

1.

a) i) When alternate port is terminated S_{11} or S_{22} are equal to Γ

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}$$

$$Z_L = Z_0 \frac{1 + \Gamma}{1 - \Gamma}$$

Impedance at port 1 $43.5 - 120j\Omega$

Impedance at port 2 $356 + 156j\Omega$ (or use smith chart)

ii) reciprocal implies that $S_{12} = S_{21}$

to be lossless $S_{11}S_{12}^* + S_{21}S_{22}^* = 0$

Let $S_{12} = Ae^{j\theta}$

Can rewrite as $S_{11}Ae^{-j\theta} + Ae^{j\theta}S_{22}^* = 0$

Since $|S_{11}| = |S_{22}|$, this implies that the phase of the 1st and 2nd terms are opposite so they cancel, so

$$\angle S_{11} - \theta = \angle S_{22}^* + \theta + 180$$

$$\theta = \frac{\angle S_{11} + \angle S_{22}}{2} + 90$$

$$\theta = 72.5^\circ$$

Also $|S_{11}|^2 + |S_{21}|^2 = 1$

So $|S_{21}| = 0.6131$

$$S_{21} = 0.61 \angle 72.5^\circ$$

iii) looking into port 1 $\Gamma = S_{11} + \frac{S_{12}S_{21}\Gamma_L}{1 - S_{22}\Gamma_L}$

for a perfect match at port 1. $\Gamma = 0$

$$-S_{11} = \frac{S_{12}S_{21}\Gamma_L}{1 - S_{22}\Gamma_L}$$

$$-S_{11}(1 - S_{22}\Gamma_L) = S_{12}S_{21}\Gamma_L$$

$$\Gamma_L = \frac{-S_{11}}{S_{12}S_{21} - S_{11}S_{22}}$$

$$\Gamma_L = 0.78 - 0.08j$$

$$Z_L = 360 - 150j$$

b) $Z_{ij} = \frac{V_i}{I_j} |_{I_k=0}$, so by inspection we have Z parameters:

$$Z_{11} = j(X_p + X_s)$$

$$Z_{22} = Z_{12} = Z_{21} = jX_p$$

Using provided equations

$$\frac{S_{11}}{S_{12}} = \frac{(Z_{11} - Z_0)(Z_{22} + Z_0) - Z_{22}^2}{2Z_{22}Z_0}$$

$$= \frac{Z_{11}Z_{22} - Z_0Z_{22} + Z_{11}Z_0 - Z_0^2 - Z_{22}^2}{2Z_{22}Z_0}$$

Let $\frac{S_{11}}{S_{12}} = a + jb$. Recall that Z_{11}, Z_{22} are imaginary so write $Z_{11} = jX_{11}$ etc

$$a + jb = \frac{-X_{11}X_{22} - Z_0jX_{22} + jX_{11}Z_0 - Z_0^2 + X_{22}^2}{2jX_{22}Z_0}$$

Taking real part

$$a = -\frac{1}{2} + \frac{X_{11}}{X_{22}}$$

Imaginary part

$$-b = \frac{-X_{11}}{2Z_0} - \frac{Z_0}{2X_{22}} + \frac{X_{22}}{2Z_0}$$

$$X_{11} = 2X_{22} \left(a + \frac{1}{2} \right)$$

$$b = \frac{2X_{22} \left(a + \frac{1}{2} \right)}{2Z_0} + \frac{Z_0}{2X_{22}} - \frac{X_{22}}{2Z_0}$$

$$\frac{S_{11}}{S_{12}} = -0.5164 - j1.1877$$

$$X_{22} = 133.2$$

$$X_{11} = -2.64$$

$$Z_{22} = Z_p = 133.2j\Omega \quad L = 7.07nH$$

$$Z_{11} = -2.64j$$

$$X_s = -135.84 \text{ C} = 0.39\text{pF}$$

2.

(a) Gain will be 26dB

At 290 we can use noise figure cascade equation.

$$F_{cas} = F_1 + \frac{1}{G_1}(F_2 - 1)$$

Convert to linear

Noise fig of 4dB atten is 4dB.

$$F_1 = 2.512$$

$$G_1 = 0.4$$

$$F_2 = 2$$

$$F_{cas} = 5.012$$

NF = 7dB

(b)

For attenuator (hot)

$$T_{e_atten} = \frac{(1 - G_1)}{G_1} T$$

$$T_{e_atten} = 514K$$

For amplifier

$$F = 1 + \frac{T_e}{T_0}$$

$$T_{e_amp} = 288.6K$$

$$T_{cas} = T_{e_atten} + \frac{1}{G_1} T_{e_amp}$$

$$T_{cas} = 1239 K$$

$$F_{cas} = 1 + \frac{T_e}{T_0}$$

$$F_{cas} = 5.27$$

Noise figure 7.22dB.

c) figure shows that the intermods are likely 3rd order products as they are 2f₂-f₁ and 2f₁-f₂. Assuming that higher order contributions to these terms are negligible.

At a 0dBm output power, the intermods are 20dB below. Since the intermods increase 3dB/dB and the fundamental 1dB/dB this means we need to increase the input power by 10dB to reach OIP₃ which is at +10dBm.

IIP₃ is OIP₃ – gain so IIP₃ = -20dBm.

d) SFDR = 2/3 (OIP₃ – Noise Floor)

If the attenuator is now after the amplifier, the OIP3 will be reduced by 4dB, so OIP3 is +6dBm.

Input noise is not at 290K so need to use noise temps to calculate.

$$T_{e_{cas2}} = T_{e_{amp}} + \frac{1}{G_{amp}} T_{e_{atten}}$$

$$T_{cas2} = 289K$$

$$kT_{in} = 10^{-18} mW$$

$$T_{in} = 72.46K$$

Total noise at input T = 361K

$$N_{out} = kTG = 1.23 \times 10^{-23} * 361 * 10^{2.6} = 1.7677 \times 10^{-18} W/Hz$$

Noise out is -147.5 dBm

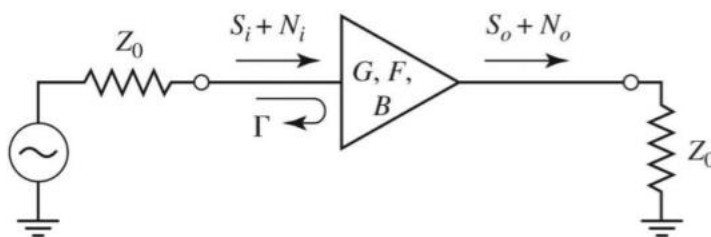
$$SFDR = 2/3(6 - 147.5)$$

$$= 102.4 \text{ dB/Hz}^{2/3}$$

e) To reduce the signal by 4dB (0.3981) we must have :

$$1 - |\Gamma|^2 = 0.4$$

$$|\Gamma| = 0.77$$



At 290K

$$N_i = kT_0 B$$

Output noise consists of noise in reduced by reflection and amplifier noise referred to amp input.

$$N_o = kT_0 GB(1 - |\Gamma|^2) + kT_0 (F_{amp} - 1)GB$$

$$S_o = (1 - |\Gamma|^2)GS_i$$

$$F_{sys} = \frac{S_i N_o}{S_o N_i} = 1 + \frac{F_{amp} - 1}{1 - |\Gamma|^2}$$

$$F_{sys} = 3.552$$

$$NF = 5.5dB$$

3. a) i)

$$T_A = \eta_{rad} T_b + (1 - \eta_{rad}) T_p$$

So if $T_A = 105K, T_p = 290K, T_b = 7K$

$$\eta_{rad} = 0.65$$

This assumes that the major contribution to the antenna is coming from the background which it is pointing at the that the contributions from side lobes which might see higher temperatures are insignificant.

ii) $e_{pol} = |\cos(\theta)|^2$

so for 20 degree miss alignment.

$$E_{pol} = |\cos(20)|^2 = 0.883$$

$$\text{Friis path loss} = \frac{G_t G_r \lambda^2}{16\pi^2 R^2} = 5.06 \times 10^{-7}$$

So overall is -63dB

iii) Reflections can increase the received power provided that they arrive in phase with one another, so a strong reflector must be present. If there is a single reflector and the path difference between the direct path and reflected path is less than one wavelength, losses can increase as R^4 (beyond the knee in the flat earth model)

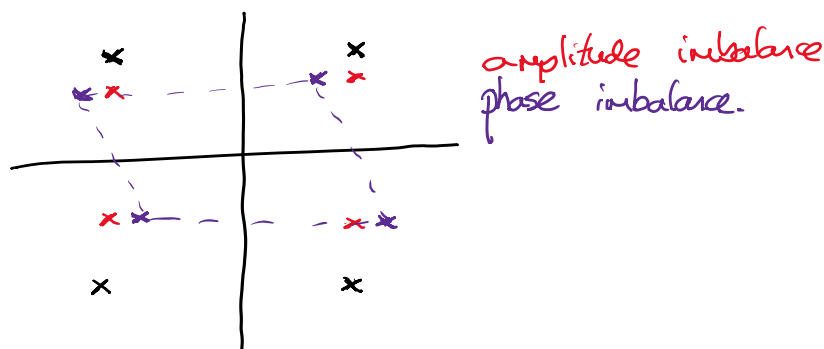
b) (i) Direct conversion – fewer components, and removes the image problem. This allows the design to potentially tune over a wide range of frequencies as a front end image rejection filter is not required.

Since the LO is at the RF frequency leakage through the mixer can be a problem. This will reflect back into the receiver from the input if mismatched (and be radiated from an antenna connected at the input) producing a strong signal at the frequency of the wanted signal. This will produce a large DC offset after mixing which might saturate ADC.

(ii) the splitter is lossy but also has an amplitude and phase imbalance. Ideally S21 and S31 should have an equal amplitude and 90 degree phase shift to separate the IQ components.

The amplitude imbalance will cause the IQ plot to become rectangular rather than square. The phase imbalance means that IQ and are not completely orthogonal which will cause a skew in the constellation. This reduces the error vector magnitude and increases the likelihood of errors.

The imperfection can be compensated by using a known preamble to characterise the receiver and then applying the inverse function.



4. a)

i) Amplifier is not unconditionally stable, stability circles required

$$C_s = 1.24 \angle 157^\circ$$

$$R_s = 0.3197$$

It is the area inside the circle which is potentially unstable.

ii) S_{12} is small so we can assume that device is unilateral and consider input and output sides independently.

Minimum noise figure is F_{min} , so 2dB and requires that we present Γ_{opt} as this is outside of the stability circle it is a valid solution. For maximum gain on the output side need a conjugate match ($\Gamma_{out} = 0.3 \angle 180$). Again this is outside the stability circle so is ok.

$$G_A = \frac{|S_{21}|^2(1 - |\Gamma_S|^2)}{|1 - S_{11}\Gamma_S|^2(1 - |\Gamma_{out}|^2)}$$

$$G_A = 23.3$$

$$Z_{opt} = Z_0 \frac{1 + \Gamma_{opt}}{1 - \Gamma_{opt}}$$

$$Z_{opt} = 27 - 22.2j \Omega$$

iii) See smith chart (next page)

For min noise need Γ_{opt} . Plotting Γ_{opt} on smith chart gives us the Z_s so we need to design a matching network which transforms Z_0 into Z_s .

Transmission line is 0.006λ .

Reflect through centre to find $0.9j$ inductive susceptance required. Measure from open (short in admittance towards source) $\rightarrow 0.13\lambda$ stub.

b) i)

For max bandwidth don't use any impedance matching networks. Since the centre of smith chart is in the stable region of both input and output side this is ok.

Bandwidth is improved as impedance matching to reduce reflections on the input and output is not required as the 4 port hybrids allow reflections to be dumped into the resistive loads so they do not have an impact on other stages.

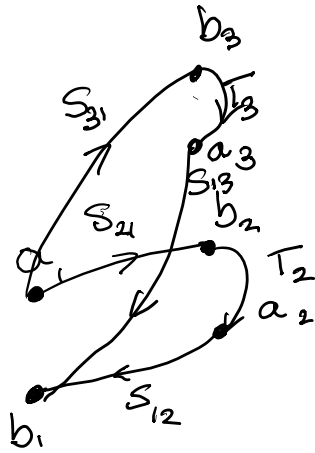
Compared to (a) the gain is reduced due to the lack of matching causing reflections, the noise figure will be worse as we are away from Γ_{opt} and the linearity is improved in terms of OIP3, IIP3 is much greater due to the reduced gain. Efficiency at a particular output will be halved since there are now 2 devices to bias, but as the P1dB will double, efficiency at P1dB will be unchanged

Compared to a single unmatched amplifier, the gain will be the same, the noise figure improves and the linearity improves.

ii) If the amplifiers are unilateral then the $\Gamma_{in} = S_{11}$

For the system.

Considering signal flow graph for S11:



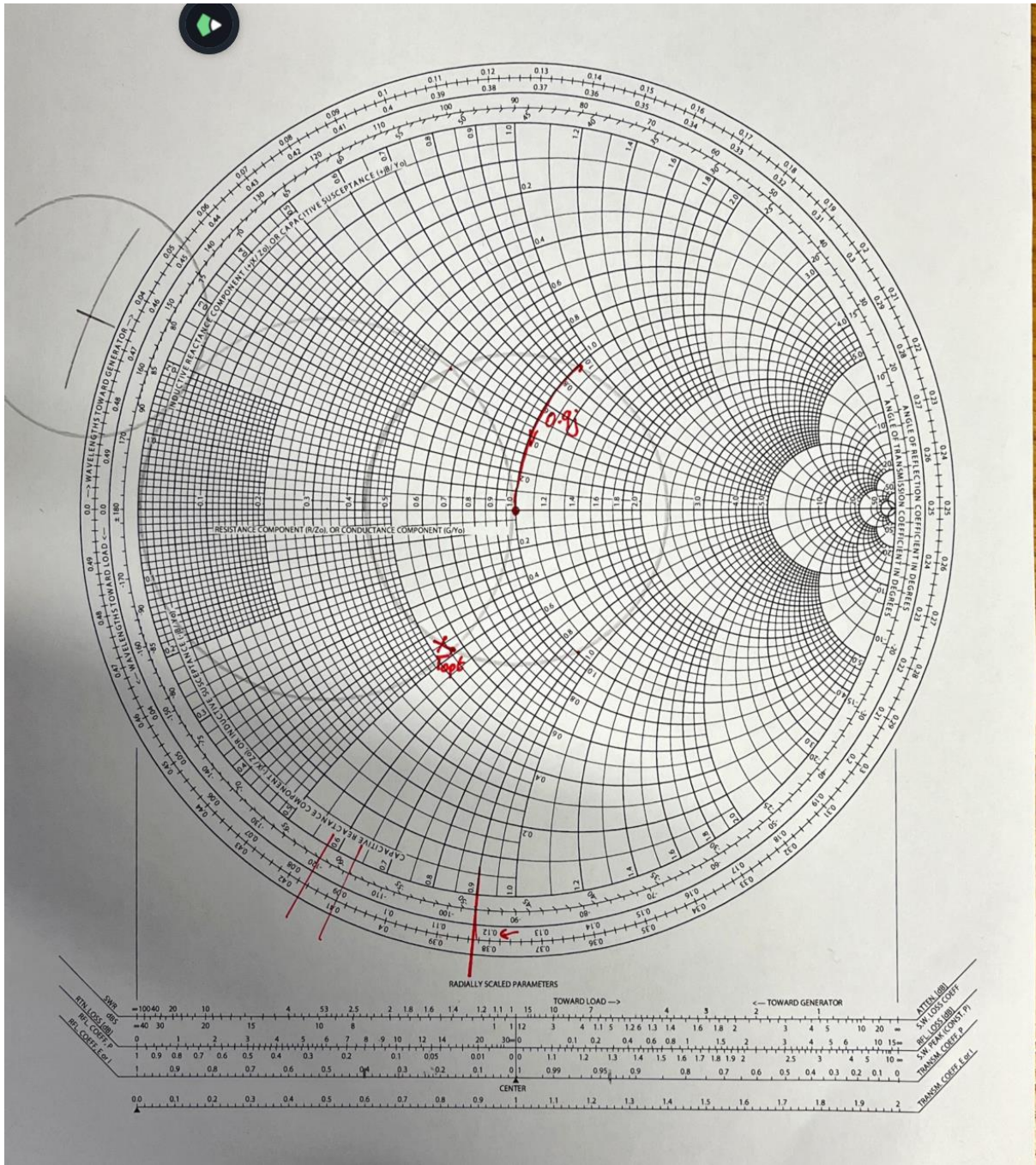
$$\Gamma = S_{31}S_{13}\Gamma_3 + S_{21}S_{12}\Gamma_2$$

$$S_{11} = 0.139 \angle -60^\circ$$

For the unmatched case the amplifier gain is simply S_{21} so the overall gain will be:

$$S_{31_hybrid}S_{13_hybrid}S_{21_amp} + S_{21_hybrid}S_{12_hybrid}S_{21_amp}$$

$$\text{Gain} = 4.98 \angle 121^\circ$$



Q1:

A very popular question. Part (a) was well answered with the phase of the S_{21} in (ii) proving to be the most difficult part. (b) was clearly trickier although most could find the Z parameters for the given circuit not many realised that the fact the z parameters were entirely imaginary helped and that careful choice of the equations to use also helped a lot.

Q2:

A popular question. Noise figures and temperatures were well answered. In the SFDR calculation many did not realise that it is the output noise which matters and the input noise was provided, and the noise figure had changed. Most used the cascaded IIP3 equation rather than a more intuitive approach considering the impact of the attenuator.

Q3:

The least popular question, but good attempts from those who chose it in the early parts. In (a) (ii) some included the radiation efficiency not realising that gain take this into account. Answers on the direct conversion receiver were often rather generic rather than addressing the specific case asked for in (b)(i). (ii) was answered better.

Q4:

This was the least well answered question overall. Most could do the straight forward stability parts, and impedance matching. The answers on the hybrid amplifier operation were disappointing with some having little idea about the principles, of those that did understand the operation, the performance was often not discussed in relation to the matching conditions specified.