Version CD/4

EGT3
ENGINEERING TRIPOS PART IIB

Wednesday 30 April 2014 9:30 to 11:00

Module 4B5

NANOTECHNOLOGY

*Answer not more than **three** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

*Write your candidate number **not** your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
Engineering Data Book

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

1 (a) The theory of quantum mechanics was devised at the turn of the twentieth century in an attempt to explain a variety of physical phenomena which could not be explained by conventional physics. Discuss this statement, using three examples based on experimental evidence, of the limitations of traditional physics. [30%]

(b) In quantum mechanics, what can wave-packets be used to represent? Consider the wave-packet formed when two electromagnetic waves of the same amplitude but slightly different frequencies are combined. Write down a mathematical expression for such a wave-packet, and show how to extract from it the phase and group velocity of the wave-packet. What constraints, if any, are placed on these velocities? How would we refine this wave-packet to further increase its degree of localisation? [20%]

(c) Consider the electromagnetic wave-packet described by:

$$E(x,t) = E_0\sqrt{\delta}e^{-\frac{\delta^2}{2}(x-ct)^2}\cos(\omega t - kx)$$

where δ is the full width at half-maximum of the spectral function, *f(k)*, which is given by the Gaussian expression:

$$f(k) = \frac{1}{\sqrt{2\pi\delta}}e^{-\frac{k^2}{2\delta^2}}$$

and c is the speed of light in a vacuum, ω is the angular frequency and k is the wavenumber.

Outline the steps involved in deriving the form of *E(x,t)*. Briefly discuss the time-evolution of a wave-packet described by the expression above and explain what would happen, and why, for a wave-packet describing a matter particle. [30%]

(d) State the Heisenberg uncertainty principle and discuss its interpretation in the context of wave-packets. [20%]

2 (a) The band-gap of GaAs is 1.45 eV, and that of $Ga_{1-x}Al_xAs$ is (1.45 + 1.247x) eV. For the case of 60% substitution of Al atoms for Ga atoms and assuming that the valence and conduction bands are symmetric about the Fermi Energy, sketch the potential as seen by an electron in a 10 nm wide quantum well formed using these materials, indicating which material is used, and where. [20%]

(b) Assuming that the electron and hole effective masses in both materials are 0.067 and 0.45 times the free-electron mass respectively, estimate the energy of the electron and hole ground states in the well. How accurate do you expect your answer to be? Which assumptions were used in arriving at your estimate, and at which point would they start to break down? [50%]

(c) Sketch the wave-function and the probability density for an electron and a hole in the ground state and first excited state of this quantum well. Sketch the expected optical absorption spectrum of this device, and comment on its characteristics. [30%]

(TURN OVER

3 (a) In conventional optical microscopy, what is the minimum feature size that can be resolved? State and explain the Rayleigh criterion with the use of a diagram. Briefly describe how this limit to resolution can be overcome. [25%]

(b) Describe the basic principle of operation of the Scanning Electron Microscope and briefly list the pros and cons of using such a microscope. [10%]

(c) Describe the basic principles of operation of scanning probe microscopes and discuss how they have been utilised in research and development. [25%]

(d) Fig. 1 shows the tip and sample in a scanning tunnelling microscope (STM). Assuming that the radius of curvature of the tip, *R*, is 9 nm and that it is 1 nm away from the sample, estimate and comment on the resolution of the STM, clearly stating any approximations. [40%]

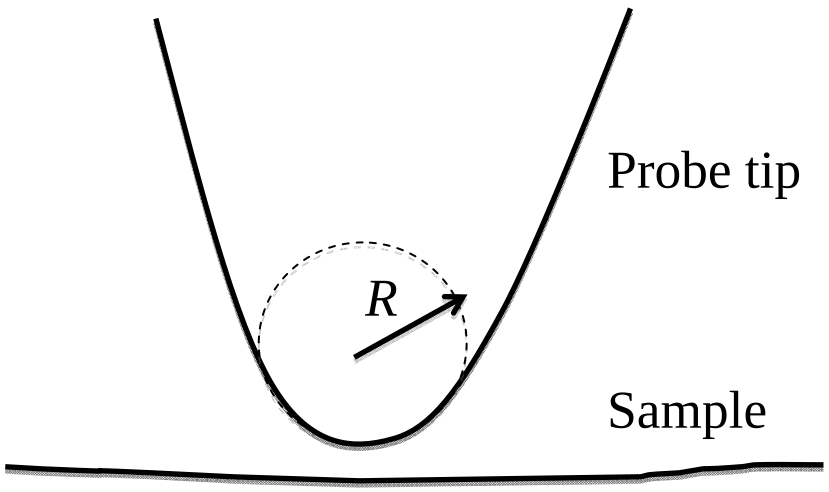

Fig. 1

4 (a) Consider a potential step from 1.2 eV down to 1 eV, which is produced in a semiconductor heterostructure. Sketch the kinetic energy versus position of an electron at 1.5 eV that is incident from the left hand side. After the electron has passed the potential step, what term can we use to describe it? [20%]

(b) Assuming now that band-bending can occur in the heterostructure, show how a 2-dimensional electron gas (2DEG) arises. Discuss the differences between 3-D and 2-D in quantum systems. [30%]

(c) What property of the 2DEG makes it so desirable from a device standpoint? How would you actually incorporate a 2DEG in a device to improve its operation? [40%]

(d) Briefly comment on the practical applications of semiconductor heterostructures in everyday devices, with the use of two examples. [10%]

(TURN OVER

5 (a) State Moore's law. Discuss briefly the advances in technology over the last seven decades that have enabled the continued increase in the speed of operation of transistors. Why is there a desire to have smaller transistors? [30%]

(b) With particular reference to field-effect transistors, name and discuss with examples, one *quantum* and one *classical* phenomenon that will hinder further reduction in size of the transistor. How would you go about trying to overcome these? [30%]

(c) A number of alternative architectures to those of conventional transistors have been proposed and are currently being studied. Why is this deemed to be necessary? Describe the basic principles of fabrication and operation of devices based on resonant tunnelling, with emphasis on size, speed and reproducibility. [40%]

END OF PAPER