4B5 2023 solutions

1.

(a) Light of wavelength 200 nm has an energy of $hc/\lambda = 6.6\text{x}10^{-34}\text{x}3\text{x}10^{8}/200\text{x}10^{-9} = 6.19$ eV. Therefore, light of this wavelength has sufficient energy to overcome the work function, and electrons will be emitted via the photoelectric effect. Their kinetic energy will be the difference between the photon energy and the work function, i.e. 6.19-5 = 1.19 eV. Converting this to kinetic energy, 1.19 eV $= ½mv^2$ yields that $v = 5.11\text{x}10^5$ m/s.

(b) Doubling the brightness will increase the intensity of the light source, which means increasing the number of photons without changing their energy. Therefore, it will double the number of electrons emitted from the surface, but it won't change their energy or velocity.

(c) The answer should include a discussion around the fact that the classical picture treats the radiation as a continuous stream of light, whereas the quantum viewpoint is that it comprises a stream of energetic photons. Each of these photons has a well-defined energy of hf. Via superposition, increasing the number of photons has the effect of increasing the net electric field, $\boldsymbol{E}$.

(d) The key assumptions are (i) the system comprises n oscillators, and there are a series of energy levels given by the quantum harmonic oscillator. The probability of occupation of any of those levels is determined by the Boltzmann distribution. Beyond that, a general principle in quantum systems is that increased confinement leads to higher energy levels. Higher temperature corresponds to higher energy, and via deBroglie's hypothesis, that means higher frequency.

2.

(a) Ohm's law tells us that $I \propto V$. Landauer showed how conduction takes place elastically in transport channels, and that the current will depend also on *T*. Ultimately, Ohm's law is that $I \propto \rho$ for a given voltage, and essentially, apart from geometric factors, $T \propto 1/l$ which is the same as saying resistance $\propto l$.

(b) The key point is that one only notices the quantisation of conductance when there is coherence. This happens in systems whose length is far less than the mean free path and where there are a small number of conductance channels. Larger systems tend to behave more diffusively. Therefore, one would expect to observe the quantum nature of conduction in cases where the system is of order a few nm in size or at temperatures approaching 0K.

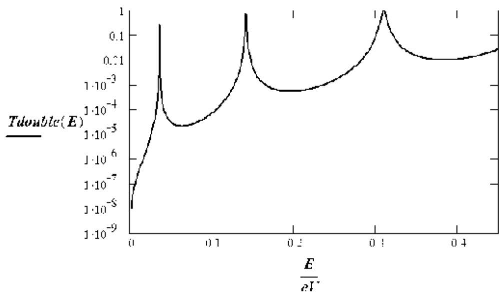
(c) The transmission probability is approximated as

$$T \sim e^{-2ka}$$

where $k = \frac{\sqrt{(2m(V-E))}}{\hbar}$

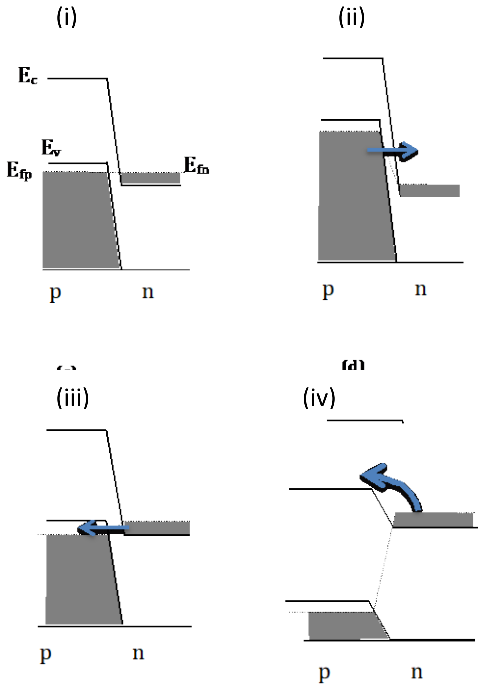
and *V-E* is the difference between the work function and the Fermi level, which essentially means 5.3 eV. This yields a *T* = 0.0088. The accuracy could be improved by considering the effect of the image potential (which will effectively lower the barrier) and of the applied potential, which will cause the barrier to be sloped.

(d) In this case, the "device" is 2 grain boundaries long and 1 wide, so it is akin to a double-barrier system. Therefore, as the size is less than the mean free path, we would expect transport to be coherent through the structure, and transmission resonances will appear. Qualitatively, we would expect the following: - peaks in transmission due to coherent scattering, which correspond to energies coinciding with bound states between the grain boundaries.



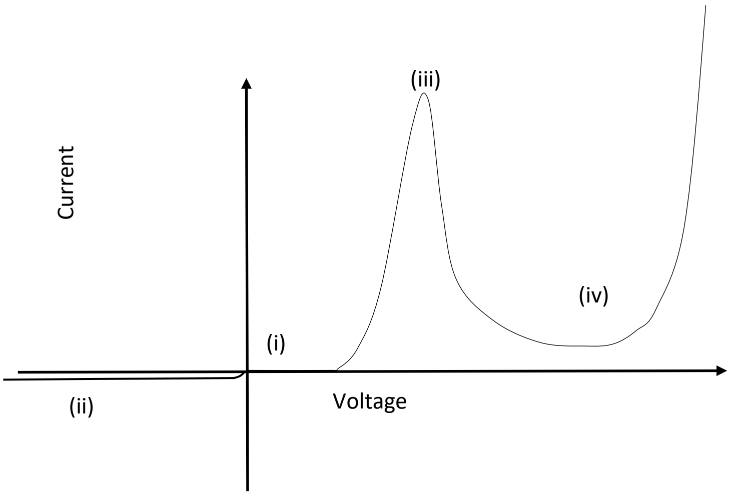
3. (a)The Esaki diode has doping levels of order 10^{25} m^{-3}. This leads the Fermi level in the *n*-type to be within the conduction band, and in the *p*-type to be within the valence band, respectively. This is what we mean by a broken band gap.

(b)

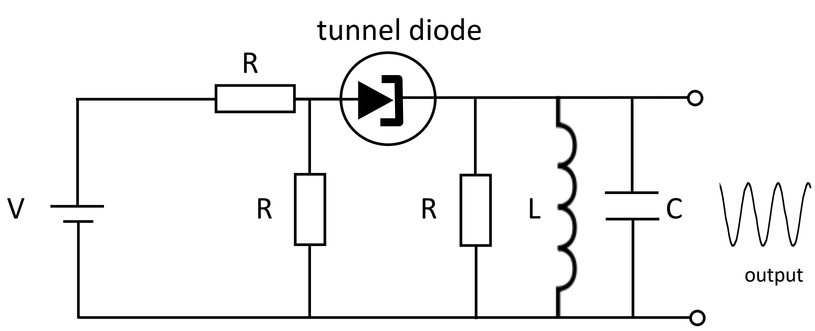


In Fig. (a), i.e. under zero applied bias: there is no net current flow, as the electron current from the conduction band of the n-type into the valence band of the p-type is balanced by the electron current from the valence band of the p-type in to the conduction band of the n-type. In Fig. (b), under reverse bias conditions, the bands on the p-type side are raised relative to the n-type side, and electrons can flow from p to n, tunneling across the depletion region. The width of this region will increase as the voltage is increased, so little current will actually flow. In Fig. (c), which is under a low forward bias, the electron-filled states in the n-type conduction band overlap with the holes in the p-type valence band and a significant current can tunnel across the depletion region, similar to Regime B in Fig. (b). In Fig. (d), as the forward bias is increased, the degree of overlap between the n-type conduction band electrons and the p-type valence band holes decreases, as more of them start to overlap with the band gap within the p-type. This has the effect of reducing the current across the depletion region as there are fewer states for the n-type electrons to tunnel into. In Fig. (e), similar to Regime (c) in Fig. (b), the current drops to its minimum value, as there is no longer any overlap between the conduction band electrons in the n-type and holes in the p-type: there are no available states for the electrons to tunnel into. The only current that can flow at this point is a small inelastic tunnel current and a small thermal diffusion current. In Fig. (f), when the applied forward bias is large enough, the height of the potential barrier between the n and p-type is low enough for a thermal diffusion current to flow over the barrier, and this becomes the dominant means of current flow.

(c)



(d) Resonant tunneling diodes initially gained a lot in interest for their potential application in oscillator circuits, particularly ones operating at high (Microwave) frequencies. The reason for this can be seen by considering the simplest possible oscillator : an LC circuit (i.e. an inductor in parallel with a capacitor). Due to the phase difference of 180 degrees between the voltage dropped across each of these, energy is effectively continually transferred from one component to the other — the circuit is an oscillator. Once the oscillations begin, if we remove the voltage driving source, the oscillations would continue indefinitely in the absence of any resistance within the circuit. However, all circuits have some resistance, so real oscillator circuits have a finite Q-factor. In principle, if we could add a negative resistance into the circuit to counteract the stray resistance of the components, we could greatly increase the circuit's Q-factor. This is done by adding a resonant tunneling diode into the LC circuit, and ensuring that it is operating in the middle of its NDR region. This is illustrated below. In recent years, the tunnel diode has been replaced by digital components which are more reliable and which have significantly better performance.



Typical circuit utilising a tunnel diode. The voltage source V is used to set the diode operating in the NDR region, and to start the oscillation. It also provides the energy to sustain the oscillation 3f the circuit. The oscillation frequency is $\cdot \left(\frac{1}{2\pi\sqrt{LC}} \right)$

4. a) Quantum supremacy is that point at which a quantum computer can solve problems that are beyond the capability of conventional devices/computers. Answer should include a mention of different types of qubit, with particular reference to superconducting Josephson junctions as the most promising, given that these are what are most commonly used.

(b) Answer should discuss two of the following: (i) electron-based systems including quantum dots, (ii) Josephson junctions, (iii) ion traps. The key points to consider are the ease with which the quantum states can be controlled, entangled and then read out. Remember diVicenzo's criteria: (a) scaleable system with well-defined qubits; (b) ability to initialise qubits to a well-defined quantum state; (c) states coherent for long enough; (d) ability to implement gates and (e) ability to measure (readout) qubit state. This poses several challenges – the qubits should have long decoherence times, which is only achieved by having only weak interactions with their environment. However, this is contrary to the requirement for readout. Discussion should focus on ease of fabrication, scaleability, complexity etc.

(c) After the measurement of first qubit being in state $|1\rangle$, we need to renormalise, so the new wavefunction is

$$\frac{1}{\sqrt{25}}(-3|10\rangle - 4i|11\rangle)$$

A subsequent measurement of the second qubit being in state $|1\rangle$ will have probability

$$\frac{1}{25}(|(-4i)^2|) = 0.64$$