EGT3
ENGINEERING TRIPOS PART IIB

Thursday 28 April $2022 \quad 2.00$ to 3.40

Module 4C15

MEMS DESIGN

Answer not more than three questions.

All questions carry the same number of marks.

The approximate percentage of marks allocated to each part of a question is indicated in the right margin.

Write your candidate number not your name on the cover sheet.
STATIONERY REQUIREMENTS
Single-sided script paper.

## SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM

CUED approved calculator allowed
Attachment: 4C15 MEMS Design data sheet (4 pages).
Engineering Data Book

10 minutes reading time is allowed for this paper at the start of the exam.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

You may not remove any stationery from the Examination Room.

## Version AAS/4

1 Figure 1 shows one of the legs of a silicon microrobot. The leg can be thought of as a cantilever with a spherical tip of radius $R=10^{-5} \mathrm{~m}$. The vertical spring constant of the cantilever is $k=100 \mathrm{~N} \mathrm{~m}^{-1}$. The robot leg makes contact with a silicon substrate and the mechanical contact between the silicon surfaces can be approximated by the elastic contact between spherical and flat surfaces. You may assume that the elastic modulus of silicon is 160 GPa and the Poisson's ratio is 0.3 .
(a) A normal load of $10^{-6} \mathrm{~N}$ is applied when the leg makes contact with the substrate. Estimate the radius of the contact spot and the mean contact pressure assuming the effects of adhesion between the surfaces can be ignored.
(b) Estimate the size of the contact spot if surface energy is considered.
(c) If the load is reduced to zero, estimate (i) the size of the contact spot and (ii) the magnitude of the force required to pull the cantilever off the surface.
(d) Estimate the magnitude of the force required to separate the junction if it became contaminated by a drop of water with surface tension $72 \mathrm{~mJ} \mathrm{~m}^{-2}$ and the contact angle is $15^{\circ}$.
(e) Comment on the susceptibility to failure due to permanent adhesion and how this might be mitigated in practice.


Fig. 1

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2 A voltage-controlled micromachined actuator consists of a spring-supported beam actuated capacitively, as shown in Fig. 2. The electrostatic actuator is fabricated such that the gap spacing between the beam and the fixed electrode changes uniformly from a value $g_{1}$ at the left-hand side of the beam to a value $g_{2}$ at the right-hand side of the beam as shown. The overlap length of the electrode is $l$ and the width of the electrode into the page is $w$. The net effective mechanical spring constant of the beam is $k$.
(a) Obtain an expression for the electrostatic force acting on the beam for the case of voltage control and a gap closing response.
(b) Formulate the conditions for pull-in.
(c) Derive an expression for the displacement of the beam at the onset of pull-in.
(d) Write down an expression for the pull-in voltage and show that this reduces to the standard expression when the gap spacing is uniform.


Fig. 2

3 A single-axis polysilicon micromachined capacitive accelerometer comprises of a suspended mass in the form of a square plate with side dimension 2 mm as shown in Fig. 3. The structural layer is $10 \mu \mathrm{~m}$ thick and the device is suspended $2 \mu \mathrm{~m}$ above the substrate. The mass is designed to be compliant along the $x$-direction with a natural frequency of 10 kHz . The motion of the proof mass is measured capacitively by employing differential parallel-plate electrodes and comb drive structures oriented as shown in Fig. 3 with 50 unit cells for each configuration employed. The electrode spacing for each set of movable and fixed electrodes is $1 \mu \mathrm{~m}$ with an overlap length of $300 \mu \mathrm{~m}$. The device is packaged in an inert atmosphere and the operating temperature is $T=300 \mathrm{~K}$. The dynamic viscosity of the gas surrounding the micromechanical device equals $1.8 \times 10^{-5} \mathrm{~kg} \mathrm{~m}^{-1} \mathrm{~s}^{-1}$ and the relative permittivity of this gas is 1 .
(a) Estimate the displacement of the proof mass for an input acceleration of $1 \mathrm{~m} \mathrm{~s}^{-2}$ acting along the $x$-direction.
(b) What is the fractional change in capacitance of the comb drive in response to an input acceleration of $1 \mathrm{~m} \mathrm{~s}^{-2}$ along the $x$-direction?
(c) What is the fractional change in capacitance of the differential parallel-plate arrangement in response to an input acceleration of $1 \mathrm{~m} \mathrm{~s}^{-2}$ along the $x$-direction?
(d) Estimate the thermo-mechanical noise limited resolution of the device.
(e) List two limitations associated with the design shown.

## Version AAS/4


moveable
fixed


Fig. 3

4 A glass-based microfluidic device consists of four fluidic ports as shown in Fig. 4. The channels have a square cross-section of dimensions $100 \mu \mathrm{~m} \times 100 \mu \mathrm{~m}$. A buffer solution consisting of two molecular species with differing electrophoretic mobilities is pumped from Port 1 to Port 2 initially, under a uniform electric field using electroosmosis. The separation between Ports 1 and 2 is 8 mm and the spacing between junctions J 1 and J 2 is 2 mm . The difference in the electrophoretic mobilities of the two molecular species is $10^{-8} \mathrm{~m}^{2} \mathrm{~V}^{-1} \mathrm{~s}^{-1}$. The glass wall potential is estimated to be 80 mV and the relative permittivity of the medium is 80 . You may assume that the dynamic viscosity of water is $10^{-3} \mathrm{~Pa} \mathrm{~s}$.
(a) Estimate the volumetric flow rate for an applied voltage of 100 V between Ports 1 and 2.
(b) Estimate the equivalent pressure difference that must be applied across Ports 1 and 2 to achieve an identical flow rate to (a) above.
(c) At a certain instant, the electric field between Ports 1 and 2 is switched off while the voltage at port 3 is set at 200 V relative to the potential of junction J1 so that a plug of fluid is transported between the channel junction towards port 3. Determine the flow velocity of the plug.
(d) After a gap of 10 s , the voltage at port 4 is set at 300 V relative to the potential of junction J2 so that a second plug of fluid is transported between J2 and Port 4. Estimate the time taken for this plug of fluid to reach Port 4 and comment on whether this plug would make it to Port 4 prior to the other plug exiting Port 3.
(e) Compare the separation distance between the two molecular species in both plugs of fluid when they have been transported a distance of 3 mm down their respective separation channels.
(f) Comment on the optimisation of device design and geometry to obtain a good separation between the two molecular species.


Port 4
Fig. 4

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