1

(a) JKR equation:

$$\frac{4E^* a^3}{3R} = P + 2\sqrt{2\pi w E^* a^3}$$

on pull-off $\left.\frac{dP}{da}\right|_{a=a_B} = 0$

or $$\frac{4E^* a_B^2}{R} = 0 + 2 \times \frac{3}{2}\sqrt{2\pi w E^* a_B}$$

or $$a_B = \sqrt[3]{\frac{9\pi w R^2}{8E^*}}$$

Substituting this back into the JKR equation:

$$\frac{4E^*}{3R} \cdot \frac{9\pi w R^2}{8E^*} = P + 2\sqrt{2\pi w E^* \cdot \frac{9\pi w R^2}{8E^*}}$$

$$P = -\frac{3\pi w R}{2}.$$

(b) For contact at the two spheres

$R = R'/2 = 5\,\mu m.$

For contact between sphere + flat $R = 10\,\mu m$

P_1 = pull-off force for top sphere

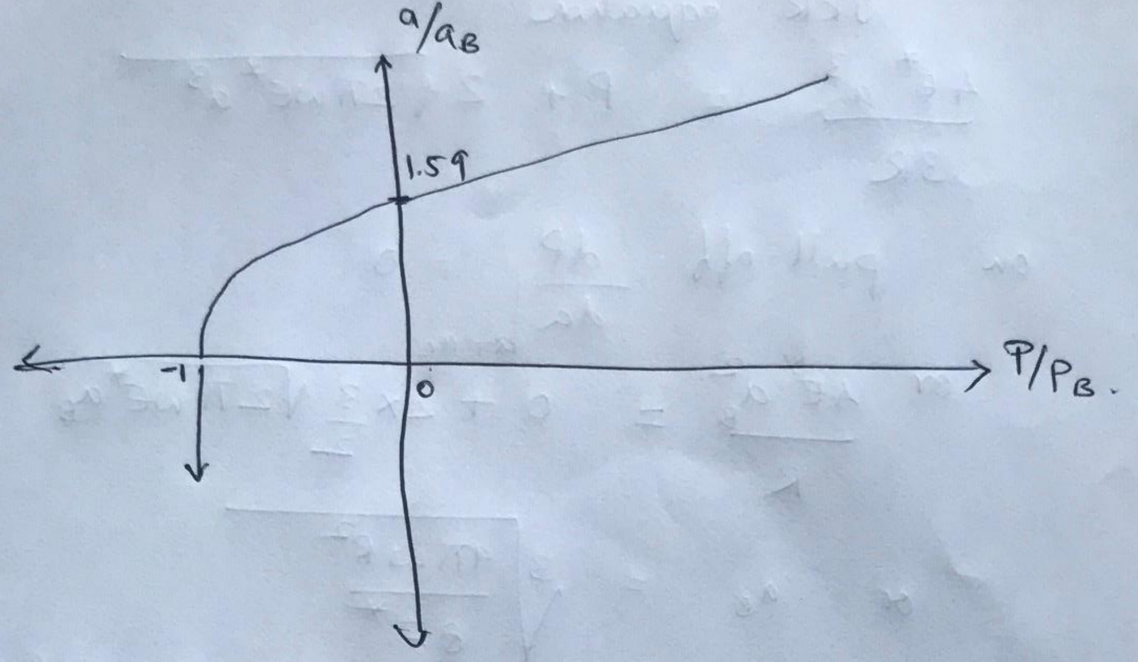
$= \frac{3}{2}\pi \times 1.28 \times \frac{10^{-5}}{2} = 30.1\,\mu N.$

P_2 = pull-off force for bottom sphere

$= 2 \times P_1 = 60.2\,\mu N.$

(c) Note that contact spot size when $P = 0$ from JKR equation

$$a_c = \sqrt[3]{\frac{9\pi w R^2}{2E^*}}$$



(d) $$P_m = \pi r^2 \times \Delta p = \pi r^2 \cdot \frac{\gamma(\cos\alpha \times 2) \times 2R}{r^2}$$

$$= 4\pi R \gamma \cos\alpha$$

is added on for the adhesive force of the lower sphere.

$$\therefore P_m = 4\pi \times 10^{-5} \times 72 \times 10^{-3} \times \cos 20^{0}$$

$$= 8.5\,\mu N.$$

P_1 remains the same and P_2 increases by $8.5\,\mu N$ i.e. $68.7\,\mu N$.

2\. (a)

$$F_x = \frac{N \epsilon_0 h \, V_p^2}{2g}$$

$$x_0 = F_x / k_x .$$

(b)

$$F_y = \frac{1}{2}\left(\frac{N \epsilon_0 h \, (l + x_0) V_p^2}{2(g-y)^2} - \frac{N \epsilon_0 h (l + x_0) V_p^2}{2(g+y)^2} \right)$$

Conditions for sideways instability

$$\left.\frac{dF_y}{dy}\right|_{y=0} = k_y .$$

$$\therefore \quad \frac{N \epsilon_0 h \, (l + x_0) V_p^2}{(g-y)^3} + \frac{N \epsilon_0 h (l + x_0) V_p^2}{(g+y)^3} = 2k_y .$$

(c)

$$\frac{N \epsilon_0 h \left(l + \frac{N \epsilon_0 h V_p^2}{2 g k_x} \right) V_p^2}{g^3} + \frac{N \epsilon_0 h \left(l + \frac{N \epsilon_0 h V_p^2}{2 g k_x} \right) V_p^2}{g^3} = 2k_y .$$

$$\left(\frac{N \epsilon_0 h}{g^2} \right)^2 \frac{V_p^4}{\cancel{2} k_x} + \frac{2 N \epsilon_0 h l \, V_p^2}{g^3} = 2k_y .$$

$$V_p^2 = \frac{ -\frac{\cancel{2} N \epsilon_0 h l}{g^3} + \sqrt{ \left(\frac{2 N \epsilon_0 h l}{g^3} \right)^2 + \frac{\cancel{8} k_y}{k_x} \left(\frac{N \epsilon_0 h}{g^2} \right)^2 } }{ \left(\frac{\cancel{2} N \epsilon_0 h}{g^2} \right)^2 \times \frac{2}{k_x} }$$

$$v_p^2 = \frac{-\frac{l}{g} + \sqrt{\left(\frac{l}{g}\right)^2 + \frac{2k_y}{k_x}}}{\left(\frac{N\epsilon_0 h}{g^2}\right)\frac{1}{k_x}}$$

$$v_p^2 = \left(\frac{g^2 k_x}{N\epsilon_0 h}\right)\left[\sqrt{\frac{2k_y}{k_x} + \left(\frac{l}{g}\right)^2} - \frac{l}{g}\right].$$

(d) Substituting for v_p in expression for x we get:

$$x_{o\,max} = \frac{1}{k_x}\left(\frac{N\epsilon_0 h}{2g}\right)\left(\frac{g^2 k_x}{N\epsilon_0 h}\right)\left[\sqrt{\frac{2k_y}{k_x} + \left(\frac{l}{g}\right)^2} - \frac{l}{g}\right].$$

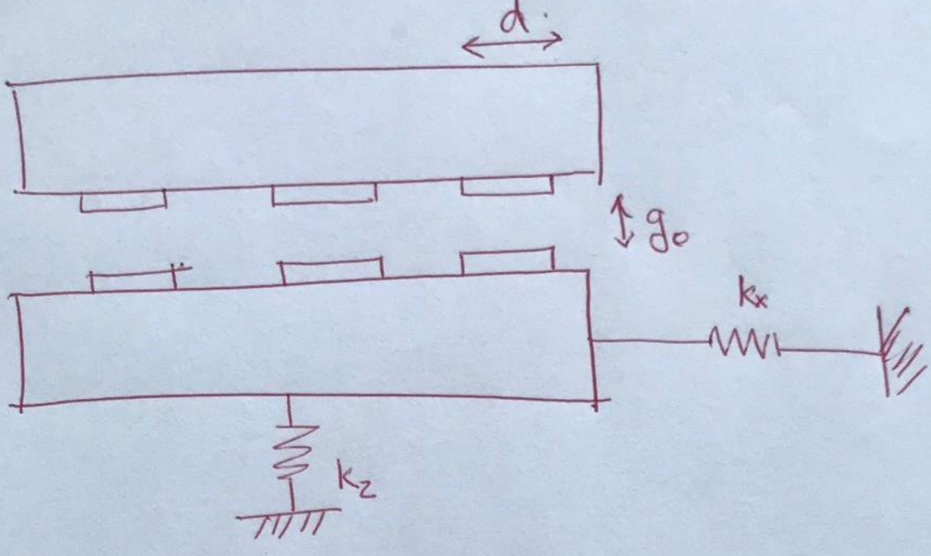
$$= \frac{g}{2}\left[\sqrt{\frac{2k_y}{k_x} + \left(\frac{l}{g}\right)^2} - \frac{l}{g}\right].$$

$$= 1\left[\sqrt{2 + (2.5)^2} - 2.5\right].$$

$$= \underline{\underline{0.37\,\mu m}}$$

Q3

(a)



$$C(x) = \frac{N \epsilon_0 w (d - x)}{g_o}$$

$$\frac{\Delta C(x)}{\Delta x} = \frac{-N \epsilon_0 w}{g_o}$$

$$\Delta x = \frac{a}{w_x^2} \quad \& \therefore \quad \Delta C(x) = -\frac{N \epsilon_0 w \cdot a}{g_o \, \omega_x^2}$$

$$\frac{\Delta C(x)}{a} = \frac{-N \epsilon_0 w}{g_o \, \omega_x^2}$$

(b)

$$\Delta C(x) = \frac{-50 \times 8.85 \times 10^{-12} \times 500 \times 10^{-6}}{10 \times 10^{-6} \times (2\pi \times 2 \times 10^{3})^2}$$

$$= -140 \times 10^{-18} F$$

(c)

$$C(z) = \frac{N \epsilon_0 w d}{(g_o - z)}$$

$$\frac{\Delta C}{\Delta z} = \frac{+N \epsilon_0 w d}{(g_o - z)^2} \quad \& \frac{\Delta C}{\Delta a_z} \approx \frac{N \epsilon_0 w d}{g_o^2 \cdot \omega_z^2}$$

(d) $$\Delta C(z) = \frac{50 \times 8.85\times10^{-12} \times 500\times 10}{10\times10\times(2\pi\times 10\times10^{3})^{2}}$$

$$= 5.60 \times 10^{-18}\ F.$$

(e) Limit to dynamic range in x is when the fingers no longer overlap:

displacement $x = d = a/\omega_x^2$

or $a_{max} = 10^{-5} \times (2\pi \times 10^{3}\times 2)^{2} = 395\ m/s^2$

$$a_{min} = \sqrt{\frac{4\times 1.38\times10^{-23}\times300\times(2\pi\times 2\times10^{3})^{3}}{10\times100}} \times \sqrt{100}$$

$$= 5.73 \times 10^{-5}\ m/s^2.$$

So possible to measure over a wide dynamic range

(f) One of the potential limitations is cross-axis coupling where acceleration in the z direction is aliased into the x-direction.

A mitigation approach would be to increase the stiffness in the z-axis by fabrication approaches e.g. thicker structural layers. Alternatively the gap spacing between the electrodes can be increased as this impacts z-axis sensitivity more than the x axis at the expense of reducing sensitivity along x-axis as well.

4 (a) $m_{eff} = \frac{\rho A l}{2} = \frac{2330 \times 10 \times 10 \times 192 \times 10^{-18}}{2} = 2.24 \times 10^{-11}\,kg$

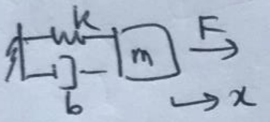
$k_{eff} = \omega^2 m_{eff} = (2\pi \times 10^7)^2 \times 2.24 \times 10^{-11}$

$= 8.84 \times 10^4$ N/m.

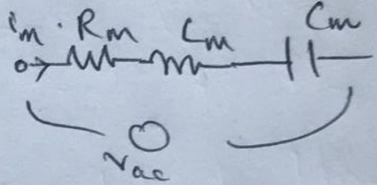
$b_{eff} = \frac{m_{eff}\,\omega}{Q} = \frac{2.24 \times 10^{-11} \times 2\pi \times 10^7}{10^5} = 1.41 \times 10^{-8}\,kg/s$

(b) Equivalent motional parameters obtained by drawing analogies between mechanical and electrical domains

Mechanical.



Electrical



$\frac{\dot{x}}{F} = \frac{1}{sm + b + k/s}$.

$\frac{i}{V_{ac}} = \frac{1}{sL_m + R_m + \frac{1}{sC_m}}$.

$\frac{i/\eta}{\eta V_{ac}} = \frac{1}{sm + b + k/s}$.

$\therefore\ i/V_{ac} = \frac{\eta^2}{sm + b + k/s}$.

where η = electromechanical transduction coefficient $= \frac{\epsilon_0 A}{g^2} . V_p = 1.77 \times 10^{-8}$

Comparing expressions we get the values of the motional parameters as:

$C_m = \eta^2/k_{eff}$, $L_m = m_{eff}/\eta^2$, $R_m = b_{eff}/\eta^2$.

$= 3.54 \times 10^{-21}$ F, $L_m = 71499$ H, $R_m = 45\,M\Omega$.

(c) i_m at resonance $\approx \frac{\eta^2 V_{ac} . Q . \omega}{k_{eff}}$

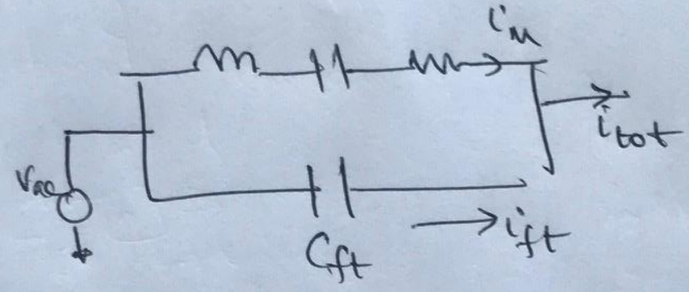
(d) The magnitude of the feedthrough current

$$i_{ft} \simeq j\omega C_{ox} v_{ac}$$

$$|i_{ft}| \approx 2\pi \times 10^{7} \times 50 \times 10^{-15} \times 0.1 \simeq 314 \, nA$$

Clearly the magnitude of the feedthrough current is much larger than the motional current at resonance.

(e)



$i_{tot} = i_m + i_{ft}$

Two approaches to mitigate feedthrough:

(1) Increase η through process + device design. This includes reducing the gaps, increasing widths + structural thicknesses and increasing the bias voltage.

(2) Reduces sources of capacitive feedthrough through careful device layout, substrate grounding and separation of drive / sense electrodes.