EGT3
ENGINEERING TRIPOS PART IIB

Tuesday 4 May 2021 1.30 to 3.10

Module 4B23

OPTICAL FIBRE COMMUNICATION

*Answer not more than **two** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

*Write your candidate number **not** your name on the cover sheet.*

STATIONERY REQUIREMENTS
Write on single-sided paper.

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed.
Attachment: 4B23 Optical Fibre Communication formula sheet (2 pages).
You are allowed access to the electronic version of the Engineering Data Books.

10 minutes reading time is allowed for this paper at the start of the exam.

The time taken for scanning/uploading answers is 15 minutes.

Your script is to be uploaded as a single consolidated pdf containing all answers.

1 A step index optical fibre has a core radius $a = 4\ \mu$m with core refractive index $n_{co} = 1.45$ and cladding refractive index $n_{cl} = 1.445$. The attenuation measured in dB/km as a function of optical wavelength λ may be modelled as

$$\alpha_{dB} = \frac{A}{\lambda^4} + \exp\left[B\left(\frac{1}{\lambda_{IR}} - \frac{1}{\lambda}\right)\right]$$

where A, B and λ_{IR} are constants.

(a) State the physical mechanism that is responsible for the term A/λ^4 in the equation above. [5%]

(b) The signal to noise ratio is 30 dB over the entire O-U band (1260 nm to 1675 nm). Estimate the total data capacity, stating any assumptions made. [25%]

(c) The attenuation follows the equation above, taking specific values of 0.3 dB/km at 1260 nm and 1675 nm, and 1 dB/km at 1770 nm. Obtain the constants A, B and λ_{IR} and hence estimate the minimum value of attenuation and the minimum attenuation wavelength.

Hint: To solve the resulting transcendental equation, rearrange it to the form

$$x_{n+1} = \frac{1}{a + b\ln(x_n)}$$

This results in a convergent solution with x_{n+1} being an improved approximation upon x_n where a and b are constants to be determined. [35%]

(d) The maximum power in the optical fibre is 1 W. Assuming nonlinear effects may be neglected, estimate to one significant figure the capacity of the O-U band operating over 25 km of optical fibre. [35%]

2 Nonlinear effects in optical fibres present a key limitation for long-haul optical fibre communication systems.

(a) In an optical fibre communication system the signal to noise ratio (SNR) as a function of power P can be written as

$$SNR = \frac{P}{a + bP^3}$$

where a represents the additive linear noise and b represents the nonlinear interactions which occur in the fibre.

(i) State what nonlinear phenomenon is modelled by the term bP^3 in the equation above. [5%]

(ii) For any nonlinear transmission system there is an optimum launch power P_{opt} at which a maximum signal to noise ratio SNR_{max} is observed. If we define $p = P/P_{opt}$ as the normalised power and $s(p) = SNR/SNR_{max}$ as the normalised SNR as a function of normalised power, show that [30%]

$$s(p) = \frac{3p}{2 + p^3}$$

Hint: You may find it helpful to first maximise the SNR as a function of P to obtain expressions for P_{opt} and SNR_{max} in terms of a and b.

(iii) Sketch $10 \log_{10}(s)$ as a function of $10 \log_{10}(p)$, highlighting salient features including the asymptotic behaviour in the regions where $p \ll 1$ and $p \gg 1$. [10%]

(b) A WDM system transmits over 4000 km of single mode optical fibre 50 WDM channels, providing a total capacity of 20 Tbit/s. Each WDM channel transmits 400 Gbit/s at 95 GBd, with the transmitter employing digital signal processing to create a rectangular Nyquist spectrum. The signal is amplified every 100 km using an EDFA with gain 20 dB, noise figure 6 dB and bandwidth 5 THz. At the operating wavelength of 1550 nm, the optical fibre has an attenuation of 0.2 dB/km, dispersion coefficient of 17 ps/nm/km and nonlinear coefficient of 1.3 $\mathrm{W}^{-1}\mathrm{km}^{-1}$.

(i) Assuming that a 100 GHz WDM grid is used, calculate the total output power required for the EDFA if the system operates at the optimal launch power with 50 WDM channels. [25%]

(ii) Due to aging, the launch power decreases by 3 dB over the lifetime of the system. If the initial launch power is optimised to minimise the SNR variation due to the degradation, estimate the corresponding initial SNR at the receiver. [30%]

3 A 800 Gbit/s signal is transmitted over 200 km of single mode optical fibre using 95 GBd PDM-32QAM. The transmitter employs digital signal processing to create a rectangular Nyquist spectrum, with the chromatic dispersion compensated digitally at the receiver. At the operating wavelength of 1550 nm, the optical fibre has an attenuation of 0.2 dB/km, dispersion coefficient of 17 ps/nm/km, a PMD coefficient of 0.1 ps/$\sqrt{\text{km}}$ and nonlinear coefficient of 1.3 $\text{W}^{-1}\text{km}^{-1}$.

(a) List the equalisation stages employed in a digital coherent receiver. [5%]

(b) Calculate the minimum number of taps N_{CD} required to compensate the chromatic dispersion in the 95 GBd signal transmitted over the 200 km link if the digital coherent receiver uses an oversampling rate of 32/31. [25%]

(c) Assuming appropriate frequency domain implementation, estimate the minimum power consumption required to realise digital chromatic dispersion compensation for the 95 GBd PDM-32QAM signal assuming the energy required to perform a complex multiply is 0.5 pJ. You may assume there are no technological restrictions regarding the FFT size. [30%]

(d) An adaptive equaliser is used to correct for the PMD present in the signal. Estimate the total power consumption of the adaptive equaliser if the outage probability due to the PMD should be less than 10^{-13}. [25%]

(e) If the adaptive equaliser is also required to compensate for ±100 ps/nm of residual chromatic dispersion how would this change the design of the digital signal processing subsystems? [15%]

END OF PAPER

Formula	Notes
$\langle S\rangle = \frac{1}{2}\frac{n}{\eta_0}\lvert E_0\rvert^2$	$\langle S\rangle$ - time averaged Poynting Vector, E_0 - complex electric field, $\eta_0 = \sqrt{\mu_0/\varepsilon_0} = 377\ \Omega$, n is refractive index.
$P(z) = P(0)\exp(-\alpha z)$	α in nepers/km, related to loss in terms of α_{dB}, the loss in dB/km via $\alpha \approx 0.23\alpha_{dB}$
$L_{eff} = [1 - \exp(-\alpha L)]/\alpha$	Effective length L_{eff} associated with L and loss α
$v_p = \frac{\omega_0}{\beta} = \frac{c}{n}$	Phase velocity v_p, for electric field $E = E_0 e^{j(\omega_0 t - \beta z)}$, where $\beta = k_0 n$ and n is refractive index and $k_0 = 2\pi/\lambda$ (note λ always refers to the wavelength in vacuo). $c = 3\times 10^8$ m/s
$v_g = \frac{d\omega}{d\beta}$	Group velocity v_g of a modulated wave $e^{j(\omega t - \beta z)}$
$n_g = c/v_g$	Group refractive index $n_g = n + \omega\frac{dn}{d\omega} = n - \lambda\frac{dn}{d\lambda}$
$D = -\frac{2\pi c}{\lambda^2}\beta_2 = -\frac{\lambda}{c}\frac{d^2n}{d\lambda^2}$	Dispersion coefficient D: $\beta_2 = \frac{d^2\beta}{d\omega^2}$. For λ =1550 nm $D\ (ps/nm/km) = -0.78\times\beta_2(ps^2/km)$
$\Delta t = D\Delta\lambda L$	Dispersion Δt, with dispersion coefficient D, spectral width $\Delta\lambda$, over a distance L. For 1550 nm 100 GHz spectrum corresponds to 0.8 nm
$V = k_0 a\sqrt{n_{co}^2 - n_{cl}^2}$	Normalised wavenumber for step index fibre. Core radius a, core refractive index n_{co}, cladding refractive index n_{cl}
$J_{m-1}(V) = 0$	Cutoff criterion for the modes. LP_{mn} is n^{th} solution of $J_{m-1}(V) = 0$ where J_m is the m^{th} order Bessel function of the first kind
$F(r) = \exp\left(-\frac{r^2}{r_0^2}\right)$	Gaussian approximation for fundamental mode with mode field radius: $r_0^2 = \frac{a^2}{\ln V}$
$\eta = \frac{2\sigma_1\sigma_2}{\sigma_1^2+\sigma_2^2}\exp\left(-\frac{\Delta^2}{\sigma_1^2+\sigma_2^2}\right)$	Overlap integral between two normalised Gaussian fields separated by Δ with the mode field radii σ_1 and σ_2

First n zeros for $J_k(x)$

k \ n	1	2	3	4	5
0	2.405	5.520	8.654	11.792	14.931
1	3.832	7.016	10.173	13.324	16.471
2	5.136	8.417	11.620	14.796	17.960
3	6.380	9.761	13.015	16.223	19.409
4	7.588	11.065	14.373	17.616	20.827
5	8.771	12.339	15.700	18.980	22.218

Formula	**Notes**
$S_{3dB} = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & j \\ j & 1 \end{bmatrix}$	Scattering matrix of a 3 dB coupler
$i = RP = R\|A\|^2$	Current in a photodiode, responsivity R, electric field amplitude A, optical power $P = \|A\|^2$
$\phi = \gamma P$	Kerr nonlinear phase shift: $\gamma = n_2 k_0 / A_{eff}$ where for optical fibres it can be assumed $n_2 = 2.6 \times 10^{-20}$ m²/W
$\frac{\partial A}{\partial z} = j\frac{\beta_2}{2}\frac{\partial^2 A}{\partial t^2}$	Effect of dispersion in retarded frame of reference. With loss and nonlinearity becomes the NLSE $\frac{\partial A}{\partial z} = -\frac{\alpha}{2}A + j\frac{\beta_2}{2}\frac{\partial^2 A}{\partial t^2} - j\gamma\|A\|^2 A$
$P(\Delta\tau > x) \approx \frac{4}{\pi}\frac{x}{\langle\Delta\tau\rangle}\exp\left(-\frac{4x^2}{\pi\langle\Delta\tau\rangle^2}\right)$	Probability that with mean DGD $\langle\Delta\tau\rangle$ the instantaneous DGD $\Delta\tau$ exceeds x
$N_{NLI} = C_{NLI} G_{TX}^3$	Nonlinear noise power density for input PSD of G_{TX} with $C_{NLI} = \frac{8\gamma^2 L_{eff}^2 \alpha}{27\pi\|\beta_2\|}\ln\left(\frac{\|\beta_2\|}{\alpha}\pi^2 B^2\right)$
$N_{ASE} = 10^{NF/10} h\nu(G-1)$	Power spectral density for ASE amplifier with gain G and noise figure NF. $h = 6.634 \times 10^{-34}$ Js and for $\lambda \approx$1550 nm, $h\nu \approx 1.3 \times 10^{-19}$ J = 0.8 eV.
$G_{opt} = \sqrt[3]{\frac{N_{ASE}}{2C_{NLI}}}$	Optimum power spectral density
$N_q = h\nu P$	PSD for quantum noise
$N_{cm} = \frac{N\log_2(N) + N}{N - N_f + 1}$	Number of complex multiplications per sample for overlap and save implementation of a filter of length N_f using N point FFT (that in turn requires $0.5N\log_2 N$ complex multiplications)
$\boldsymbol{h} := \boldsymbol{h} - \mu\frac{\partial\|\epsilon\|^2}{\partial\boldsymbol{h}^*}$	Stochastic gradient update for taps $\boldsymbol{h}$ with error ϵ and convergence parameter μ
$C = 1 - H_2(p_b)$	Capacity of binary symmetric channel where $H_2(p_b) = -(1-p_b)\log_2(1-p_b) - p_b\log_2 p_b$
$C = B\log_2(1 + SNR)$	Shannon capacity (for one polarisation), with bandwidth B and signal to noise ratio SNR

4B23 Optical Fibre Communication 2020/21 numerical answers

1
(a) –
(b) 1.2 Pbit/s
(c) $A = 0.756\ \mu\text{m}^4$, $B = 47\ \mu\text{m}$, $\lambda_{IR} = 1.775\ \mu\text{m}$, $\lambda = 1.540\ \mu\text{m}$, $\alpha_{dB} = 0.152\ \text{dB/km}$
(d) 2 Pbit/s

2.
(a) -
(b) (i) $114\ \text{mW}$, (ii) 8.5 dB

3.
(a) -
(b) 255
(c) 1.34 W
(d) 0.4 W
(e) -