Version CD/4

EGT3
ENGINEERING TRIPOS PART IIB

29 April 2021 09:00 to 10:40

Module 4B5

QUANTUM AND NANO-TECHNOLOGIES

Answer not more than ***three*** *questions.*

All questions carry the same number of marks.

The ***approximate*** *percentage of marks allocated to each part of a question is indicated in the right margin.*

Write your candidate number ***not*** *your name on the cover sheet and at the top of each answer sheet.*

STATIONERY REQUIREMENTS
Write on single-sided paper.

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed.
Attachment: 4B5 Quantum Technologies Formula sheet (2 pages).
You are allowed access to the electronic version of the Engineering Data Books.

10 minutes reading time is allowed for this paper at the start of the exam.

The time taken for scanning/uploading answers is 15 minutes.

Your script is to be uploaded as a single consolidated pdf containing all answers.

1 (a) The theory of quantum mechanics was developed in the early 20th century in order to explain several phenomena that could not be explained using conventional physics. Discuss this statement using two examples of where the limitations of classical physics were overcome by the use of quantum mechanics. [30%]

(b) (i) What is the physical significance of wave-packets? [10%]

(ii) A wave-packet is formed by combining two electromagnetic waves of the same amplitude and slightly different frequencies. Describe this wave-packet mathematically and show how to determine its group and phase velocity. [15%]

(iii) Briefly discuss the difference between both velocities and what their significance is. How can we create a wave-packet with a greater degree of localisation? [10%]

(c) Consider the electromagnetic wave-packet described by:

$$E(x,t) = E_o\sqrt{\delta}e^{-\frac{\delta^2}{2}(x-ct)^2}cos(\omega t - kx)$$

where δ is the width of the spectral function, which is a Gaussian.

(i) Why do we use Gaussian functions for this purpose? [10%]

(ii) Describe the steps involved in deriving $E(x,t)$. [10%]

(iii) How does this relate to Heisenberg's uncertainty principle? Briefly discuss how the uncertainty principle, which is quantum in origin, can apparently be arrived at using this sort of classical approach. [15%]

2 (a) Consider a material which has a bond stiffness of 520 N/m for small atomic displacements. Determine the vibrational energies of the atoms for the ground state and the first two excited states in eV, assuming that the atomic mass is 10^{-26} kg. [40%]

(b) (i) Explain why only certain values of vibrational energy are allowed in this system given that within the framework of classical mechanics, a continuum of values of energy are allowed. [20%]

(ii) How is this disparity between classical and quantum mechanics resolved? [10%]

(iii) What is the consequence of the ground state having an energy greater than zero? [10%]

(c) Sketch the form of the probability density of the ground state and the first excited state and compare them to what is expected on the basis of classical mechanics. [20%]

3 (a) Describe the physical significance of wave-functions in quantum mechanics and what properties they must have. Discuss the meaning of the rules we need to apply when solving boundary-value problems. [20%]

(b) A beam of electrons with a kinetic energy of 50 eV is passed through a negatively charged hollow metal cylinder which is at an electrical potential of 40 eV. We can assume that this cylinder is infinitely long. Describe the motion of the electrons through this system and compare to what one would expect classically. Determine the fraction of electrons that will be reflected at the entrance to the cylinder. [40%]

(c) If, instead of being infinitely long, this cylinder is only 1 nm long, determine the fraction of the incident beam that will be reflected. [40%]

4 (a) An electron is placed in a 1-dimensional infinitely deep potential well of width 20 nm. The effective mass of electrons in the material used in the well is 10% of the free-electron mass. Write an expression for the normalised wavefunctions for this system and calculate the energies of the ground and first excited states in eV. [15%]

(b) In practice, potential wells are not infinitely deep but have a finite depth. How are they created in semiconductors and what are typical values for well width and depth? Describe the concept, construction and operation of a device where a potential well is utilised. [35%]

(c) Sketch the form of the probability density of the ground state and the first excited state for potential wells that are (i) infinite and (ii) finite in depth, and comment on the differences. [25%]

(d) Using Heisenberg's uncertainty principle, estimate the ground state energy of the above infinitely deep quantum well, stating any assumptions made. How can you obtain a more accurate estimate using this concept? [25%]

5 (a) (i) What is the *quantum supremacy*? [10%]

(ii) In terms of practical applications, what are currently the most promising systems we can use to create a functional quantum computer? [20%]

(b) Discuss the comparison between a classical and a quantum computer, with particular reference to the rules they follow and what each can and cannot do. [40%]

(c) Describe the principle of operation of superconducting qubits and discuss their advantages for the experimental realisation of a quantum computer. [30%]

END OF PAPER

NUMERICAL ANSWERS:

2. (a) 74.8 meV, 224.4 meV, 374 meV

3. (b) R = 0.144

   (c) R = 0.1

4. (a) 9.3 meV, 37.2 meV

   (d) 0.94 meV