4B23 Optical Fibre Communication 2020/21 worked solutions version 1.3

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a) The term A/λ^4 is due to Rayleigh scattering
b) at $\lambda = 1260$ nm, $f = \frac{3\times10^8}{1260\times10^{-9}} = 2.38 \times 10^{14}$ Hz

at $\lambda = 1675$ nm, $f = \frac{3\times10^8}{1675\times10^{-9}} = 1.79 \times 10^{14}$ Hz

therefore total bandwidth is 59 THz

Assuming the capacity is given by Shannon and polarisation multiplexing is employed then

$C = 2B\log_2(1 + SNR) = 2 \times 59 \times 10^{12} \times \log_2(1 + 1000) = 1.17 \times 10^{15} = 1.2$ Pbit/s

c) Loss at 1260 nm is purely due to Rayleigh scattering. If λ is measured in μm then

$$A = 0.3 \times (1.26)^4 = 0.756\ \mu\text{m}^4$$

At $\lambda = 1.675$ μm $\alpha_{dB} = 0.3$ dB/km so

$$0.3 = \exp\left(B\left[\frac{1}{\lambda_{IR}} - \frac{1}{1.675}\right]\right) + \frac{0.756}{(1.675)^4}$$

Therefore

$$\exp\left(B\left[\frac{1}{\lambda_{IR}} - \frac{1}{1.675}\right]\right) = 0.3 - \frac{0.756}{(1.675)^4} = 0.204$$

And likewise at $\lambda = 1.770$ μm $\alpha_{dB} = 1$ dB/km so

$$1 = \exp\left(B\left[\frac{1}{\lambda_{IR}} - \frac{1}{1.770}\right]\right) + \frac{0.756}{(1.770)^4}$$

Therefore

$$\exp\left(B\left[\frac{1}{\lambda_{IR}} - \frac{1}{1.770}\right]\right) = 1 - \frac{0.756}{(1.770)^4} = 0.923$$

Dividing the two equations gives

$$\frac{\exp\left(B\left[\frac{1}{\lambda_{IR}} - \frac{1}{1.770}\right]\right)}{\exp\left(B\left[\frac{1}{\lambda_{IR}} - \frac{1}{1.675}\right]\right)} = \exp\left(B\left[\frac{1}{1.675} - \frac{1}{1.770}\right]\right) = \frac{0.923}{0.204}$$

Hence

$$B = 31.2 \times \ln(4.525) = 47\ \mu\text{m}$$

To obtain λ_{IR} we note

$$\exp\left(47\left[\frac{1}{\lambda_{IR}}-\frac{1}{1.770}\right]\right)=0.923$$

i.e.

$$\frac{1}{\lambda_{IR}}=\frac{1}{1.770}+\frac{1}{47}\ln 0.923=\frac{1}{1.775}$$

Hence $\lambda_{IR}=1.775$ µm

Hence

$$\alpha_{dB}=\frac{0.756}{\lambda^4}+\exp\left(47\left[\frac{1}{1.775}-\frac{1}{\lambda}\right]\right)$$

Differentiating w.r.t. λ gives

$$\frac{d\alpha_{dB}}{d\lambda}=-\frac{4\times 0.756}{\lambda^5}+\frac{47}{\lambda^2}\exp\left(47\left[\frac{1}{1.775}-\frac{1}{\lambda}\right]\right)$$

Setting the derivative equal to zero gives

$$\frac{4\times 0.756}{\lambda^3}=47\ \exp\left(47\left[\frac{1}{1.775}-\frac{1}{\lambda}\right]\right)$$

Hence

$$47\left[\frac{1}{\lambda}-\frac{1}{1.775}\right]=\ln(\lambda^3\times 15.54)$$

i.e.

$$\lambda=\frac{1}{\frac{1}{47}\ln(\lambda^3\times 15.54)+\frac{1}{1.775}}=\frac{1}{0.6218+0.06383\ln\lambda}$$

First estimate for $\lambda=1.55$ µm
Second estimate for $\lambda=1.539$ µm
Third estimate for $\lambda=1.540$ µm
Fourth estimate for $\lambda=1.540$ µm

Value of attenuation at $\lambda=1.540$ µm is

$$\alpha_{dB}=\frac{0.756}{(1.54)^4}+\exp\left(47\left[\frac{1}{1.775}-\frac{1}{1.54}\right]\right)=0.152\ \text{dB/km}$$

d) To estimate the capacity we assume Shannon capacity

$$C=2B\log_2(1+SNR)$$

And $SNR=\frac{P}{2h\nu B_{eff}}$

And we take $h\nu=0.75$ eV at $\lambda=1.675$ µm and $h\nu=1$ eV at $\lambda=1.260$ µm

Minimum attenuation is 0.152 dB/km and the maximum attenuation is 0.3 dB/km

Initial power is 1W = 30 dBm. Hence P varies between 22.5 dBm=178 mW and 26.2 dBm=417 mW

Hence worst case capacity with $h\nu = 1$ eV and P=178 mW

Giving

$$C = 2 \times 59 \times 10^{12} \times \log_2\left(1 + \frac{0.178}{2 \times 1.6 \times 10^{-19} \times 59 \times 10^{12}}\right) = 1.56 \times 10^{15} = 2 \text{ Pbit/s}$$

Likewise best case capacity with $h\nu = 0.75$ eV and P=417 mW

$$C = 2 \times 59 \times 10^{12} \times \log_2\left(1 + \frac{0.417}{2 \times 0.75 \times 1.6 \times 10^{-19} \times 59 \times 10^{12}}\right) = 1.75 \times 10^{15} = 2 \text{ Pbit/s}$$

Hence to one significant figure we estimate the capacity to be 2 Pbit/s

Assessor's comments

This question was answered 13 of the 22 candidates. It dealt with fibre capacity and attenuation. Part (a) was straightforward recall and most correctly identified the term and being due to Rayleigh scattering. For part (b), generally it was answered correctly, albeit some students failed to convert the SNR from decibels into linear units in order to use Shannon's formula and several students used a narrow band approximation for wavelengths in the region of 1550 nm to convert the bandwidth of the O-U band into THz, which resulted in typically errors of 10% or more in the calculated capacity. For part (c), most students managed to extract parameters to allow both the value and wavelength associated with minimum attenuation, making the necessary approximations in order to solve the problem. The final part (d) was without a doubt the most challenging with only a couple of students able to give a reasonable estimate of the capacity. While some students noted that the SNR would be limited by shot noise, they failed to use the attenuation profile (or even that it would be 0.3 dB/km or less which was given in part (c), to calculate the SNR at the receiver (after 25 km). None of the students used the approach mentioned in lectures of looking at the best case and worst-case SNR at the receiver to bound the capacity (and for this case it was deliberately chosen such that both would give the same capacity to 1 s.f.)

2.

a)

i) bP^3 represents the Kerr nonlinearity, i.e. the power dependent refractive index

ii) First note that to determine the optimum easier to work with the noise to signal ratio – the reciprocal of the SNR so

$$\frac{1}{SNR} = \frac{a}{P} + bP^2$$

To determine the optimum we differentiate w.r.t. P to give

$$\frac{d}{dP}\left(\frac{1}{SNR}\right) = -\frac{a}{P^2} + 2bP$$

At the optimum the derivative is zero and hence

$$\frac{a}{P^2} = 2bP$$

i.e.

$$P^3 = \frac{a}{2b}$$

Therefore

$$P_{opt} = \sqrt[3]{\frac{a}{2b}}$$

hence

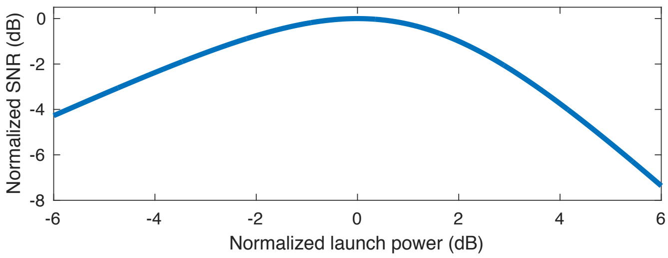
$$SNR_{max} = \frac{P_{opt}}{a + bP_{opt}^3}$$
$$= \frac{P_{opt}}{a + b\frac{a}{2b}}$$
$$= \frac{2P_{opt}}{3a}$$

Therefore

$$s = \frac{SNR}{SNR_{max}}$$
$$= \frac{P}{a + bP^3} \times \frac{3a}{2P_{opt}}$$
$$= \frac{3ap}{2a + 2bP^3}$$
$$= \frac{3ap}{2a + 2bP_{opt}^3 p^3}$$
$$= \frac{3ap}{2a + 2b\frac{a}{2b}p^3}$$
$$= \frac{3p}{2 + p^3}$$

As required

iii) for $p \ll 1$ $s \approx 3/2p$ so $10\log_{10}(s) = 10\log_{10}(p) + 10\log_{10}(3/2)$ so gradient of 1 dB/dB. Likewise for $p \gg 1$ $s \approx 3/p^2$ so $10\log_{10}(s) = -20\log_{10}(p) + 10\log_{10}(3)$ so gradient of -2 dB/dB



b)

i) $\alpha_{dB} = 0.2$ dB/km (so $\alpha = 0.0461$ km^{-1}), D=17 ps/nm/km (so $|\beta_2| = 17/0.784 = 21.6$ ps^2/km), $B_o = 5$ THz, $\gamma = 1.3$ W^{-1}km^{-1}, $L = 100$ km (so $L_{eff} = 21.5$ km).

$$C_{NLI} = \frac{8\gamma^2 L_{eff}^2 \alpha}{27\pi|\beta_2|} \ln\left(\frac{\pi^2|\beta_2|B_o^2}{\alpha}\right) = 1.833 \times 10^{24}\, J^{-2} = 1.833\ \ (pJ)^{-2}$$

$$N_{ASE} = 10^{NF/10} h\nu(G-1) = 5.124 \times 10^{-17}\, J = 5.124 \times 10^{-5}\, pJ$$

$$PSD_{opt} = \sqrt[3]{\frac{5.124 \times 10^{-17}}{2 \times 1.833 \times 10^{24}}} = 2.409 \times 10^{-14}\, J = 0.02409\, pJ$$

Total power is $50 \times 0.02416 \times 0.095 = 114$ mW

ii) If we neglect the degradation we note
after 1 span $SNR_{opt} = \frac{0.02416}{3/2 \times 5.124 \times 10^{-5}} = 314 = 25.0$ dB
and hence after 40 spans $SNR_{dB} = 25 - 10\log_{10}(40) = 9$ dB
to take into account the degradation we define use equation (3) with p at the start of life and $p/2$ at the end of life and set the SNR equal so

$$\frac{3p}{2+p^3} = \frac{3(p/2)}{2+(p/2)^3}$$

Hence

$$2 + p^3 = 2\left(2 + \frac{p^3}{8}\right)$$

i.e.

$$p^3\left(1 - \frac{1}{4}\right) = 2$$

So $p^3 = 8/3$ and hence

$$s = \frac{3\sqrt[3]{8/3}}{2 + 8/3} = 0.89 = -0.5 \text{ dB}$$

Hence the initial SNR is 8.5 dB

Assessor's comments

This question was answered 11 of the 22 candidates and was concerned with optical fibre communication systems operating in the nonlinear propagation regime. Part (a) was generally answered well albeit somewhat surprisingly asking the students to sketch how the SNR changed as a function of power on a double log scale caused some issues, even through in lectures it was only ever show in this way (with asymptotic gradient 1 dB/dB in the linear regime and -2 dB/dB in the nonlinear regime). For part (b) in general the straightforward part i) was generally answered well albeit in many cases having determined the optimum power spectral density, they failed to determine the total power but determining the power per wavelength and then multiplying this by the number of wavelengths. The second part of (b) namely (b) (ii) was answered well by the handful of students who realised that the question meant using the result given in part (a) and setting the initial SNR to the final SNR once the power had been reduced by 3 dB. Occasionally the SNR at the end of 100 km span rather than the required 40x100 km spans was given resulting in a 16 dB error in the resulting SNR.

3.

a) Equalisation stages are

- Coherent detection equalisation
- Static channel equalisation
- Adaptive channel equalisation

b) Rectangular Nyquist spectrum means that a 95 GBd signal will occupy 95 GHz of optical spectrum. At 1550 nm, 100 GHz occupies 0.8 nm so 95GHz corresponds to 0.76 nm and the chromatic dispersion from 200 km is

$$17 \times 0.76 \times 200 = 2584 \text{ ps}$$

The sampling rate is $95 \times \frac{32}{31} = 98$ GSa/s therefore the number of tap to span 2584 ps is

$$N_{CD} = 1 + 2584 \times 10^{-12} \times 95 \times 10^{9} \times \frac{32}{31} = 255 \text{ taps}$$

c) Using the overlap and save algorithm with an N point FFT the number of complex multiplies per sample N_{cm} is

$$N_{cm} = \frac{N \log_2(N) + N}{N - N_{CD} + 1}$$

Given $N_{CD} = 255$ we expect the minimum value of N to be 512 which gives

$$N_{cm} = \frac{512 \times 9 + 512}{512 - 255 + 1} = 19.8$$

Given there are no technological limitations regarding the FFT size let us consider $N = 1024$ which gives

$$N_{cm} = \frac{1024 \times 10 + 1024}{1024 - 255 + 1} = 14.6$$

Increasing to $N = 2048$ gives

$$N_{cm} = \frac{2048 \times 11 + 2048}{2048 - 255 + 1} = 13.7$$

Increasing to $N = 4096$ gives

$$N_{cm} = \frac{4096 \times 12 + 4096}{4096 - 255 + 1} = 13.9$$

Hence the optimum value of N is 2048. The power consumption per polarisation is

$P = 13.7 \times 0.5 \times 10^{-12} \times 98 \times 10^{9} = 0.67\ W$ and hence for two polarisations the total power consumption is 1.34 W.

d) PMD coefficient is 0.1 $\mathrm{ps}/\sqrt{\mathrm{km}}$ and therefore for 200 km the mean DGD is $\langle\Delta\tau\rangle = 0.1 \times \sqrt{200} = 1.41$ ps

From data sheet $P(\Delta\tau > x) \approx \frac{4}{\pi}\frac{x}{\langle\Delta\tau\rangle}\exp\left(-\frac{4x^2}{\pi\langle\Delta\tau\rangle^2}\right)$

With $\frac{x}{\langle\Delta\tau\rangle} = 5\ \ P(\Delta\tau > x) = 10^{-13}$

Hence span in samples is $5 \times 1.41 \times 10^{-12} \times 98 \times 10^{9} = 0.7$

Only need two taps so for the 2x2 MIMO equaliser total of 8 taps so 8 complex multiplications

Total power would be $8 \times 98 \times 10^{9} \times 0.5 \times 10^{-12} = 0.4$ W

N.B. In addition to this there would be power required for the updating which would be a similar magnitude if updating at the symbol rate, however PMD varies much more slowly – typically kHz so can reduce update rate by several orders of magnitude, so power required for update is negligible and hence the overall power is approximately 0.4 W

e) With 100 ps , 100×0.76=76 ps and hence number of taps is

$$76 \times 10^{-12} \times 95 \times 10^{9} \times \frac{32}{31} = 7.4$$

Simple estimate would be that power consumption goes up by 4 to 1.6 W but would be better to modify the DSP design to have an 8-tap equalizer per polarization requiring 0.4 W so 1.2 W in total (with the MIMO equalizer still only having 2 taps).

Assessor's comments

This question was answered by 20 of the 22 candidates. The first half of the question, parts (a) to (c) where generally answered well with most students able to correctly determine the number of FIR filter taps, then correctly optimising the FFT size to minimise the number of complex multiplications required and then subsequently calculated the power required. While there were occasional errors such as using the symbol rate as opposed to the sample rate, or forgetting to double to total power to account for the two polarisations in general the part (c) was answered well by most students. Part (d) presented challenges to most students. While most were able to calculate the peak DGD corresponding to the given outage probability and hence deduce that two taps were required for each of the filters in the adaptive equalisers, a significant number of students calculated the power consumption assuming an FFT method was being used, even when this indicated the number of complex multiplications increased compared to implementing directly in the time domain (which should have been used). In addition, a number of students merely doubled the power when considering the two polarisations rather than multiplying by four to take into account the adaptive equaliser structure used to compensate for PMD. The final part of the question asked students to consider how they could modify the DSP structure if the adaptive equaliser also needed to compensate for residual chromatic dispersion. While some students correctly calculated that the number of taps required to compensate the residual chromatic dispersion, no one correctly identified that the optimal solution from a power consumption point of view would be to modify the DSP to have adaptive chromatic dispersion compensating filters for each polarisation followed by the previous MIMO adaptive equaliser to correct for PMD.