Version AAS/3

EGT3
ENGINEERING TRIPOS PART IIB

Thursday 27 April 2023 2.00 to 3.40

Module 4C15

MEMS DESIGN

*Answer not more than **three** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

*Write your candidate number **not** your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper.

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
Attachment: 4C15 MEMS Design data sheet (4 pages).
Engineering Data Book

10 minutes reading time is allowed for this paper at the start of the exam.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

You may not remove any stationery from the Examination Room.

1 Two identical silicon microspheres of 10 μm radius are stacked vertically on a silicon substrate as shown in Fig. 1. A micromanipulator applies a vertical force to the stack in order to move the spheres off the surface. The adhesion between the surfaces can be modelled by the JKR equation. You may assume that the elastic modulus of silicon is 160 GPa and the Poisson's ratio is 0.3.

(a) Starting with the JKR equation, derive expressions for the pull-off force and size of the contact on pull-off. [20%]

(b) Estimate the magnitudes of the forces required to move both spheres off the surface assuming that the adhesive forces between the surfaces dominate. [30%]

(c) Sketch a graph of the applied load required to separate the spheres from each other versus the contact spot size for the case above, noting salient values. [30%]

(d) The interface between the lower sphere and the silicon substrate is now contaminated by a drop of water with surface tension 72 mJ m^{-2} and a contact angle of 20^{o}. Estimate the forces required to move the silicon spheres in this case. [20%]

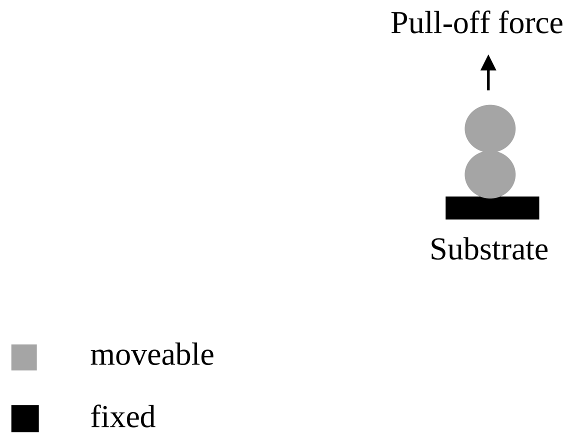

Fig. 1

2 A compliant mass-spring system is constrained to translate in-plane with spring constants k_x and k_y in the x- and y- directions respectively. A voltage-controlled comb drive actuator is employed to actuate this system as shown in Fig. 2 with a dc voltage V_p applied between the mass and the fixed electrode. The nominal uniform gap spacing between the electrodes is g, the structural thickness of the layers is h, the number of comb finger gaps is N, and the comb finger overlap length with no voltage applied is l.

(a) Obtain an expression for the electrostatic force for motion in the x-direction. [10%]

(b) The system is susceptible to a sideways instability along the y-direction. Formulate the conditions for instability along this direction. [20%]

(c) Derive an expression for the sideways instability voltage as a function of the governing parameters. [40%]

(d) Obtain a value for the displacement of the mass along the x-axis when sideways instability is first observed for $k_x = k_y = 10$ N m^{-1}, $h = 10$ µm, $g = 2$ µm, $l = 5$ µm. [30%]

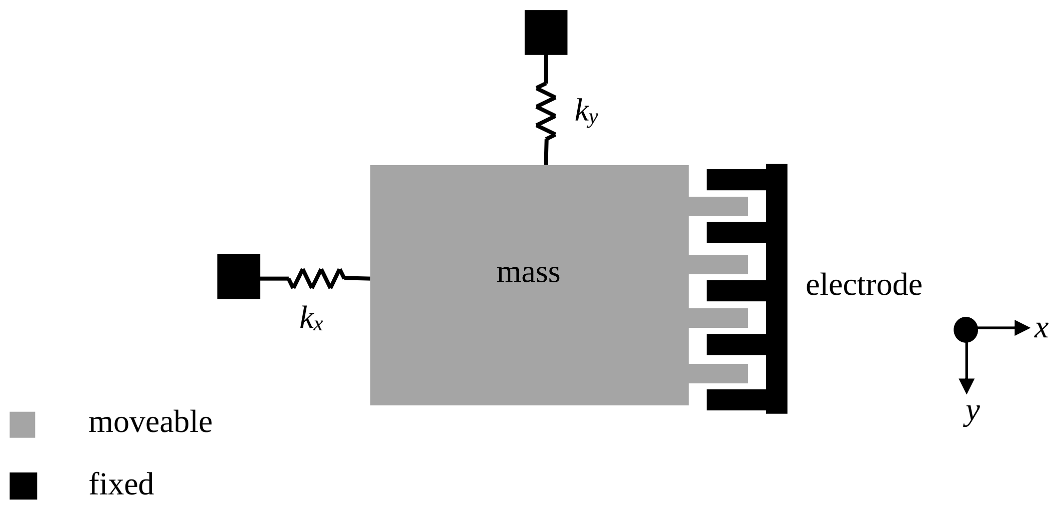

Fig. 2

3 A silicon micromachined capacitive accelerometer consists of a suspended mass and a fixed substrate each consisting of a set of 50 patterned electrodes separated by an air gap. The nominal capacitive gap between the mass and the fixed substrate is 10 μm and the overlap width is 10 μm as well. The electrodes extend 500 μm into the page with perfect alignment and overlap between the two sets of electrodes when no external acceleration is applied as shown in Fig. 3. The primary sensing axis is along the *x*-direction with a natural frequency of 2 kHz. The proof mass of value 10^{-6} kg is also compliant along the *z*-direction with a natural frequency of 10 kHz. The operating temperature is 300 K and viscosity of air is 1.8 x 10^{-5} kg m^{-1} s^{-1}.

(a) Estimate the change in capacitance for an input acceleration of 1 m s^{-2} acting along the *x*-direction. [20%]

(b) Estimate the change in capacitance for an input acceleration of 1 m s^{-2} acting along the *z*-direction. [20%]

(c) Estimate the maximum acceleration that can be sensed along the *x*-direction assuming that the limit is set by the condition when the sense electrodes no longer overlap. [20%]

(d) Estimate the thermo-mechanical noise limited resolution of the device for sensing in the *x*-direction. [20%]

(e) Discuss the limitation associated with cross-axis sensitivity and provide a potential design change to address this limitation. [20%]

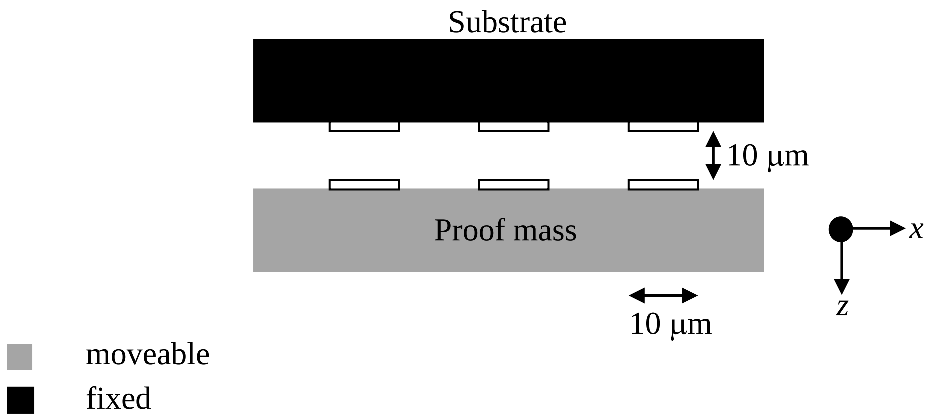

Fig. 3

4 A top view of a 10 μm thick free-free beam polysilicon length-extensional mode microresonator is shown in Fig. 4. The microresonator is actuated capacitively using a parallel plate electrode separated by a gap spacing of 1 μm and width equal to the resonator width of 10 μm. A second identical electrode is located on the opposite end of the resonator to sense the motional current. The natural frequency of the fundamental extensional mode of the resonator is 10 MHz with a Quality factor of 10^5 in vacuum. A dc polarisation voltage of 20 V is employed for both drive and sense transduction. The length of the microresonator is 192 μm and the effective modal mass is half the total device mass. The density of polysilicon is 2330 kg m^{-3}.

(a) Estimate the effective mass, spring constant and damping constant for the device. [20%]

(b) Derive expressions for the motional parameters of the device and estimate their nominal values. [30%]

(c) Estimate the motional current of the device if an ac voltage of magnitude equal to 100 mV is applied to the drive electrode at the resonance frequency. [20%]

(d) A feedthrough capacitance of 50 fF is observed between the drive and sense ports. Compare the magnitude of the feedthrough current relative to the motional current. [20%]

(e) Provide two distinct approaches to reduce the impact of capacitive feedthrough on the measured response. [10%]

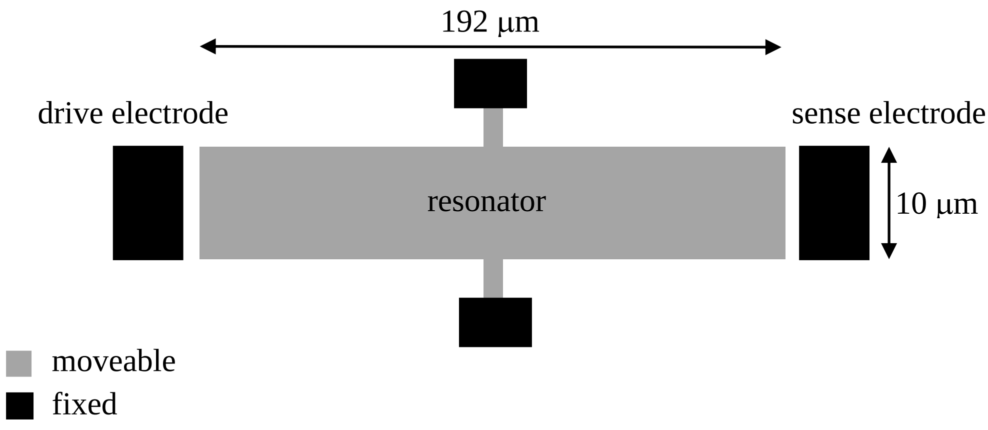

Fig. 4

END OF PAPER

THIS PAGE IS BLANK

ENGINEERING TRIPOS Part IIB

Module 4C15 Data Sheet

Elastic Hertzian point contact under load P

Reduced radius R given by $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$ (Suffixes 1, 2 refer to the two bodies in contact)

Contact modulus E^* given by $\frac{1}{E^*} = \frac{1-\nu_1^2}{E_1} + \frac{1-\nu_2^2}{E_2}$

Radius of contact circle $a = \left\{\frac{3PR}{4E^*}\right\}^{1/3}$

Maximum contact pressure $p_0 = \frac{3P}{2\pi a^2} = \left\{\frac{6PE^{*2}}{\pi^3 R^2}\right\}^{1/3}$

Mean contact pressure $\bar{p} = \frac{2}{3} p_0$

Approach of distant points $\delta = \frac{a^2}{R} = \left\{\frac{9P^2}{16RE^{*2}}\right\}^{1/3}$

Maximum shear stress is of magnitude $0.31 p_0$ and at depth $0.48a$.

Lennard-Jones potential between point atoms

$$U(r) = -\frac{C}{r^6} + \frac{D}{r^{12}} = -4U_0\left\{\left(\frac{1.12r}{r_0}\right)^{-6} - \left(\frac{1.12r}{r_0}\right)^{-12}\right\}$$

where U_0 is bond energy and r_0 is bond length, i.e. spacing at which $U(r)$ is minimum.

Smooth surface adhesion $p(h) = \frac{8w}{3h_0}\left\{\left(\frac{h}{h_0}\right)^{-3} - \left(\frac{h}{h_0}\right)^{-9}\right\}$

w is the work of adhesion, in principle $w = \gamma_1 + \gamma_2 - \gamma_{12}$

Elastic spherical contact with adhesion, JKR $\frac{4E^* a^3}{3R} = P + 2\sqrt{2\pi w E^* a^3}$

Pressure drop across meniscus $\Delta p = \frac{\gamma}{r}$ for each liquid/vapour interface

Yield stress in shear $k \approx H/6$

Archard wear, dimensional wear rate $\propto \dfrac{\text{pressure} \times \text{sliding speed}}{\text{hardness } H}$

SURFACE ENERGIES AT ROOM TEMPERATURE*

High energy solids

Material	Surface energy mJ m^{-2}
NaCl	160
Al_2O_3	641
Si	1280
Al	1120
Ag	1440
Fe	2400
W	4490

Low energy solids

Material	Surface energy mJ m^{-2}
nylon	46.5
polyvinyl chloride	38.9
polystyrene	33.0
polyethylene	30.4
paraffin wax	25.0
PTFE	18.3
Diamond-Like-Carbon	25–40

Liquids

Material	Surface energy mJ m^{-2}
water	73.1
benzene	28.8
n-pentane	16.0
n-octane	21.6
n-dodecane ($C_{12}H_{26}$)	25.5
n-hexadecane ($C_{16}H_{34}$)	27.6
n-octadecane ($C_{18}H_{38}$)	28.0
Fomblin Zdol	20~25

* from: Adamson, A. W., *Physical Chemistry of Surfaces*, Wiley (1990) and Israelachvili, J., *Intermolecular and Surface Forces*, Academic Press (1992)

Electrostatic forces

$F_{PP} = \frac{\varepsilon A V^2}{2g^2}$; magnitude of the electrostatic force for a gap-closing parallel-plate actuator where ε is the permittivity for the medium between the plates, A is the area of overlap, g is the gap spacing between the electrodes, V is the voltage applied.

$V_{PI} = \sqrt{\frac{8kg_0^3}{27\varepsilon A}}$; pull-in voltage for a gap-closing parallel-plate actuator where k is the effective spring constant, g_0 is the initial gap spacing with 0V applied, ε is the permittivity for the medium between the plates, and A is the area of overlap.

$F_{COMB} = \frac{\varepsilon t V^2}{2g}$; magnitude of the electrostatic force generated between a pair of electrodes arranged in the form of a comb drive where ε is the permittivity for the medium between the plates, t is the structural thickness, g is the gap spacing between the electrodes, V is the voltage applied.

Thermo-mechanical Noise

$\overline{F}_n = \sqrt{4k_B T b}$ in units of $N/\sqrt{Hz}$; analytical expression for the thermo-mechanical force noise spectral density where k_B is the Boltzmann constant, T is the temperature, and b is the damping constant.

Damping constants

$b = \frac{\eta A}{h}$; analytical expression for damping constant determined by Couette flow where η is the dynamic viscosity of the fluid, A is the area of overlap of the two surfaces and h is the constant gap spacing between the surfaces.

$b = \frac{96\eta L W^3}{\pi^4 h^3}$; analytical expression for damping constant determined by squeeze film effects where η is the dynamic viscosity of the fluid, L is the overlap length (long dimension), W is the width (short dimension), and h is the nominal gap spacing between the surfaces.

Equivalent Circuit parameters for resonators

$L_m = m/\eta^2$; where m is the effective mass and η is the transduction parameter.

$C_m = \eta^2/k$; where k is the effective stiffness and η is the transduction parameter.

$R_m = b/\eta^2$; where b is the damping constant and η is the transduction parameter.

$\eta = \frac{V_P \varepsilon A}{g^2}$; transduction parameter for a parallel-plate electrode geometry with a symmetric drive and sense configuration, where ε is the permittivity for the medium between the plates, A is the electrode overlap area, g is the gap spacing between the electrodes, V_P is the DC polarization voltage applied across the electrode(s) and the resonator.

Microfluidics

$\Delta P = \frac{12\eta L}{Wh^3} Q$; relationship between pressure drop (ΔP) and volumetric flow rate (Q) for Poiseuille Flow in a microchannel with rectangular cross-section, where η is the dynamic viscosity of the fluid, L is the channel length, W is the channel width and h is the channel height.

$Q = \frac{\pi a^4}{32\eta} \frac{\Delta P}{L}$; relationship between volumetric flow rate (Q) and pressure drop (ΔP) for Poiseuille Flow in a microchannel with circular cross-section, where η is the dynamic viscosity of the fluid, a is the channel radius and L is the channel length.

$U_0 = -\frac{\sigma_w \mathsf{E}_x L_D}{\eta} = -\frac{\varepsilon\zeta}{\eta} \mathsf{E}_x$; Electroosmotic plug flow velocity (U_0) as a function of the glass wall charge (σ_w), the zeta potential (ζ), magnitude of the Electric field (E_x) for electroosmotic transport, dynamic viscosity (η), Debye Length (L_D), permittivity of the fluid medium (ε).

$v_{ep} = \mu_{ep} \mathsf{E}_x$; electrophoretic velocity (v_{ep}) as a function of the electrophoretic mobility (μ_{ep}) and the magnitude of the Electric field (E_x).