4B24 Radio Frequency Systems 2020-2021 Crib

1. (a)

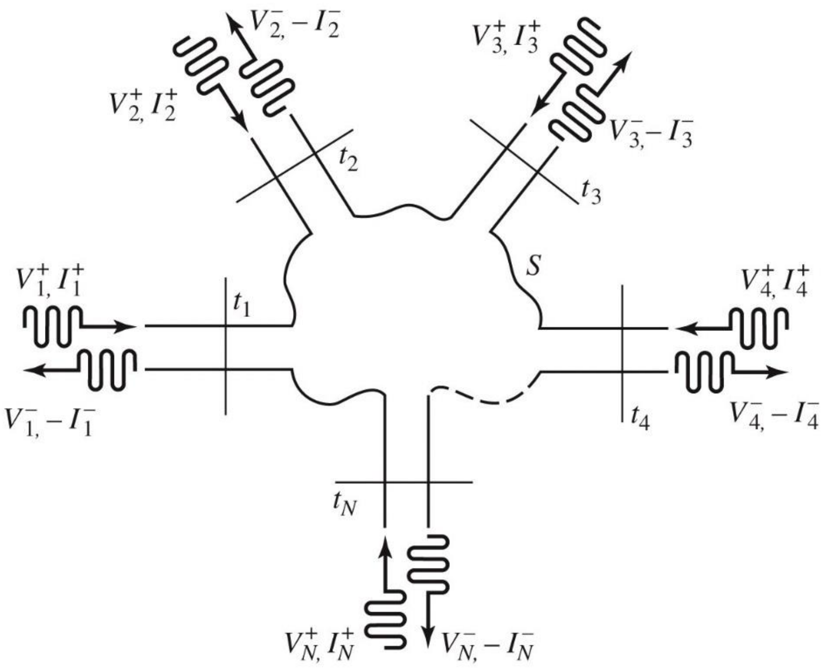
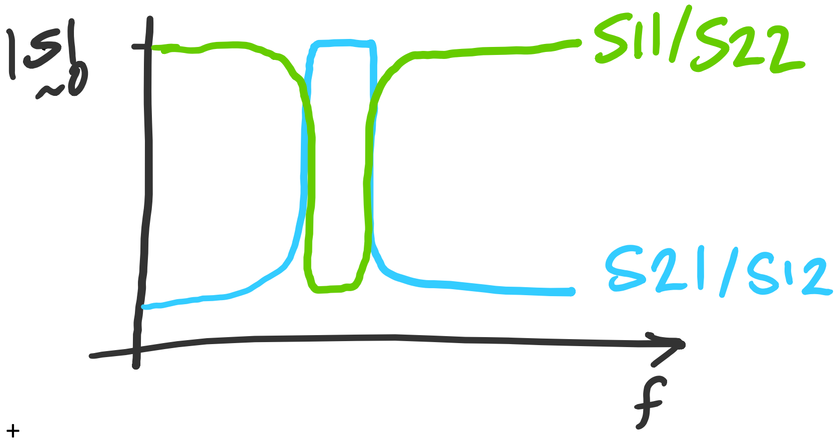


Figure 4.5

$$S_{ij} = \frac{V_i^-}{V_j^+}\bigg|_{V_k^+=0\ for\ k\neq j}$$

V_i^- is the voltage out of the I'th port
V_j^+ is the voltage into the j'th port
$V_k^+ = 0$ implies that the other ports are all terminated in matched impedances so that there is no voltage incident on the port as a result of any reflections.
S21 and S12 represent the transmission through the filter. S11 and S22 represent reflection in the filter.



+

b)
Impedance parameters:

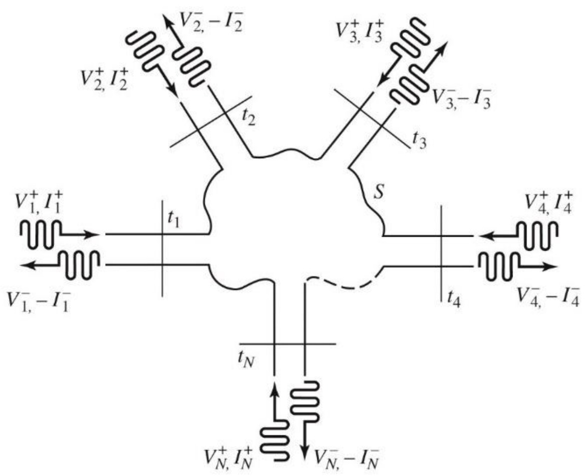


Figure 4.5

$$Z_{i,j} = \left.\frac{V_i}{I_j}\right|_{I_k=0 \; for \; k\neq j}$$

Z11 – open circuit on the output. ZA and ZB appear in series.
Z11=ZA+ZB
Z21 drive current into port 1 and look at open circuit voltage on port 2
Z21 = ZB
Clearly reciprocal so Z22=Z11 and Z12=Z21

c)
All elements except S21,S23 and S31 are zero.
Signal incident on port 1 appears losselessly at port 2
Signal incident on port 2 appears losselessly at port 3
Signal incident on port 3 appears losselessly at port 1
Clearly not reiciprocal.
Mismatch at port 2 will result in signal incident from port 1 appearing at port 2, reflecting off mismatch and entering port 2, so overall signal incident on port 1 will appear at port 3.
Signals from port 3 to port 1 will be unaffected.

d)
Terminate Z02 section with Z02 to get V+2=0

$$V_1 = (jX + Z_{02})I$$

$$V_1^+ + V_1^- = (jX + Z_{02})\frac{V_1^+ - V_1^-}{Z_{01}}$$

$$V_1^+\left(1 - \frac{jX + Z_{02}}{Z_{01}}\right) = V_1^-\left(-1 + \frac{jX + Z_{02}}{Z_{01}}\right)$$

$$S_{11} = \frac{jX + Z_{02} - Z_{01}}{jX + Z_{02} + Z_{01}}$$

By similar argument or inspection:

$$S_{22} = \frac{jX + Z_{01} - Z_{02}}{jX + Z_{02} + Z_{01}}$$

S12 – Terminate port 1 with Z01.

$$V_2 = (jX + Z_{01})I_1$$

$V_2^+ + V_2^- = (jX + Z_{01})\,\frac{-V_1^-}{Z_{01}}$ (remember $V_1^+ = 0$ by definition.)

$$I_1 + I_2 = 0$$
$$\frac{-V_1^-}{Z_{01}} + \frac{V_2^+ - V_2^-}{Z_{02}} = 0$$
$$V_2^- = V_2^+ - \frac{Z_{02}}{Z_{01}} V_1^-$$
$$(jX + Z_{01}) \frac{-V_1^-}{Z_{01}} = V_2^+ - \frac{Z_{02}}{Z_{01}} V_1^- + V_2^+$$

$$V_1^- \left(-\frac{jX + Z_{01}}{Z_{01}} + \frac{Z_{02}}{Z_{01}} \right) = 2V_2^+$$
$$S_{12} = \frac{2Z_{01}}{Z_{02} - jX - Z_{01}}$$
$$S_{21} = \frac{2Z_{02}}{Z_{02} - jX - Z_{01}}$$

2 (a)

New frequencies are 3rd order intermodulation products which arise due to 3rd order non-linearies which result in $2\omega_1 - \omega_2$ and $2\omega_2 - \omega_1$ so the tones are equally spaced as shown. For a single stage, 3rd order nonlinearity will result in equal power in the 3rd orders. For cascaded stages each stage may be non-linear. In the case of multiple stages, the 2nd stage with amplify the input IM products and contribute its own. If there is a long phase delay between the stages, the 1st and 2nd stage IM products may not add in phase, so it could be the case that the phase shift is larger at higher frequencies, reducing the coherent sum.

(b)

$$Y = \frac{N_1}{N_2} = \frac{T_1 + T_e}{T_2 + T_e}$$
$$T_e = \frac{T_1 - YT_2}{Y - 1}$$
$$15.83dB = 38.28$$
$$T_e = 1162.88$$
$$F = 1 + \frac{T_e}{T_0}$$

$T_0 = 290K$ by definition of noise figure. NF only valid for this input noise temperature.

$F = 5$ or 7dB.

(c)

View as power from source, R and power added by lossy line.

$$N_o = kTB = GkTB + GN_{add}$$

Solving gives

$$N_{add} = \frac{1 - G}{G} kTB = (L - 1)kTB$$

So we can have $T_e = (L - 1)T$

(d)

(i) $\Gamma = \frac{30-50}{30+50} = -\frac{1}{4}$

(ii) Available gain – ratio of power available from port 2 to power available from source.
Loss of line 1 = 2dB = 1.58
Loss of line 2 = 1dB = 1.26
Total gain = (1-0.25^2)*1/1.58*1/1.26 = 0.469 or -3.28dB

(iii) Let $N_i = kT_0B$ (by definition for noise figure)
Contributions from:
noise in reduce by reflection and loss of L1 and L2
noise from L1 reduced by reflection and L2
noise from L2.

$$N_0 = \frac{kT_0B(1-|\Gamma|^2)}{L_1L_2} + \frac{(L_1-1)}{L_1L_2}kTB(1-|\Gamma|^2) + \frac{L_2-1}{L_2}kTB$$

$$S_0 = \frac{1-|\Gamma|^2}{L_1L_2}S_i$$

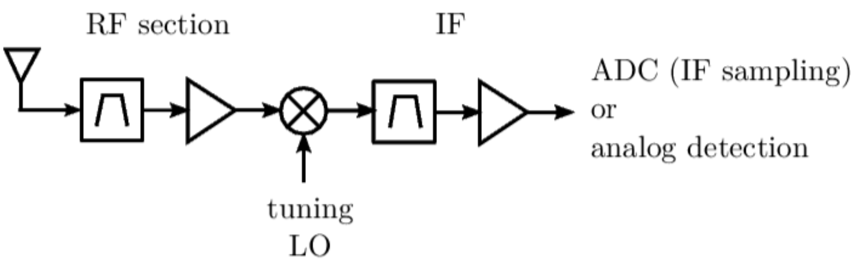
$$F = \frac{S_iN_o}{S_oN_i} = \frac{\left(\frac{kT_0B(1-|\Gamma|^2)}{L_1L_2} + \frac{(L_1-1)}{L_1L_2}kTB(1-|\Gamma|^2) + \frac{L_2-1}{L_2}kTB\right)}{\frac{1-|\Gamma|^2}{L_1L_2}kT_0B}$$

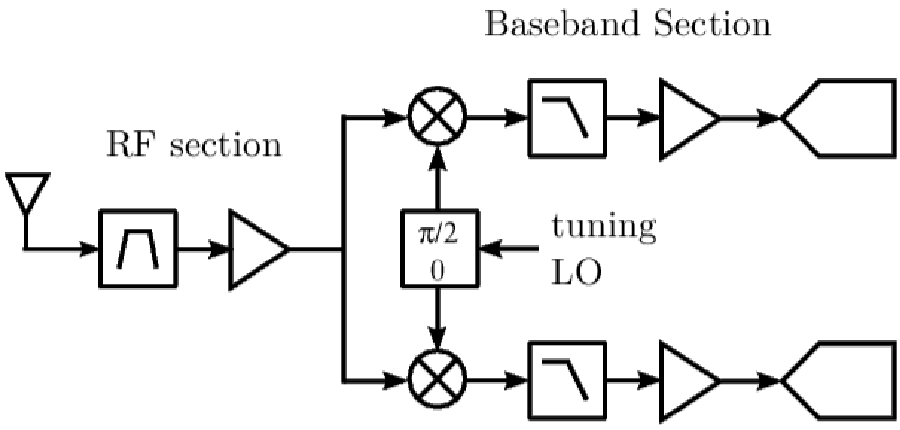
$$F = 1 + \frac{(L_1-1)T}{T_0} + \frac{(L_2-1)L_1}{1-|\Gamma|^2}\frac{T}{T_0} = 2.16$$

Or 3.34dB

3 (a) Where there is a single reflection from the ground, the freespace and reflected waves interfere with one another resulting in fading. Beyond the break point, at large separations and for vertical polairation with a reflection co-efficient of -1, this results in a loss which increases as R^4. Beyond the breakpoint, this is independent of frequency, but the breakpoint at which the loss changes to be proportional to frequency squared (and range squared) scales with frequency. Higher frequency -> greater range with r^2 loss.

(b) (i) Advantage of superheterodyne is that the majority of filtering is carried out at a relatively low IF where the fractional bandwidth is significant so filters don't require a very high Q. Also allows flexibility by tuning the LO so that a single fixed filter can be used for multiple channels. For this application the RF bandwith is 400MHz, so the IF must be greater than 200MHz. This is rather high!
Half IF problem – might want double the IF so half IF is also out of band and won't result in components at the IF due to 2nd order non-linearity.
Direct conversion receiver. No image problem, simpler components, reduced ADC requirements, but 2 chains needed to capture complex signal.





(iii) Treat as loss-less antenna and attenuator:

$$T_A = \eta_{rad} T_b + (1 - \eta_{rad}) T_p = 355K$$

$N = kTB = 4.899 \times 10^{-21} W$ or -173dBm/Hz

Note that it isn't appropriate to use a specific bandwidth – the channel filter will be much greater and the antenna must have a bandwidth from at least 400MHz, but is likely to be higher.

(c) (i) radar cross section: $\sigma = \frac{P_s}{S_t}$ Ps is power scattered, St is the incident plane wave.

$$P_S = G P_r \Gamma$$

$$P_r = S_t A_e = \frac{S_t \lambda^2 G}{4\pi}$$

So $\Delta\sigma = \frac{P_S}{S_t} = \frac{\lambda^2 G^2 |\Gamma_A - \Gamma_B|^2}{4\pi}$

$$\Gamma = \frac{Z_1 - Z_{ant}}{Z_1 + Z_{ant}}$$

$$\Gamma_1 = -0.8182, \Gamma_2 = 0.333$$

$$\lambda = 0.1m$$

$\Delta\sigma$ =0.0042m^2

The modulation is a combination of ASK and PSK.

(ii) maximise the rcs with $\Gamma_a = -1$ and $\Gamma_b = +1$ ie. Open and short (or any other point opposite sides of smith chart. Rcs = 0.01256m^2 (approx. a 3x increase).

The losses from freespace will go a r^4, so for the same tx power and detected signal the range impoves by 3^0.25 or about 32%.

4 (a) If the amplifier is not unilateral then $\Gamma_{in} \neq S_{11}$ and $\Gamma_{out} \neq S_{22}$. In such a case the input and output must be matched simultaneously as Γ_{in} will depend on Γ_L and Γ_{out} depends on Γ_s.

(b) (i)

$K - \Delta$ test, unconditionally stable if and only if

$K = \frac{1-|S_{11}|^2-|S_{22}|^2+|\Delta|^2}{2|S_{12}S_{21}|} > 1$ and $|\Delta| = |S_{11}S_{22} - S_{12}S_{21}| < 1$

$$|\Delta| = 0.2359$$
$$K = -0.16$$

Since K<1 the amplifier is only conditionally stable.

$C_L = \frac{(S_{22}-\Delta S_{11}^*)^*}{|S_{22}|^2-|\Delta|^2} R_L = \left|\frac{S_{12}S_{21}}{|S_{22}|^2-|\Delta|^2}\right|$

$$C_L = 1.035\angle -21°$$
$$R_L = 0.0831$$

Since $|S_{11}|$ and $|S_{22}|$ are both <1, centre of chart is stable.
Max gain $G_T = G_s G_0 G_L$, conjugate match input and output.
S_{12} close to zero so can make unilateral assumption.

$$G_L = \frac{1}{1-|S_{22}|^2} = 7.4$$
$$G_S = \frac{1}{1-|S_{22}|^2} = 1.5625$$
$$G_0 = |S_{21}|^2 = 25$$
$$G_T = 289\ (24.6dB)$$

$S_{22}^* = 0.95\angle -20°$ since 1.035-0.0831=0.9519 this is right on the boundary of stability, but is just about stable.
(S_{11}^* is fine but not asked for)

ii) $G_T = G_S G_0 G_L$
For direct connection to 50 ohms on input side $G_S = 1$
For max gain, $G_L = \frac{1}{1-|S_{22}|^2} = 7.4$

$$G_0 = |S_{21}|^2 = 25$$

Total gain = 22.7dB
Noise figure:

$$\Gamma_{opt} = \frac{\frac{1}{Y_{opt}} - 1}{\frac{1}{Y_{opt}} + 1}$$

$\Gamma_S = 0$ by definition if input is direct to 50 ohms.

$$\mathrm{N} = \frac{\left|\Gamma_S - \Gamma_{opt}\right|^2}{1-|\Gamma_S|^2} = 0.8681$$
$$F = \frac{N\frac{4R_N}{Z_0}}{\left|1+\Gamma_{opt}\right|^2} + F_{min}$$
$$F = 2.7212\ (4.34dB)$$

iii) For 20dB gain = 100

$$G_L = \frac{100}{G_0} = 4$$
$$g_L = \frac{4}{7.4} = 0.5404$$
$$C_L = \frac{g_L S_{22}^*}{1-(1-g_L)|S_{22}|^2}, R_L = \frac{\sqrt{1-g_L}(1-|S_{22}|^2)}{1-(1-g_L)|S_{22}|^2}$$
$$C_L = 0.8343 \angle -20°$$
$$R_L = 0.1520$$

To minimise match, pick the location on the smith chart which is closest to the origin.

$$\Gamma_L = 0.6823\angle -20°$$

Either by calculation or reading from smith chart $Z_M = 150 - 120j$

iv) With no input matching network present, $\Gamma_S = 0$ – no reflection from IMN looking towards 50 ohm source. As a result the $\Gamma_s\Gamma_{in}$ term goes to zero which is the only term which is effected by the S_{12} value. Another way to look at this is that there is no reflection from the source so no reverse components are reflected back into the amplifier.

Examiners comments:

Q1:

A popular question. Answers for parts (a)-(c) were generally good, but many struggled with the mixture of line impedances in part (d) trying to solve it from the point of view of reflections (which would require an infinite geometric series) rather than using S parameters from 1st principles.

Q2:

Early parts were well answered, although many missed the multi-stage hint in (a). (b) and (c) were generally well done, although a few used a circular argument for part (c). Part (d) generally started well, but the complexity arising from the mis match in part (iii) which prevents the common expressions for cascaded noise figure being applied.

Q3:

Generally well answered. Some confused the channel bandwidth and tuning range in determining the IF for the superhet. The backscatter was generally well done. Common mistakes were with the signs of the reflection coefficient and failing to notice that both phase and amplitude are modulated in c(i), but that phase only modulation will be optimum in (ii)

Q4:

Unpopular with polarised responses. Some near perfect solutions and some trying to pick up the odd mark here and there. A common problem was not realising the small S12 value.