Version LS/2

EGT3
ENGINEERING TRIPOS PART IIB

Monday, 28 April 2025   9.30 to 11.10

Module 4B5

QUANTUM AND NANO-TECHNOLOGIES

*Answer not more than **three** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

*Write your candidate number **not** your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
Attachment: 4B5 Quantum and Nano-technologies data sheet (2 pages)
Engineering Data Book

10 minutes reading time is allowed for this paper at the start of the exam.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

You may not remove any stationery from the Examination Room.

1 (a) Describe the double slit experiment, carried out with an electron beam, and define the critical parameters in the setup. Calculate the slit size required for an electron beam with energy of 1 eV. [10%]

(b) Discuss what the experiment proves and explain why. [10%]

(c) Consider carrying out the same experiment using a single-electron gun (only one electron at a time is sent towards the double slits).

(i) Describe what you expect to observe in the experiment and why. [10%]

(ii) Describe what you expect to observe if you add to your setup a device that is able to tell you which slit the electron has gone through. [10%]

(iii) Discuss why the results obtained require a quantum mechanical description and how the observed phenomenon can be used in quantum technologies. [20%]

(d) Now consider that you are utilising the same setup, you can adjust the slit size (state what dimension you will set your slit to and why), and instead of a beam of electrons, you send a red laser onto the upper slit and a green laser onto the lower slit. Describe what you expect to observe in your experiment and why. [20%]

(e) Explain how the same physical phenomenon is utilised in x-ray spectroscopy, what information it allows to acquire and how. [20%]

2 (a) Define a semiconductor heterostructure, draw its schematic, discuss what growth techniques can be used to create it, and what a heterostructure can be used for. [15%]

(b) Discuss why doping the semiconductors in a heterostructure can be useful and describe possible drawbacks of the process in the device performances. [20%]

(c) Define a two-dimensional electron gas, show in a sketch (including critical dimensions) how it can be obtained, and describe what its properties are. [20%]

(d) Provide an example of a device implementing a two-dimensional electron gas and describe, qualitatively, its performances, compared to a device where doping is implemented instead. [20%]

(e) What are the orders of magnitude of mobility and operating frequency that can be typically reached with High Electron Mobility Transistors? How do they compare to values obtained with bulk heterostructures? [25%]

3 (a) Consider an electron impinging on a potential barrier whose energy is higher than the electron energy.

(i) Discuss, qualitatively, the process taking place, as a function of barrier thickness (provide drawings to elucidate your descriptions). [10%]

(ii) Why does such behaviour require a quantum mechanical description of the electron? [10%]

(b) Describe the principles of operation of a Scanning Tunnelling Microscope. [30%]

(c) When using the Wentzel-Kramers-Brillouin approximation, a thin potential barrier has a transmission coefficient of the form e^{-2kx}, where x is the width of the barrier.

(i) Explain when such an approximation holds. [10%]

(ii) What is the change in the measured current, in a Scanning Tunnelling Microscope, for a variation in the tip-sample distance of 0.5 Å, assuming that $k = 1$ Å^{-1}? [10%]

(iii) Considering the result obtained in part (c)(ii), what requirements are imposed in the construction of a well-performing Scanning Tunnelling Microscope? [10%]

(d) Describe the process behind field emission and explain how field emission can be utilised in displays. [20%]

4 A quantum cascade laser (whose operation principle is schematically shown in Fig. 1) is a device that utilises electronic transitions taking place in the conduction band of quantum wells. Considering quantum wells with two bound states, the radiative transition (vertical arrow) takes place from the first excited state (2) to the ground state (1). By applying an electric field, the bound state 1 of the quantum well on the left can align with the excited state 2 of the quantum well on the right and tunnelling can take place: the electron in the ground state can tunnel (horizontal arrow) into the first excited state of the neighbouring quantum well and the light emission process can take place again. Such structure is repeated several times so that an electron can emit several photons.

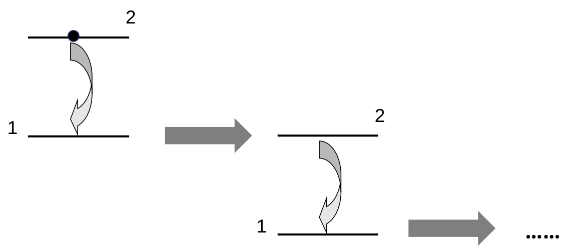

Fig. 1

(a) Explain how you would ensure that, in such a device, electrons are present in the conduction band. [10%]

(b) Discuss what parameters you would need to optimise to ensure optimal performances of the laser and why. [10%]

(c) Considering that the quantum wells are made of alternating layers of GaAs and $Al_{0.4}Ga_{0.6}As$, semiconductors that have band gaps of 1.42 eV and 1.92 eV, respectively, what is the upper bound on the emission wavelength that can be reached? [20%]

(d) If we want to realise a laser emitting at a wavelength of 10 μm, what is the required quantum well thickness? State any approximations made in your calculations. [20%]

(e) What parameter(s) can one modify, in order to vary the emission wavelength of a quantum cascade laser? [20%]

(f) Assuming that you could realise a similar laser based on quantum dots instead of quantum wells, what do you expect the main differences to be, if any, and why? [20%]

END OF PAPER

4B5 Quantum Technologies Formula sheet – 2 pages

$-\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2} + V(x)\psi(x) = E\psi(x)$	1-D time-independent Schrödinger equation, $V(x)$ is the potential, E is the total energy
$\hat{E} = i\hbar\frac{\partial}{\partial t}$	Quantum Energy Operator
$\hat{\boldsymbol{p}} = -i\hbar\frac{\partial}{\partial x}$	Quantum Momentum Operator
$\widehat{K.E.} = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}$	Quantum Kinetic Energy Operator
h	Planck's constant = 6.626 x 10^{-34} Js
$\hbar$	Reduced Planck's constant = 1.05 x 10^{-34} Js
e	Electron charge = 1.6 x 10^{-19} C
eV	1 eV = 1.6 x 10^{-19} J = 1.6 x 10^{-19} kg m^2 s^{-2}
m	Free electron rest mass = 9.11 x 10^{-31} kg
c	Speed of light in vacuum = 2.998 x 10^8 ms^{-1}
k_B	Boltzmann's constant = 1.38 × 10^{-23} J/K
$E_n = \left(n + \frac{1}{2}\right)\hbar\omega_c$	Spectrum of energy levels in quantum harmonic oscillator, where ω_c is the natural resonant frequency of the system in rad s^{-1}

$E_n = \frac{n^2 h^2}{8mL^2}$

Spectrum of energy levels in an infinite quantum well of width L

Prefix Tera = 10^{12}

Prefix Atto = 10^{-18}

$\Delta x(t) = \sqrt{\Delta x_0 + \left(\frac{\hbar t}{m\Delta x_0}\right)^2}$

Width (standard deviation) of matter wave-packet of initial width Δx_0 as a function of time, t.

$\frac{d\lfloor\psi(x)\rfloor^2}{dt} = \frac{i\hbar}{2m}\nabla.\boldsymbol{j}$

Quantum continuity equation, where $\boldsymbol{j}$ = probability flux.

$\{\psi^*(\nabla\psi) - (\nabla\psi^*)\psi\}$

1-D Probability flux.

$T \cong e^{-\frac{2}{\hbar}\int_A^B \sqrt{2m(V(x)-E)}dx}$

WKB approximation to transmission probability, T.

The integration limits are the classical turning points, A & B.