Version CD/03

EGT3

ENGINEERING TRIPOS PART IIB

Monday 24th April 2023   9.30 to 11:10

Module 4B5

QUANTUM AND NANO-TECHNOLOGIES

*Answer not more than **three** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

*Write your candidate number **not** your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
Attachment: 4B5 formula sheet (2 pages).
Engineering Data Book

10 minutes reading time is allowed for this paper at the start of the exam.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

You may not remove any stationery from the Examination Room

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1. Light of wavelength 200 nm is incident on a sample which comprises a Ni surface of workfunction 5.0 eV. The sample is placed in a vacuum chamber, and next to it is an apparatus which can be used to measure the kinetic energy of any electrons emitted from the surface.

(a) Will any electrons be emitted from this surface as a result of the incident radiation? If so, what kinetic energy will they possess, and how fast will they be travelling?

[25%]

(b) If the brightness of the light source is doubled, what effect will that have on the velocity of any emitted electrons?

[10%]

(c) In classical terms, the energy of an electromagnetic wave is described using the Poynting vector, which states that the energy per unit time passing through a unit area is **E**×**H**, where **E** and **H** are the electric and magnetic field strength, respectively. In quantum terms, the energy of a photon of frequency f Hz is given by hf, where h is Planck's constant. Discuss how these disparate views can be reconciled.

[35%]

(d) In trying to understand the absorption and emission of electromagnetic radiation from objects, the concept of a "black body" was introduced. Planck derived an expression for the energy density (this is reproduced in the formula sheet):

$$\varepsilon(f) = \frac{8\pi h f^3}{c^3} \frac{hf}{e^{nhf/k_B T} - 1}$$

Describe the key assumptions made in deriving this formula and explain why an increase in temperature has the effect of pushing the peak energy density towards higher frequencies.

[30%]

2. The Landauer formula for electrical conduction shows that the electric current I due to a potential difference V across a system of quantum transmission probability T is of the form

$$I = \frac{2e^2}{h} TV$$

(a) How does this relate to Ohm's law and the concept of electrical resistivity?

[10%]

(b) The formula above suggests that electrical conductance is quantised. Given that this is not our everyday experience, under what experimental conditions would one expect to observe this?

[20%]

(c) Consider a grain boundary in a polycrystalline gold film, where the work function of the gold is 5.3 eV, and the width of the grain boundary is 0.2 nm. Using the approximation that the transmission probability through a barrier of width a is

$$T \sim e^{-2ka}$$

where k is the propagation constant within the barrier, estimate the transmission probability of the grain boundary, clearly stating any assumptions made. How could you improve the accuracy of this calculation?

[35%]

(d) Now consider the situation whereby a device is fabricated which consists of a gold nanowire of length 20 nm and width 5 nm, and through which we can pass an electric current. If the mean grain size is 10 nm and the bulk mean free path is 40 nm, would you expect to observe any quantum effects at room temperature? If so, how would you expect them to manifest? Draw a qualitative sketch of what the transmission probability versus energy could look like for such a device, labelling the salient features.

[35%]

(TURN OVER

3. The Esaki diode is created by heavily doping a conventional *p-n* junction, resulting in a broken band gap and a depletion region approx. 10 nm wide, across which charge carriers can tunnel.

(a) Explain, with the aid of a band diagram, what is meant by a broken band gap.

[10%]

(b) Draw the band diagrams (to include both the conduction and valence bands) which correspond to each of the following bias conditions in an Esaki diode, and indicate the states between which current flows:

(i) Zero bias;

[10%]

(ii) Reverse bias;

[10%]

(iii) Small forward bias at which the maximum current is obtained;

[10%]

(iv) Large forward bias.

[10%]

(c) Sketch the form of the current-voltage characteristic for this device, indicating each of the scenarios in part (b) above.

[25%]

(d) Which feature of this diode is especially useful from a practical perspective? Describe a situation where one would wish to incorporate such a diode within a circuit and explain how to ensure it operates effectively.

[25%]

4. Within the past few years, quantum computing has become a reality, and quantum supremacy was reached in 2019.

(a) Explain what is meant by the term *quantum supremacy*.

[10%]

(b) There are a number of two-state systems that can be used to implement qubits, largely based on the use of either electrons or photons. Describe two systems currently used, and discuss their relative advantages and disadvantages, within the context of DiVincenzo's 5 criteria.

[60%]

(c) Consider the situation where two qubits are used in a quantum computer. They are initialised in the quantum state

$$\frac{1}{\sqrt{30}}\left(|00\rangle + 2i|01\rangle - 3|10\rangle - 4i|11\rangle\right)$$

A measurement reveals that the first qubit is in the state $|1\rangle$. What is the state of the entire system after this measurement? What is the probability that a subsequent measurement of the second qubit will reveal a $|1\rangle$?

[30%]

END OF PAPER

4B5 Quantum Technologies Formula sheet – 2 pages

$-\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2} + V(x)\psi(x) = E\psi(x)$ — 1-D Time-independent Schrödinger equation, $V(x)$ is the potential, E is the total energy.

$\hat{E} = i\hbar\frac{\partial}{\partial t}$ — Quantum Energy Operator.

$\hat{\boldsymbol{p}} = -i\hbar\frac{\partial}{\partial x}$ — Quantum Momentum Operator.

$\widehat{K.E.} = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}$ — Quantum Kinetic Energy Operator.

h — Planck's constant = 6.626 x 10^{-34} Js

$\hbar$ — Reduced Planck's constant = 1.05 x 10^{-34} Js

e — Electron charge = 1.6 x 10^{-19} C

m — Free electron rest mass = 9.11 x 10^{-31} kg

c — Speed of light in vacuo = 2.998 x 10^{8} ms^{-1}

$E_n = \left(n + \frac{1}{2}\right)\hbar\omega_c$ — Spectrum of energy levels in Quantum Harmonic Oscillator, where ω_c is the natural frequency ($\sqrt{\frac{k}{m}}$) of the system in rad s^{-1}

$E_n = \frac{n^2h^2}{8mL^2}$ — Spectrum of energy levels in infinite Quantum Well of length L.

$\Delta x(t) = \sqrt{\Delta x_0 + \left(\frac{\hbar t}{m \Delta x_0}\right)^2}$	Width (standard deviation) of matter wave- Packet of initial width Δx_0 as a function of time, t.
$\frac{d\lfloor\psi(x)\rfloor^2}{dt} = \frac{i\hbar}{2m}\nabla j$	Quantum continuity equation, where j = probability flux.
$-\frac{i\hbar}{2m}\{\psi^*(\nabla\psi) - (\nabla\psi^*)\psi\}$	1-D Probability flux.
$T \cong e^{-\frac{2}{\hbar}\int_A^B \sqrt{2m(V(x)-E)}dx}$	WKB approximation to Transmission Probability, T. Integration limits are the Classical turning points, A & B.