EGT3
ENGINEERING TRIPOS PART IIB

Friday 29 April 2022 14.00 to 15.40

Module 4B5

QUANTUM AND NANO-TECHNOLOGIES

*Answer not more than **three** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

*Write your candidate number **not** your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
Attachment: 4B5 formula sheet (2 pages)
Engineering Data Book

10 minutes reading time is allowed for this paper at the start of the exam.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

You may not remove any stationery from the Examination Room.

1 (a) A number of experiments in the early 1900s led to the development of quantum theory. In practice, while it was straightforward to demonstrate the wave nature of particles, it was far more challenging to convincingly demonstrate the particle nature of light. Describe and discuss the two main experiments which led to the conclusion that light can be considered to have a particle nature. Your answer should include a description of the experimental findings and how they were interpreted. [35%]

(b) X-rays of wavelength 10^{-11} m encounter a metal foil at normal incidence. If we were to position a spectrometer in such a way as to collect the X-rays transmitted at 60^o to the incident beam, what wavelength(s) would we observe? [25%]

(c) The Klein-Gordon equation was arrived at by applying the principle of complementarity to Einstein's mass-energy relationship, yet it was later shown to be incapable of explaining the behaviour of non-relativistic particles. This led to the development of Schrödinger's equation. Describe the differences between the two equations and why the interpretations of their solutions are mutually incompatible. [20%]

(d) Within the framework of the Copenhagen interpretation of quantum mechanics, explain the physical significance of the wavefunction, ψ and its square. [20%]

2 Electrons at an energy E = 1 eV are incident from the left on the one-dimensional potential step of height V, which is 0.5 eV less than E, as shown below in Fig. 1.

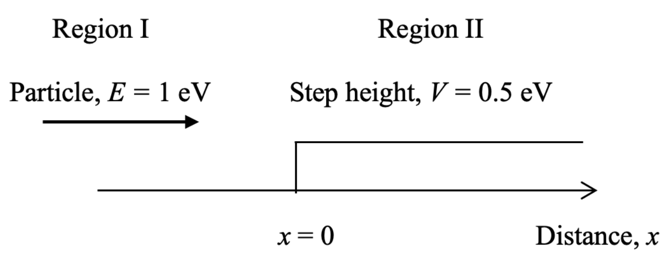

Fig. 1

(a) Write down the time-independent single-particle wave-functions for regions I and II. Hence calculate the reflection coefficient for this potential step. [30%]

(b) Consider the case where the above potential step occurs in a semiconductor heterostructure, and where the electron effective mass in region II is 40% of that in region I. Calculate the new reflection coefficient, stating any assumptions made. [40%]

(c) Now consider the case where we have a potential barrier which is higher than the incident electron energy as shown below in Fig. 2. Find an approximate value of d at which the electron probability density is 20% of the value at $x = 0$, stating any assumptions made. You may assume an electron effective mass of 0.1 m_e. Briefly discuss how you would improve the precision of this calculation. [30%]

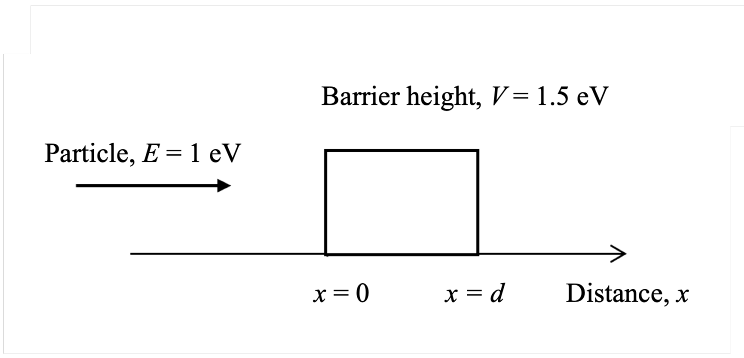

Fig. 2

3 Consider a device with the potential profile as shown below in Fig. 3 where there is one bound state of energy E_1 within the potential well of height V. E_f is the Fermi energy of both sides.

(a) What is this device called? Sketch what happens to this potential profile as a voltage is applied to the right-hand side (such that it lowers the potential there), while keeping the left-hand side at ground potential. [20%]

(b) Describe what happens to the current as the applied voltage is increased, taking into account the relationship between current and transmission probability. Sketch the form of current vs voltage characteristic for the device, labelling the salient features, and indicate the potential profile corresponding to the different regimes of operation. [30%]

(c) Sketch the wave-functions for an electron travelling through this device for the case where the applied voltage is such that the Fermi energy on the left-hand side

(i) matches E_1, and

(ii) is larger than E_1 but lower than V. [20%]

(d) Describe what would happen to the bound states if instead of having one potential well as in Fig. 3, we had two potential wells very close together. How many electrons could such a structure contain? How will the bound states evolve as

(i) we introduce more wells, or

(ii) we change the spacing between the two wells? [30%]

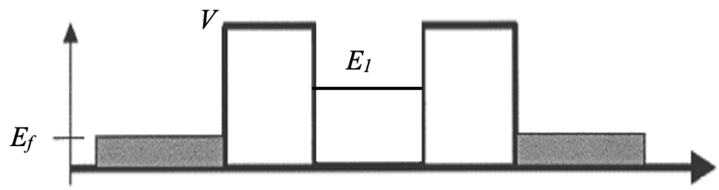

Fig. 3

4 (a) Explain what is meant by band engineering and why it is employed. [10%]

(b) What is a heterojunction? Describe what happens to the electron bands when a piece of GaAs is brought into contact with a piece of GaAlAs which has a larger band gap. In what way is the resulting 2-dimensional electron gas (2-DEG) fundamentally different from the sea of electrons in a bulk semiconductor? Sketch the first three sub-bands of a 2-DEG, stating any assumptions made. [30%]

(c) We would like to fabricate a quantum-well laser using a semiconductor heterostructure. The quantum well is 10 nm wide and is formed in a semiconductor with a band gap of 1.1 eV. Given that the electron and hole effective masses are 0.06 m_e and 0.3 m_e, respectively, where m_e is the free electron mass, estimate the emission wavelength of this laser. How accurate do you expect this answer to be? How would you refine your calculation? [50%]

(d) If we were to replace the quantum well with a quantum dot, in what ways would the laser's operation be different? [10%]

5 (a) Describe, in detail, how the electrical conductance of a metallic wire evolves as its dimensions decrease from 100 nm down to a single atom, with reference to surface specularity, mean grain size and electronic mean free path. At what size would you expect to observe quantum effects directly at room temperature? [40%]

(b) Describe the principle of operation of high electron mobility transistors (HEMTs), with sketches of their band structure. Design a HEMT which takes advantage of hot electrons to further increase its speed, and draw its basic structure, ensuring that the various layers are clearly labelled, and their function is made clear. [40%]

(c) Explain how the band structure of graphene gives rise to its unusual electrical properties, and what the challenges are in terms of creating high-mobility graphene-based transistor structures with high on/off ratio. Your answer should include how those challenges can be addressed. [20%]

END OF PAPER

4B5 Quantum Technologies Formula sheet – 2 pages

$-\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2} + V(x)\psi(x) = E\psi(x)$	1-D Time-independent Schrödinger equation, $V(x)$ is the potential, E is the total energy.
$\hat{E} = i\hbar\frac{\partial}{\partial t}$	Quantum Energy Operator.
$\widehat{\boldsymbol{p}} = -i\hbar\frac{\partial}{\partial x}$	Quantum Momentum Operator.
$\widehat{K.E.} = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}$	Quantum Kinetic Energy Operator.
h	Planck's constant = 6.626 x 10^{-34} Js
$\hbar$	Reduced Planck's constant = 1.05 x 10^{-34} Js
e	Electron charge = 1.6 x 10^{-19} C
m	Free electron rest mass = 9.11 x 10^{-31} kg
c	Speed of light in vacuo = 2.998 x 10^{8} ms^{-1}
$E_n = \left(n + \frac{1}{2}\right)\hbar\omega_c$	Spectrum of energy levels in Quantum Harmonic Oscillator, where ω_c is the natural frequency ($\sqrt{\frac{k}{m}}$) of the system in rad s^{-1}
$E_n = \frac{n^2h^2}{8mL^2}$	Spectrum of energy levels in infinite Quantum Well of length L.

$\Delta x(t) = \sqrt{\Delta x_0 + \left(\frac{\hbar t}{m \Delta x_0}\right)^2}$	Width (standard deviation) of matter wave- Packet of initial width Δx_0 as a function of time, t.
$\frac{d\lfloor \psi(x) \rfloor^2}{dt} = \frac{i\hbar}{2m} \nabla j$	Quantum continuity equation, where j = probability flux.
$-\frac{i\hbar}{2m} \{\psi^* (\nabla \psi) - (\nabla \psi^*) \psi\}$	1-D Probability flux.
$T \cong e^{-\frac{2}{\hbar} \int_A^B \sqrt{2m(V(x) - E)} dx}$	WKB approximation to Transmission Probability, T. Integration limits are the Classical turning points, A & B.