $\lambda = 1480$ nm. A C-band PDM transceiver with an output power of 0 dBm, transmits 1.6 Tbit s^{-1}, using a symbol rate of 200 GBd and achieves the Shannon limit. The transceiver internally generates a noise-to-signal ratio (NSR) of -23 dB. Fibre nonlinearity may be neglected.

(i) Use the Shannon limit to calculate maximum permitted NSR at the receiver. [10%]

(ii) Calculate the maximum system length assuming the link generates the maximum tolerable additive NSR between the transmitter and the receiver. [30%]

(iii) It is proposed to remotely pump the EDFA at the midpoint, transmitting the pump (λ=1480 nm) with the data channels located in the C-band. To avoid fibre damage the transmitted pump power is limited to 2.5 W. The efficiency of the EDFA limits the EDFA maximum output power to 40% of the received pump power. Estimate the maximum total link length and the optimal EDFA output power assuming all 22 wavelength division multiplexed channels in the C-band are used. [30%]

3 A 1.6 Tbit s^{-1} signal is transmitted over 400 km of single mode optical fibre with a symbol rate of 200 GBd. The polarisation division multiplexed transmitter employs digital signal processing to create a rectangular Nyquist spectrum, with the chromatic dispersion compensated digitally at the receiver. At the operating wavelength of 1550 nm, the optical fibre has an attenuation of 0.15 dB km^{-1}, a dispersion coefficient of 17 ps nm^{-1} km^{-1}, a polarisation mode dispersion (PMD) coefficient of 0.1 ps $km^{-1/2}$ and a nonlinear coefficient of 1.3 W^{-1} km^{-1}.

(a) Briefly outline the equalisation stages employed in a digital coherent receiver. [10%]

(b) Calculate the minimum number of taps N_{CD} required to compensate the chromatic dispersion in the 200 GBd signal transmitted over the 400 km link if the digital coherent receiver uses an oversampling rate of 64/63. [20%]

(c) Assuming appropriate frequency domain implementation, estimate the minimum power consumption required to realise digital chromatic dispersion compensation for the 200 GBd signal assuming the energy required to perform a complex multiply is 0.1 pJ. [30%]

(d) An adaptive equaliser is used to correct for the PMD present in the signal. Estimate the minimum total power required for the equaliser if the outage probability due to PMD should be less than 10^{-12}. [20%]

(e) Conventionally the chromatic dispersion and the adaptive filter are partitioned. As the adaptive filter increases in length however it will become more efficient to combine the chromatic dispersion compensation into the adaptive filter. For this 200 GBd receiver operating over 400 km estimate the number of taps in the adaptive filter when this transition occurs. [20%]

END OF PAPER

Formula	Notes
$\langle S \rangle = \frac{1}{2}\frac{n}{\eta_0}\lvert E_0\rvert^2$	$\langle S \rangle$ - time averaged Poynting Vector, E_0 - complex electric field, $\eta_0 = \sqrt{\mu_0/\varepsilon_0} = 377\ \Omega$, n is refractive index.
$P(z) = P(0)\exp(-\alpha z)$	α in nepers/km, related to loss in terms of α_{dB}, the loss in dB/km via $\alpha \approx 0.23\alpha_{dB}$
$L_{eff} = [1-\exp(-\alpha L)]/\alpha$	Effective length L_{eff} associated with L and loss α
$v_p = \frac{\omega_0}{\beta} = \frac{c}{n}$	Phase velocity v_p, for electric field $E = E_0 e^{j(\omega_0 t - \beta z)}$, where $\beta = k_0 n$ and n is refractive index and $k_0 = 2\pi/\lambda$ (note λ always refers to the wavelength in vacuo). $c = 3 \times 10^8$ m/s
$v_g = \frac{d\omega}{d\beta}$	Group velocity v_g of a modulated wave $e^{j(\omega t - \beta z)}$
$n_g = c/v_g$	Group refractive index $n_g = n + \omega\frac{dn}{d\omega} = n - \lambda\frac{dn}{d\lambda}$
$D = -\frac{2\pi c}{\lambda^2}\beta_2 = -\frac{\lambda}{c}\frac{d^2 n}{d\lambda^2}$	Dispersion coefficient D: $\beta_2 = \frac{d^2\beta}{d\omega^2}$. For λ =1550 nm $D\ (ps/nm/km) = -0.78 \times \beta_2 (ps^2/km)$
$\Delta t = D\Delta\lambda L$	Dispersion Δt, with dispersion coefficient D, spectral width $\Delta\lambda$, over a distance L. For 1550 nm 100 GHz spectrum corresponds to 0.8 nm
$V = k_0 a\sqrt{n_{co}^2 - n_{cl}^2}$	Normalised wavenumber for step index fibre. Core radius a, core refractive index n_{co}, cladding refractive index n_{cl}
$J_{m-1}(V) = 0$	Cutoff criterion for the modes. LP_{mn} is n^{th} solution of $J_{m-1}(V) = 0$ where J_m is the m^{th} order Bessel function of the first kind
$F(r) = \exp\left(-\frac{r^2}{r_0^2}\right)$	Gaussian approximation for fundamental mode with mode field radius: $r_0^2 = \frac{a^2}{\ln V}$
$\eta = \frac{2\sigma_1\sigma_2}{\sigma_1^2+\sigma_2^2}\exp\left(-\frac{\Delta^2}{\sigma_1^2+\sigma_2^2}\right)$	Overlap integral between two normalised Gaussian fields separated by Δ with the mode field radii σ_1 and σ_2

First n zeros for $J_k(x)$

k \ n	1	2	3	4	5
0	2.405	5.520	8.654	11.792	14.931
1	3.832	7.016	10.173	13.324	16.471
2	5.136	8.417	11.620	14.796	17.960
3	6.380	9.761	13.015	16.223	19.409
4	7.588	11.065	14.373	17.616	20.827
5	8.771	12.339	15.700	18.980	22.218

Formula	**Notes**
$S_{3dB} = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & j \\ j & 1 \end{bmatrix}$	Scattering matrix of a 3 dB coupler
$i = RP = R\|A\|^2$	Current in a photodiode, responsivity R, electric field amplitude A, optical power $P = \|A\|^2$
$\phi = \gamma P$	Kerr nonlinear phase shift: $\gamma = n_2 k_0 / A_{eff}$ where for optical fibres it can be assumed $n_2 = 2.6 \times 10^{-20}$ m²/W
$\frac{\partial A}{\partial z} = j\frac{\beta_2}{2}\frac{\partial^2 A}{\partial t^2}$	Effect of dispersion in retarded frame of reference. With loss and nonlinearity becomes the NLSE $\frac{\partial A}{\partial z} = -\frac{\alpha}{2}A + j\frac{\beta_2}{2}\frac{\partial^2 A}{\partial t^2} - j\gamma\|A\|^2 A$
$P(\Delta\tau > x) \approx \frac{4}{\pi}\frac{x}{\langle\Delta\tau\rangle}\exp\left(-\frac{4x^2}{\pi\langle\Delta\tau\rangle^2}\right)$	Probability that with mean DGD $\langle\Delta\tau\rangle$ the instantaneous DGD $\Delta\tau$ exceeds x
$N_{NLI} = C_{NLI} G_{TX}^3$	Nonlinear noise power density for input PSD of G_{TX} with $C_{NLI} = \frac{8\gamma^2 L_{eff}^2 \alpha}{27\pi\|\beta_2\|}\ln\left(\frac{\|\beta_2\|}{\alpha}\pi^2 B^2\right)$
$N_{ASE} = 10^{NF/10} h\nu(G-1)$	Power spectral density for ASE amplifier with gain G and noise figure NF. $h = 6.634 \times 10^{-34}$ Js and for $\lambda \approx$1550 nm, $h\nu \approx 1.3 \times 10^{-19}$ J = 0.8 eV.
$G_{opt} = \sqrt[3]{\frac{N_{ASE}}{2C_{NLI}}}$	Optimum power spectral density
$N_q = h\nu P$	PSD for quantum noise
$N_{cm} = \frac{N\log_2(N) + N}{N - N_f + 1}$	Number of complex multiplications per sample for overlap and save implementation of a filter of length N_f using N point FFT (that in turn requires $0.5N\log_2 N$ complex multiplications)
$\boldsymbol{h} \coloneqq \boldsymbol{h} - \mu\frac{\partial\|\epsilon\|^2}{\partial \boldsymbol{h}^*}$	Stochastic gradient update for taps $\boldsymbol{h}$ with error ϵ and convergence parameter μ
$C = 1 - H_2(p_b)$	Capacity of binary symmetric channel where $H_2(p_b) = -(1-p_b)\log_2(1-p_b) - p_b\log_2 p_b$
$C = B\log_2(1 + SNR)$	Shannon capacity (for one polarisation), with bandwidth B and signal to noise ratio SNR