1. (a) Describe the double slit experiment, carried out with an electron beam, and define the critical parameters in the setup. Calculate the slit size required for an electron beam with energy of 1 eV.

   A coherent beam of electrons, carrying the same energy and travelling in the same direction, is sent through two identical slits, with size comparable to the electronic wavelength and spaced by a distance that allows the diffracted waves emerging from the slits to spatially overlap (about 100 times the slit size can be a safe guess). A screen that can detect electrons is placed behind the double slit at a distance at least 10 times the slit separation (safe guess).

   The slit size needs to be comparable to the wavelength of the incoming beam. The electronic wavelength can be calculated using the De Broglie relationship:
   $\lambda = h/(mv)$
   Kinetic energy = $\frac{1}{2} mv^2 = 1$ eV -> $v = \sqrt{(2eV/m)} \approx 6.0 \times 10^5$ m/s
   $\lambda \approx 1.2$ nm

   [10%]

   (b) Discuss what the experiment proves and explain why.

   The observation of diffraction and interference (bright and dark fringes on the screen where electrons are/are not collected respectively) proved the wave-like nature of electrons, since such behaviour can only be explained by considering the superposition and constructive and destructive interference of waves.

   [10%]

   (c) Consider carrying out the same experiment using a single-electron gun (only one electron at a time is sent towards the double slit).

   (i) Describe what you expect to observe in the experiment and why.

   the same interference pattern as in the case of multiple electrons. This is due to the fact that a single electron behaves like a wave that can travel through both slits giving rise to diffraction and interference

   [10%]

   (ii) Describe what you expect to observe if you add to your setup a device that is able to let you know which slit the electron has gone through.

   No interference pattern

   [10%]

(iii) Discuss why the results obtained require a quantum mechanical description and how the observed phenomenon can be used in quantum technology applications.

Which path information: when unknown, the electron can be described as being in a superposition between a wave travelling through one slit and a wave travelling through the other one. When we know which path the electron has travelled, the superposition collapses and we observe particle-like behaviour.
Superposition can be used in quantum computing, quantum communication and quantum imaging: a qubit can be obtained by superimposing two or more states, information can be secured by transmitting an entangled state (when read out occurs the sender will know this has happened instantaneously), ghost imaging can be achieved by monitoring entangled photons that have not interacted with the object that is being imaged.

[20%]

(d) Now consider that you are utilising the same setup, you can adjust the slit size (state what dimension you will set your slit to and why), and instead of a beam of electrons, you send a red laser onto the upper slit and a green laser onto the lower slit. Describe what you expect to observe in your experiment and why.

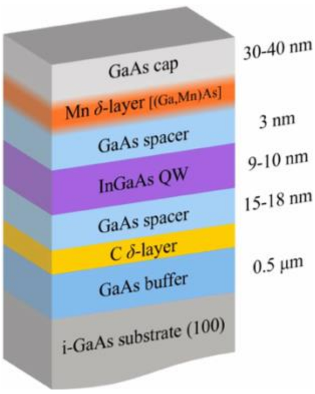
Red laser has a wavelength of about 630 nm, green laser of about 530 nm. The slits can both be set to about 600 nm.
The two laser beams will diffract but no interference will be observed since the waves emerging from the slits are not coherent.

[20%]

(e) Explain how the same physical phenomenon is utilised in x-ray spectroscopy, what information it allows to acquire and how.
X-rays have wavelengths comparable to atomic spacing in crystals: when the x-rays interact with the atomic lattice, the waves are diffracted and interfere. By analysing the interference pattern, it is possible to calculate the atomic spacing.

[20%]

2. (a) Define a semiconductor heterostructure, draw its schematic, discuss what growth techniques can be used to create it, and what a heterostructure can be used for.



A semiconductor heterostructure is created when layers of different semiconductors (typically with different bandgaps) are grown sequentially. A standard growth technique is Molecular Beam Epitaxy that allows the deposition of single monolayers of semiconductors in a controlled, high vacuum environment.
Such heterostructures can be used to confine charges, for instance by creating potential wells (like quantum wells).

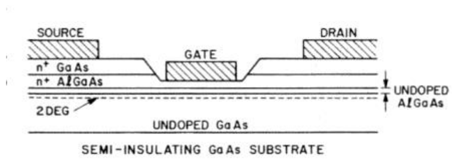
[15%]

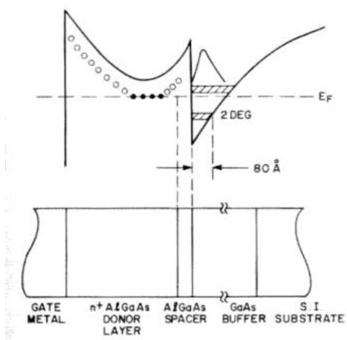
(b) Discuss why doping the semiconductor layers composing a heterostructure can be useful, and describe potential drawbacks of the doping process in the device performances.

Doping is used to increase/activate charge mobility by introducing electrons in the conduction band or holes in the valence band of a semiconductor. A potential drawback is an increase in scattering events due to the presence of contaminants that can reduce charge mobility/increase resistance.

[20%]

(c) Define a two-dimensional electron gas, show in a sketch (including critical dimensions) how it can be obtained, and describe what its properties are.





The electrons in the GaAs are in a triangular well, and are confined to a sheet parallel to the surface, just a few nm thick. This is known as a two-dimensional electron gas.
The GaAs layer is undoped (in practice it is often very lightly doped), so electrons have a very high mobility.

[20%]

(d) Provide an example of a device implementing a two-dimensional electron gas and describe, qualitatively, its performances, compared to a device where doping is implemented instead.

Laser pumped by a flow of electron from a two-dimensional electron gas. Higher repetition rates can be achieved, thanks to the higher mobility of carriers, as opposed to the case of doped semiconductors where the impurities can reduce the mobility of the carriers.

[20%]

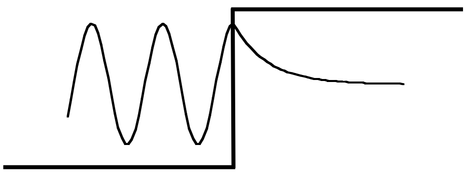
(e) What are the orders of magnitude of mobility and operating frequency that can be typically reached with High Electron Mobility Transistors? How do they compare to values obtained with bulk heterostructures?

High Electron Mobility Transistors: mobilities up to $\approx 10^7$ cm^2/V s, operation frequencies up to $\approx$ 1000 GHz.
Bulk semiconductors: mobilities of the order of $\approx 10^3$ cm^2/V s, operation frequencies up to $\approx$ 1 GHz.

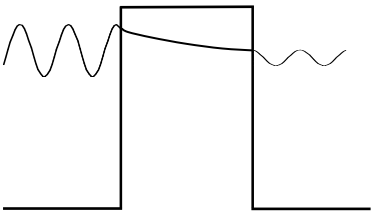
[25%]

3. (a) Consider an electron impinging on a potential barrier whose energy is higher than the electron energy.

   (i) Discuss, qualitatively, the process taking place, as a function of barrier thickness (provide drawings to elucidate your descriptions)



“Thick” (or infinite) barrier: exponential decay of the electronic wave



“Thin” barrier: tunnelling of the electronic wave through the barrier

[10%]

   (ii) Why does such behaviour require a quantum mechanical description of the electron?

   Such behaviour can only be explained by considering that the electron has a wave-like nature.

[10%]

(b) Describe the principle of operation of a Scanning Tunnelling Microscope.

A Scanning Tunnelling Microscope is based on the concept of quantum tunnelling. When the tip is brought very near to the surface to be examined, a bias voltage applied between the two allows electrons to tunnel through the vacuum separating them. The resulting tunnelling current is a function of the tip position, applied voltage, and the local density of states of the sample.
As the tip is moved across the surface in a discrete x–y matrix, the changes in surface height and population of the electronic states cause changes in the tunnelling current.
Digital images of the surface can be formed: in constant-height mode where changes of the tunnelling current are mapped directly, in constant-current mode where the voltage that controls the height (z) of the tip is recorded while the tunnelling current is kept at a predetermined fixed level.

[30%]

(c) When using the Wentzel-Kramers-Brillouin approximation, a thin potential barrier has a transmission coefficient of the form e^{-2kx}, where x is the width of the barrier.

(i) Explain when such an approximation holds

In this approximation, the wavefunction is recast as an exponential function, semi-classically expanded, and then the amplitude is taken to be changing slowly. This generally holds for potential barriers large compared to the electron energy.

[10%]

(ii) What is the change in the measured current, in a Scanning Tunnelling Microscope, for a variation in the tip-sample distance of 0.5 Angstrom, assuming that k = 2 Angstrom^{-1}?

$e^{-2*0.5} = e^{-1}$

hence about 1 order of magnitude change in the current.

[10%]

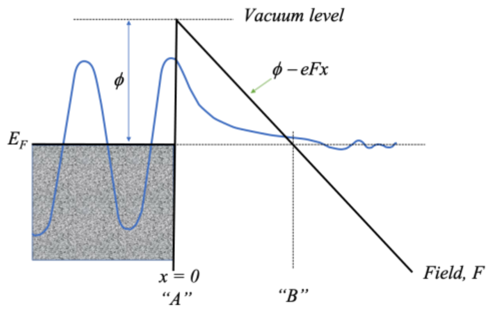
(iii) Considering the result obtained in (ii), what requirements are imposed in the construction of a well-performing Scanning Tunnelling Microscope?

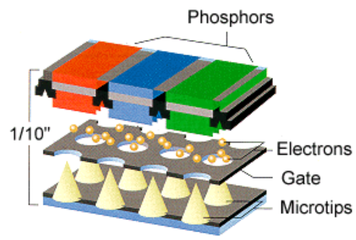
Very low vibrations and very high current sensitivity, with low noise.

[10%]

(d) Describe the process behind field emission and explain how field emission can be utilised in displays.

Field emission is an extreme form of tunnelling where, when applying a high voltage to a metal tip, electrons are ejected: those electrons can then be directed onto a fluorescent screen, resulting in the illumination of specific pixels.





The potential as seen by an electron is V(x) = Φ – eFx, where F = electric field, x is the distance from the metal surface, and Φ is the metal work function.

[20%]

4. (a) Explain how you would ensure that, in such a device, electrons are present in the conduction band.

   Generally, by doping the barriers in the quantum well. Optical pumping could also be used but it would make the device no longer monolithic.

   [10%]

   (b) Discuss what parameters you would need to optimise to ensure optimal performances of the laser and why.

   All quantum wells need to be identical, so that the emission is monochromatic. Barriers need to be thin enough to ensure electron tunnelling between quantum wells but also thick enough to avoid hybridisation of the confined electronic states. Doping needs to be optimised to ensure that carriers are present in the conduction band.

   [10%]

   (c) Considering that the quantum wells are made of alternating layers of GaAs and $Al_{0.4}Ga_{0.6}As$, semiconductors that have band gaps of 1.42 eV and 1.92 eV, respectively, what is the upper bound on the emission wavelength that can be reached?

   Considering a very large potential well, say 100 nm wide, the longest emission wavelength would be, assuming that the electron mass can be approximated with the electron free mass:
   $\Delta E = 3h^2/(8mL^2) \rightarrow \lambda \approx 100$ mm

   [20%]

   (d) If we want to realise a laser emitting at a wavelength of 10 μm, what is the required quantum well thickness? State any approximations made in your calculations.

   We use the infinite well approximation, so the energy of the bound states is provided by: $E = h^2n^2/(8mL^2)$
   We assume that the electron mass can be approximated with the electron free mass.
   $\Delta E = 3h^2/(8mL^2) \rightarrow L = \sqrt{(3h^2/8m\Delta E)}$
   $\Delta E = 0.124$ eV -> L = 3 nm

   [20%]

   (e) What parameter(s) can one modify, in order to vary the emission wavelength of a quantum cascade laser?

   By varying the semiconductor composing the quantum well, one can vary the depth of the confining potential, therefore the position of the energy states and the number of bound states. By varying the quantum well thickness, one can modify the energy position of the bound states. By applying an electric field, the energy of the bound states can be varied via the Stark shift, therefore modifying the transition energy (small variations). [20%]

(f) Assuming that you could realise a similar laser based on quantum dots instead of quantum wells, what do you expect the main differences to be, if any, and, in case, why?

A quantum dot would provide three-dimensional confinement of the carriers. This would result in sharper emission lines, different confinement energies, stronger oscillator strengths (also considering the different density of states), and different selection rules for the optical transitions.

[20%]