4B24 Crib 2021/22 Version 2

1. a)

Scattering paramters – defined for consistent directions of voltage waves into and out of ports. Allows for scaling to any number of ports. Directly and easily measure accurately by terminating other ports and measuring voltages when a signal is applied to one port.
ABCD parameters, defined with in terms of voltage and currents with directions swapped between ports 1 and 2 to allow a simple cascade of devices by multiplication of the matrixies, however limits to only 2 port devices. Can be calculated from 2 port S-parameters since current is difficult to measure.

b)

Terminated TL:

$$Z_{in} = Z_0 \frac{Z_L + Z_l j\tan(\beta l)}{Z_l + Z_L j\tan(\beta l)}$$

S_{11} of 0.5∠30° with 50Ω-> $Z_{in} = 97.6 + 65j$

User Z_l for characteristic impedance of the line.

$$97.6 + 65j = Z_l \frac{Z_L + Z_l j\tan(\beta l)}{Z_l + Z_L j\tan(\beta l)}$$

$$(97.6 + 65j)(Z_l + (100 - 100j)j\tan(\beta l)) = Z_l(100 - 100j) + Z_l^2 j\tan(\beta l)$$

$$(97.6 + 65j)(Z_l + 100\tan(\beta l) + (100j\tan(\beta l))) = Z_l(100 - 100j) + Z_l^2 j\tan(\beta l)$$

Real parts:

$$97.66(Z_l - 100\tan(\beta l)) + 6500\tan\beta l = Z_l 100$$

$$2.33Z_l - 3266\tan\beta l = 0$$

$$\tan\beta l = \frac{2.33}{3266} Z_l$$

Imaginary parts:

$$65j * (Z_l + 100\tan\beta l) + 97.66 * 100j\tan\beta l = -100Z_l j + Z_l^2 j\tan(\beta l)$$

$$0 = -165Z_l + Z_l^2\tan(\beta l) - 16266\tan\beta l$$

$$0 = -165Z_l + Z_l^3(7.134E^{-4}) - 11.6Z_l$$

$$Z_l^2 = 24755$$

$Z_l = 497\Omega$

$$\tan\beta l = 0.355$$

$$\beta l = 0.34$$

*Length is 0.34/2*pi = 0.0543 lambda*

c) (i)

Scattering matrix:

$$\begin{matrix} V_1^- \\ V_2^- \\ V_3^- \end{matrix} = \frac{1}{\sqrt{2}}\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}\begin{matrix} V_1^+ \\ V_2^+ \\ V_3^+ \end{matrix}$$

$$V_3^+ = \Gamma_3 V_3^-$$

$$V_3^- = \frac{1}{\sqrt{2}}(V_1^+ + V_2^+)$$

$$V_1^- = \frac{1}{\sqrt{2}}\left(V_2^+ + \frac{\Gamma_3}{\sqrt{2}}(V_1^+ + V_2^+)\right)$$

$$V_2^- = \frac{1}{\sqrt{2}}\left(V_1^+ + \frac{\Gamma_3}{\sqrt{2}}(V_1^+ + V_2^+)\right)$$

$$\begin{matrix} V_1^- \\ V_2^- \end{matrix} = \frac{1}{2}\begin{bmatrix} \Gamma_3 & \sqrt{2}+\Gamma_3 \\ \sqrt{2}+\Gamma_3 & \Gamma_3 \end{bmatrix}\begin{matrix} V_1^+ \\ V_2^+ \end{matrix}$$

ii) available gain is $\frac{V_2^-}{V_1^+}$ when $V_1^- = V_2^+ = 0$ (matched load). $= \frac{\sqrt{2}+\Gamma_3}{2}$ max when $\Gamma_3 = 1$ which is when P3 is open.

Impedance at P1 can be found considering resulting reflection seen at P1: $\frac{V_1^-}{V_1^+} = \frac{\Gamma_3}{2} = 0.5$

$$\frac{1}{2} = \frac{Z_{P1} - Z_0}{Z_{P1} + Z_0}$$

$$Z_{P1} = 150\Omega$$

iii)

$$10 + 10j \rightarrow \Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}$$

$$\Gamma_3 = -0.62 + 0.27j$$

For a 50 ohm impedance on P1, $V_1^- = 0$

Impedance on port 2 will present a reflection at this port of Γ_2, $V_2^+ = \Gamma_2 V_2^-$

Using result of (ii)

$$0 = \frac{1}{2}(\Gamma_3 V_1^+ + (\sqrt{2}+\Gamma_3)V_2^+)$$

$$V_2^- = \frac{1}{2}\left((\sqrt{2}+\Gamma_3)V_1^+ + (\Gamma_3)V_2^+\right)$$

Rearrange and sub for V_2^+

$$V_2^- = \frac{1}{2}\left((\sqrt{2}+\Gamma_3)V_1^+ + (\Gamma_3)\Gamma_2 V_2^-\right)$$

$$V_2^-\left(1 - \frac{\Gamma_2\Gamma_3}{2}\right) = \frac{V_1^+}{2}(\sqrt{2}+\Gamma_3)$$

$$V_2^- = \frac{\frac{V_1^+}{2}(\sqrt{2}+\Gamma_3)}{\left(1 - \frac{\Gamma_2\Gamma_3}{2}\right)}$$

$$V_2^- = \frac{V_1^+(\sqrt{2}+\Gamma_3)}{(2 - \Gamma_2\Gamma_3)}$$

$$0 = (\Gamma_3 V_1^+ + (\sqrt{2}+\Gamma_3)\Gamma_2 V_2^-)$$

Substitute

$$0 = (\Gamma_3 V_1^+ + (\sqrt{2}+\Gamma_3)\Gamma_2 \frac{V_1^+(\sqrt{2}+\Gamma_3)^2}{(2 - \Gamma_2\Gamma_3)}$$

$$\Gamma_2 = \frac{-2\Gamma_3}{(\sqrt{2}+\Gamma_3)^2 - \Gamma_3^2}$$

$$\Gamma_2 = -0.175 + 1.68i$$

Since the required $|\Gamma_2| > 1$ this is not possible with a passive match.

A very popular question. In (a) a common mistake was to simply define the parameters rather than consider their application. Most had the correct method in (b) although not all realised that Zin could be calculated from the provided S11, some became confused between the characteristic impedance at which the S parameters had been provided and the characteristic impedance of the line to be found. (c) (i) was mostly well answered, in (ii) some stopped after finding P3 should be open and didn't find the impendance presented by P1. Part (iii) was tricky but produced a few very good answers. A worrying misconception was that an impedance had to be real.

2.

a) (i)

2^{nd} harmonics, 2GHz, 3GHz, 3.01GHz
2^{nd} order intermodulation 2.5GHz, 2.505GHz, 500MHz, 505MHz, (3.505GHz), 5MHz
3^{rd} order harmonics 3GHz, (4.5GHz, 4.515GHz)
3^{rd} order intermodulation:

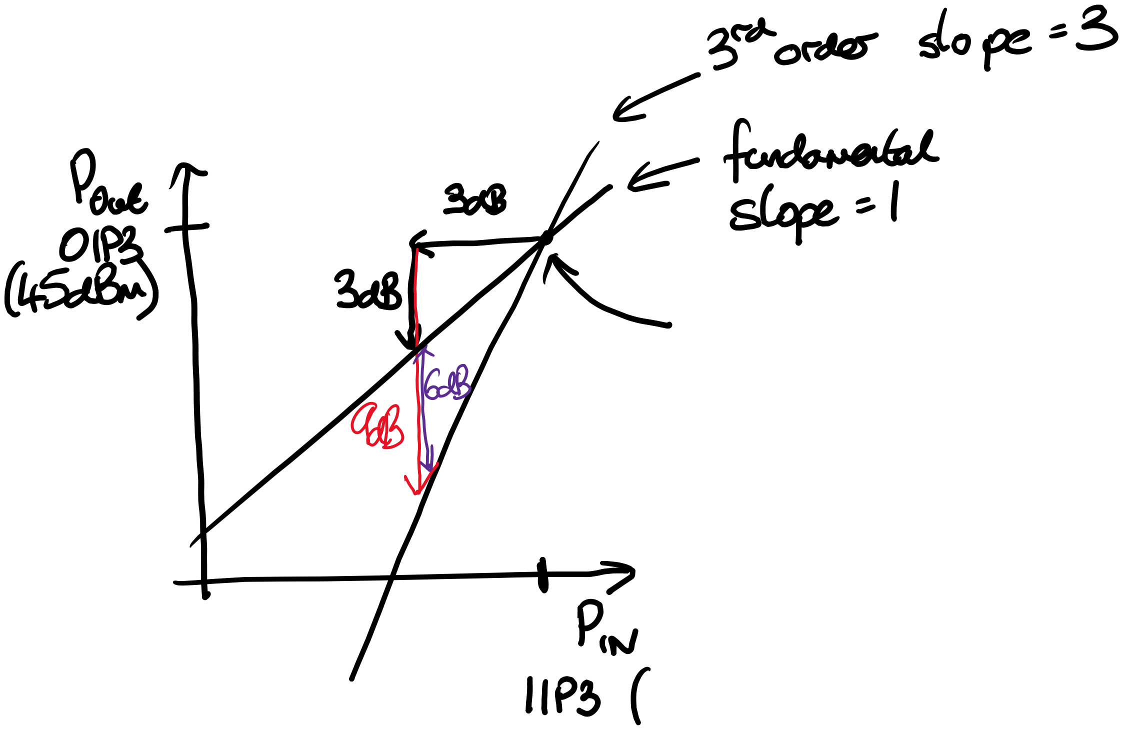
2*1 – 1.5
2*1 -1.505
2*1.5 -1
2*1.505-1
2*1.5-1.505
2*1.505-1.5

(Also 3^{rd} order due to f1+f2-f3 etc)

(ii) Convert IIP3 to OIP3:

OIP3 =IIP3+gain = 45dBm

For every 3dB reduction in fundamental, intermods reduce by 9dB, so giving 6dB ratio improvement between fundamental and 3^{rd} order intermods.



At OIP3, the intermods are equal power to the fundamental, so to get 30dB ratio we need 30/6 = 5 6dB improvements.

5*3dB back off from OIP3

45dBm-(15)=30dBm output power.

(iii) 3Ghz arises due to 2nd order and 3rd order harmonics.

For 30dBm output at the 1.5GHz, we have +10dBm input at 1.5GHz and +13dBm input at 1GHz.

2nd harmonics are 6dB greater than 2nd intermods.
3rd harmoncis are ~9dB greater than 3rd intermods

Power due to 2nd orders: 1dB reduction in fundamental from IIP2 reduces distortion 2dB.
Input is 30-10=20dB back off from IIP2, so intermods 40dB below OIP2, but 6dB greater for harmonics.

50-20*2+6=16dBm

Power due to 3rd orders
25-13 = 12dB back off from OIP3.

This is 3dB more power than (ii) so intermods are 9dB higher, we also see 9dB due to harmonic rather than intermod -> 18dBm.

Since 2nd and 3rd orders are approx size need to consider voltage sum as the worst case is when they happen to sum in phase.

20*log10(10^(18/20)+10^(16/20)) = 23.1dBm

(iv) wanted signal +10dBm at input, 30dBm at output, so for SNR of 100dB, must have 70dBm noise in 1MHz BW at the output.

Corresponds to a -130dBm/Hz noise.

3rd order SFDR – 2/3(OIP3-Noise)=117dB/Hz^(2/3)

(2nd order SFDR ½(OIP2-Noise) = 80 dB/Hz^(1/2))

(b)
(i)Since $S_{12} \sim 0$ we can ignore the load.

$S_{11} = 0.5$ is the reflection coefficent

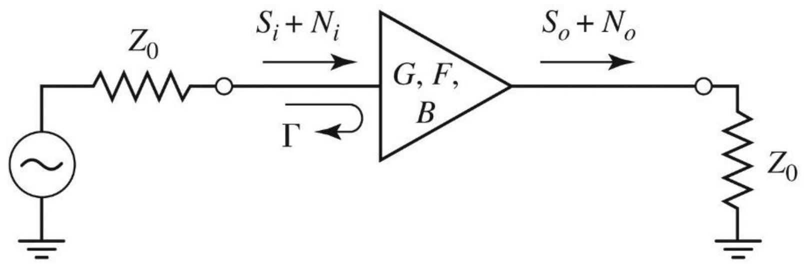
Amp input impedance = 50*3= 150 ohms.

Noise figure will be measured for 50 Ohms so the noise temperature just requires conversion from noise figure.

$$F = 1 + \frac{T_e}{T_0}$$

For $T_0 = 290K \rightarrow T_e = 627K$

iii)



$$N_i = kT_0B$$
$$N_o = kT_0GB(1 - |\Gamma|^2) + kT_eGB$$
$$S_o = G(1 - |\Gamma|^2)S_i$$
$$F_m = \frac{S_iN_o}{S_oN_i} = 1 + \frac{F-1}{1-|\Gamma|^2}$$

The noise figure of 5dB is recorded under the condition that there is a $\Gamma = 0.5$, so if a lossless transmission line is used to eliminate the reflection (e.g. quarter wave match), the noise figure can be reduced to 4.18dB.

Neither popular nor well answered. Most could find a the harmonics and intermodulation products. The rest of part (a) relied on scaling of the harmonic and fundamental powers away from OIP3, many drew plots of different gradient lines for Pin and Pout in dB, but few put this to good use. In (iii) few realised that noise could be found from the provided SNR. (b) was better answered particularly (i) and (ii). In part (iii) many confused the variation in NF due to the reflection of the feeding line with the variation in in an amplifiers intrinsic noise figure due to the impedance presented to it.

3. (i) $\Gamma = \frac{Z_{IC}-Z_{ant}^*}{Z_{IC}+Z_{ant}}$

Using values from the Q this gives:

$$\Gamma = 0.36 + 0.42j, 0.286 + 0.507j, 0.1947 + 0.466j, 0.243 + 0.367j$$

Looking at the values, there is a large common component of reflection

To show QPSK – the values could be roughly plotted.

Otherwise subtract the mean (0.26+0.44j) then look at arg angle:

$$0.0725\angle - 14°, 0.07\angle + 72°, 0.0744\angle 160°, 0.077\angle - 106°$$

(so very close to QPSK when the angle offset is removed)

(ii) Average differential cross section:

$\Delta\sigma = \frac{P_s}{S_t} = \frac{\lambda^2 G^2 (|\Gamma_1 - \Gamma_2|^2 + |\Gamma_1 - \Gamma_3|^2)}{8\pi} = 0.0094\text{m}^2$

Note that Gr is not squared as we have the reader tx in EIRP.

$$R^4 = \frac{P_t G_{reader} \lambda^2 \Delta\sigma}{(4\pi)^3 P_r}$$

$$R = 9.8m$$

Assumes that the polarisation of the antennas is matched to each other and the antenna directivity is aligned.

(iii) Need to have a phase sensitive system – so anything which preserves I and Q. High carrier leakage is most easily removed at DC so direct conversion preferred to superhet.

(iv) Becomes BPSK modulation. (with an offset constant power).

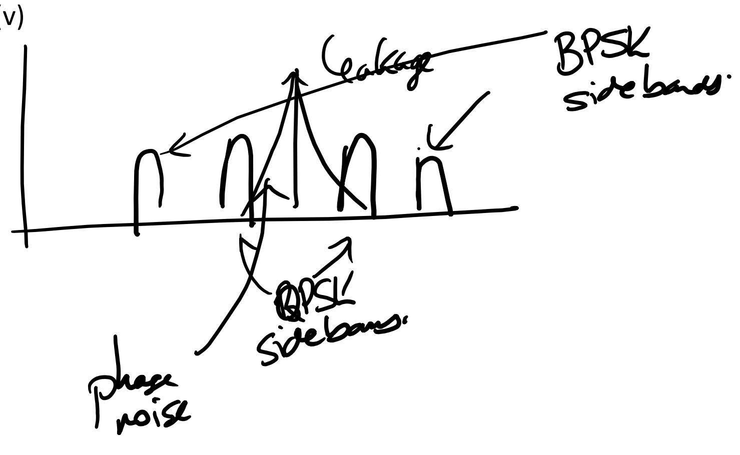
For QPSK and BPSK Eb/No are the same so provided the bit rate is the same we get the same BER (note the symbol rate is doubled).

$\Delta\sigma = \frac{P_s}{S_t} = \frac{\lambda^2 G^2 |\Gamma_1 - \Gamma_2|^2}{4\pi}$=0.0063

So Pr reduces by 0.0063/0.0094 = 0.667 = -1.76dB

New sensitivity requirement -71.76dBm.

(v)



For the same bit rate, the symbol rate of QPSK will be half which gives greater BW effiency. For a phase noise limited system though, the noise will be greater closes to the carrier, so the QPSK system will experience greater noise. Given the relatively small decrease in the backscatter RCD for the BPSK system, the reduction in phase noise by moving twice as far from the carrier is likely to be much greater.

An unpopular question. Most found the modulation format, and had a good go at estimating the range and could suggest a phase sensitive receiver architecture. The comparison of the QPSK system and BPSK system was less good although again, many saw that this is BPSK. In (v) many could sketch the shape of the phase noise, but few noticed that the noise would be lower for the higher symbol rate BPSK system.

4. (given on datasheet)

$$G = \frac{|S_{21}|^2(1-|\Gamma_L|^2)}{(1-|\Gamma_{in}|^2)|1-S_{22}\Gamma_L|^2} G_A = \frac{|S_{21}|^2(1-|\Gamma_S|^2)}{|1-S_{11}\Gamma_S|^2(1-|\Gamma_{out}|^2)}$$

$$G_T = \frac{|S_{21}|^2(1-|\Gamma_S|^2)(1-|\Gamma_L|^2)}{|1-\Gamma_S\Gamma_{in}|^2|1-S_{22}\Gamma_L|^2}$$

$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}$ and $\Gamma_S = \frac{Z_S - Z_0}{Z_S + Z_0}$

$\Gamma_{in} = \frac{V_1^-}{V_1^+} = S_{11} + \frac{S_{12}\Gamma_L S_{21}}{1-S_{22}\Gamma_L}$

$\Gamma_{out} = \frac{V_1^-}{V_1^+} = S_{22} + \frac{S_{12}\Gamma_S S_{21}}{1-S_{11}\Gamma_S}$

Power gain is the power delivered into the load to the power taken from the source. Reduces to $|S_{21}|^2$ when $\Gamma_L = 0$ which implies $Z_L = Z_0$, and $S_{11} = 0$, so the load must be matched to Z_0 and the impedance looking into port 1 should also be 50. If unilateral S_12 is zero, but no change.

Available gain is the maximum power delivered into the optimum load to the power available from the source (if matched). Reduces to $|S_{21}|^2$ when $\Gamma_S = 0$ which implies $Z_S = Z_0$, and $S_{22} = 0$. If unilateral S_12 is zero, but no change

Transducer gain is the power delivered into the load to the power available from the source. Reduces to $|S_{21}|^2$ when $\Gamma_S = 0$ which implies $Z_S = Z_0$, and $S_{22} = S_{11} = 0$

(ii) $G = \frac{|S_{21}|^2(1-|\Gamma_L|^2)}{(1-|S_{11}|^2)|1-S_{22}\Gamma_L|^2}$
Z_s doesn't matter. But want $\Gamma_l = S_{22}^*$

$G = \frac{|S_{21}|^2}{(1-|S_{11}|^2)(1-|S_{22}|^2)}$

$$G_T = \frac{|S_{21}|^2(1-|\Gamma_S|^2)(1-|\Gamma_L|^2)}{|1-\Gamma_S S_{11}|^2|1-S_{22}\Gamma_L|^2}$$

Maximum when we have a conjugate match on the input and the output. Note that S_{12} is small so $\Gamma_s = S_{11}^*, \Gamma_l = S_{22}^*$

$$Z_L = Z_0 \frac{1+\Gamma_l}{1-\Gamma_l}$$

$$Z_l = 57.7 - 272j\ \Omega$$

$Z_s = 23.5 - 44.1\, j\ \Omega$

iii) K=0-.38, Delta = 0.51 -> unconditionally stable.

iv)For cascaded amplifiers K Delta test needs to be applied separately to each rather than to the overall cascade so that Γ_1 can be included.

K Delta test assumes that a passive impedance match is present so that the reflection at the input or output is always less than 1. In this case the second amplifier is presenting a reflection which would not be possible with a passive match alone and is greater than 1, so the K-Delta test is not valid

Alternative approaches are to use stability circles or consider the Gain of the loop form by reflections between the amplifiers.

$$\Gamma_{out} = S_{22} + \frac{S_{12}\Gamma_S S_{21}}{1 - S_{11}\Gamma_S}$$

$|\Gamma_S| < 1$ as this is a passive match on the input.

$$\Gamma_{out} = S_{22} + \frac{\Gamma_S * 0.05\angle 135}{1 - 0.6\angle 90 * \Gamma_S}$$

Amplifier is close to unilateral so the 2nd term is quite small. So for a reasonable approximation

$$\Gamma_{out} = S_{22}$$

Therefore we have a loop with a gain of $0.93\angle 20 * 1.25\angle 90$ which gives use an overall gain of greater than 1 so signals will grow and lead to oscillation.

To stabalise the combination need to introduce loss between the amplifiers such that this loop gain is less than 1. The required loss will be 10*log10(1/(1.25*9.3)= -0.65dB

A popular question. Most could give the definition for each gain, but some didn't seem to look at the equations to consider the conditions for reducing to $|S_{21}|^2$. In (ii) the conjugate match was generally well done as was the KΔ test. In part (iv) many assumed this was similar to the coursework and missed that KΔ is only valid for $|\Gamma| < 1$ which was not the case here.