Version LS/2

EGT3
ENGINEERING TRIPOS PART IIB

Friday 3 May 2024   9.30 to 11.10

Module 4B5

QUANTUM AND NANO-TECHNOLOGIES

Answer not more than ***three*** *questions.*

All questions carry the same number of marks.

The ***approximate*** *percentage of marks allocated to each part of a question is indicated in the right margin.*

Write your candidate number ***not*** *your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
Attachment: 4B5 Quantum and Nano-technologies data sheet (2 pages).
Engineering Data Book

10 minutes reading time is allowed for this paper at the start of the exam.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

You may not remove any stationery from the Examination Room.

1 (a) Describe the two main postulates of quantum mechanics and write down the expression for the wave-function of a free particle, propagating in space along the direction *r*, as a function of time *t*. Define all parameters used. [20%]

(b) Define the energy, momentum and kinetic energy operators and describe what they can be used for.
Remembering that the position operator $\hat{x} = x$, and that the commutator:

$$[\hat{A}, \hat{B}] = \hat{A}\hat{B} - \hat{B}\hat{A}$$

calculate $[\hat{x},\hat{p}]\Psi(x)$. [25%]

(c) Assuming the presence of a generic potential *V(r)*, write down the time-dependent Schroedinger equation and compare it to the wave equation. [10%]

(d) Explain what boundary conditions need to be imposed when considering the propagation of a wave-packet described by the Schroedinger equation and explain their physical meaning. [25%]

(e) Describe, qualitatively, the transmission probability for an electron impinging on a potential barrier higher than its energy, in comparison to the classical mechanics case. Draw a schematic of the wave propagation when the barrier is thin enough for the transmission probability to be different from zero, and explain your sketch. [20%]

2 You are asked to design the active region for a laser with an emission wavelength of 850 nm, using, as active medium, quantum wells based on GaAs and $Al_{0.4}Ga_{0.6}As$, semiconductors that have band gaps of 1.424 eV and 1.92 eV, respectively.

(a) Sketch the valence and conduction band edges of the structure, assuming that the hole first bound state has the same confinement energy (measured from the band edge) as the electron first bound state, and that the valence and conduction band potential wells have the same depth.

[10%]

(b) State what conditions need to be met to assume that you can use the infinite potential well approximation, and verify if this approximation holds.

[10%]

(c) Considering that the optically active transition is the one between the first bound states (S-states) and assuming that the mass of the electron and the hole can be both approximated with the free electron mass, calculate the quantum well thickness required to obtain the desired emission wavelength.

[20%]

(d) How many bound states are present in the conduction band? [15%]

(e) Sketch the wave-functions of the conduction band electronic bound states of the quantum well and discuss their physical meaning and symmetry. [15%]

(f) Consider applying an electric field along the growth axis of the quantum well; sketch the modification of the band diagram for two opposite field orientations. [15%]

(g) Describe qualitatively the effect of the electric field in the energy level positions and explain how the applied electric field will modify the laser emission wavelength. [15%]

3 (a) Consider a crystalline solid where atoms are arranged in a periodic lattice. Assuming that the system can be described as a series of coupled harmonic oscillators with a parabolic confining potential, define the concept of phonons and provide the expression for their energy states. [40%]

(b) Consider a solid where the phonon natural frequency is 10 THz. What is the temperature needed to excite the first three phononic energy states? [20%]

(c) Discuss the role of phonons in electrical conduction. [10%]

(d) Discuss the role of phonons in light absorption by indirect semiconductors. [15%]

(e) What is the lowest energy state of a phonon? Discuss the physical meaning of it. [15%]

4 (a) You are given a single-electron transistor, composed of a circular central island of 10 nm in diameter, a gate, a source and drain electrodes. Draw a schematic of the device geometry that you expect to be investigating. [10%]

(b) First of all, you need to image your device via microscopy tools. Discuss what dimensions you would be able to see with an optical microscope, and state the most suitable visible wavelength that you would choose, and explain the reasoning behind your choice. [15%]

(c) You then decide to image your sample with a Scanning Electron Microscope. Discuss the main differences in terms of operation principles and resolution that can be achieved, compared to the optical microscope, and explain what dimensions you expect to be able to image and why. [20%]

(d) Considering that the capacitance of the device is 10 attoFarad, calculate the energy required to add an electron to the circular central island, assuming that the charging process is ideal (no losses and no resistances are taken into account). [15%]

(e) Sketch a plot of the source-drain current versus the gate voltage, and discuss the behaviour that you expect to observe, for different temperatures, around approximately T = 10 K, 100 K, 300 K. [25%]

(f) Discuss the importance of single electron transistors in sensing, spin-polarised current control, and quantum computing. [15%]

END OF PAPER

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4B5 Quantum Technologies Formula sheet – 2 pages

$-\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2} + V(x)\psi(x) = E\psi(x)$	1-D time-independent Schrödinger equation, $V(x)$ is the potential, E is the total energy.
$\hat{E} = i\hbar\frac{\partial}{\partial t}$	Quantum Energy Operator.
$\hat{\boldsymbol{p}} = -i\hbar\frac{\partial}{\partial x}$	Quantum Momentum Operator.
$\widehat{K.E.} = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}$	Quantum Kinetic Energy Operator.
h	Planck's constant = 6.626 x 10^{-34} Js
$\hbar$	Reduced Planck's constant = 1.05 x 10^{-34} Js
e	Electron charge = 1.6 x 10^{-19} C
m	Free electron rest mass = 9.11 x 10^{-31} kg

c	Speed of light in vacuum = 2.998 x 10^8 ms^{-1}
k_B	Boltzmann's constant = 1.38×10^{-23} J/K
$E_n = \left(n + \frac{1}{2}\right)\hbar\omega_c$	Spectrum of energy levels in Quantum Harmonic Oscillator, where ω_c is the natural resonant frequency of the system in rad s^{-1}
$E_n = \frac{n^2h^2}{8mL^2}$	Spectrum of energy levels in infinite Quantum Well of length L.

Prefix Tera = 10^{12}

Prefix Atto = 10^{-18}

$\Delta x(t) = \sqrt{\Delta x_0 + \left(\frac{\hbar t}{m\Delta x_0}\right)^2}$	Width (standard deviation) of matter wave-packet of initial width Δx_0 as a function of time, t.
$\frac{d\lfloor\psi(x)\rfloor^2}{dt} = \frac{i\hbar}{2m}\nabla j$	Quantum continuity equation, where j = probability flux.
$\{\psi^*(\nabla\psi) - (\nabla\psi^*)\psi\}$	1-D Probability flux.
$T \cong e^{-\frac{2}{\hbar}\int_A^B \sqrt{2m(V(x)-E)}dx}$	WKB approximation to Transmission Probability, T. Integration limits are the Classical turning points, A & B.