Version AAS/6

EGT3
ENGINEERING TRIPOS PART IIB

Wednesday 23 April 2014   9.30 to 11

Module 4C15

MEMS DESIGN

Answer not more than ***three*** *questions.*

All questions carry the same number of marks.

The ***approximate*** *percentage of marks allocated to each part of a question is indicated in the right margin.*

Write your candidate number ***not*** *your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
Attachment: 4C15 MEMS Design data sheet (3 pages).
Engineering Data Book

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

1 (a) Summarise briefly the idealisations that are made in the Hertz theory of elastic contact. [15%]

(b) The JKR theory of contact between a sphere of radius R and a plane pressed together by a normal force P takes into account both elasticity, through the contact modulus E^*, and any adhesive forces due to surface energy through an allowance for the work of adhesion w. The relation between P and the approach δ can be approximated by the expression

$$P = \frac{4}{3} E^* R^{1/2} \left(\delta - 2\delta_c\right)\left(\delta + \delta_c\right)^{1/2}$$

where δ_c is the 'adhesion distance' and is given by

$$\delta_c = \frac{3}{2} R^{1/3} \left(w / E^*\right)^{2/3}$$

(i) Show that this reduces to the result for Hertzian contact at large compressions but, unlike the Hertz theory, that adhesive force can occur over a range of values of δ which should be specified. [20%]

(ii) A spherical glass indenter with radius $1\,\mathrm{mm}$ is brought into contact with a plane polymer surface. Relevant values are $E^* = 3\,\mathrm{MPa}$ and $w = 80\ \mathrm{mJ\,m^{-2}}$. If surface energy effects are important, sketch a curve showing the relation between load P and approach δ. Indicate how this compares with Hertz theory for values of δ less than 5 micron. [25%]

(iii) A spherical particle of the same polymer, which has density $1200\ \mathrm{kg\,m^{-3}}$, is placed on a clean, dry glass microscope slide. Estimate the maximum size of the particle that would remain attached when the slide is inverted. [25%]

(iv) How might your estimate in part (iii) be influenced if either the slide were inadequately cleaned or the experiment carried out in an atmosphere saturated with water vapour? [15%]

2 A voltage-controlled parallel-plate electro-static actuator of nominal capacitance C_0 is connected in series with a fixed capacitor C_1 and in parallel with another fixed capacitor C_2 as shown in Fig. 1 such that $C_1=C_0/n$ and $C_2=C_0/m$ with no voltage applied. The variable parallel-plate capacitor C_0 consists of a spring-supported moveable plate that moves towards the fixed plate during actuator operation. The nominal air-gap with no voltage applied between the fixed and moveable plates is g and the nominal area of the plates is A. The spring is assumed to be linear and of value k.

(a) Derive an expression for the electrostatic force generated by the parallel-plate actuator as a function of the applied voltage V and the displacement x of the moveable plate. [30%]

(b) Under static equilibrium conditions, derive an expression for the maximum distance over which the actuator can move stably. Show that the maximum distance travelled by the moveable plate during stable operation equals to one-third of the nominal gap spacing as $n \to 0$. [40%]

(c) What is the minimum value of n such that the actuator operates stably over the entire gap? Discuss the physical interpretation. [30%]

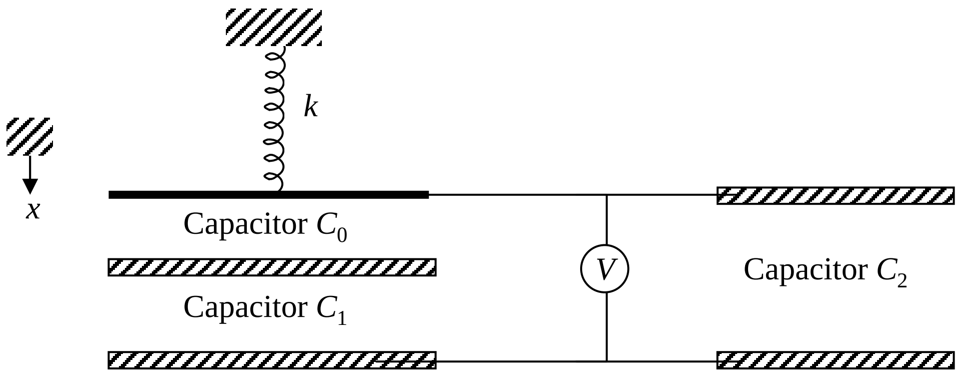

Fig. 1

3 A surface-micromachined polysilicon square plate resonator of side dimension L and uniform thickness h is driven into the square extensional mode characterised by an effective mass given by $M_{eff} = \rho L^2 h$ and an effective stiffness given by $K_{eff} = \pi^2 E h$ where E and ρ are the Young's Modulus and density of polysilicon respectively. The edge displacement of the plate in the square extensional mode may be assumed to be uniform in all in-plane directions. The overlap length of each surrounding parallel-plate electrode is equal to L as shown in Fig. 2 and the nominal gap spacing between each electrode and the suspended square plate is equal to 1 μm. You may assume that the values for E and ρ are 160 GPa and 2330 kg m^{-3} respectively.

(a) Estimate the natural frequency for free vibration in the square extensional mode for L = 1 mm. Comment on design approaches to enable scaling of natural frequency to a value greater than 1 GHz. [20%]

(b) Sketch an equivalent electrical circuit model for the resonator showing a feed-through capacitor across the resonator terminals. Hence, write down an expression for the electrical admittance of the resonator as a function of the motional parameters. [20%]

(c) The resonator is actuated by biasing all four surrounding parallel-plate electrodes at a dc voltage of 100 V and an ac voltage of 0.1 V with respect to the square plate. Obtain numerical estimates of the resonator motional parameters and the amplitude of motion at resonance for the case L = 1 mm, h = 10 μm and a Quality factor of 10^5. [40%]

(d) Discuss the origins of temperature-induced drift of the natural frequency and describe two approaches used to passively correct for this temperature dependence. [20%]

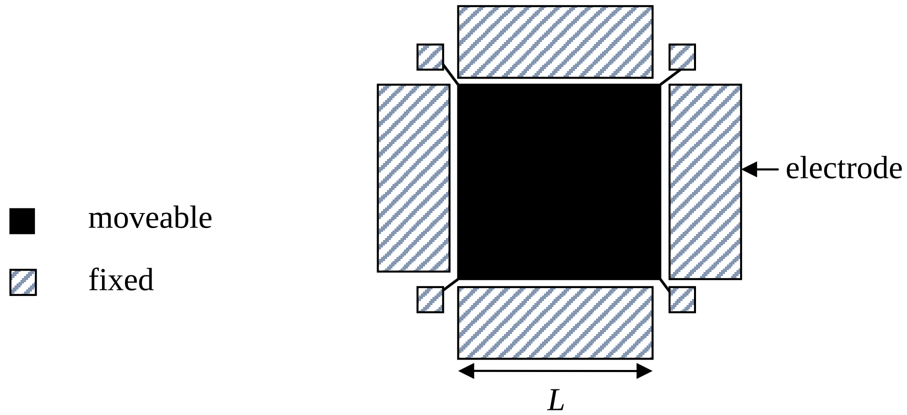

Fig. 2

4 A single-axis surface micromachined accelerometer is designed consisting of a mass m supported by a linear spring of value k in parallel with a dashpot of value b constraining the motion of the mass along the sensitive axis. A capacitive sensing scheme is employed consisting of a differential parallel-plate sensing electrode arrangement. The total nominal sense capacitance is C_s and the nominal gap spacing between the electrodes is 1 μm.

(a) Derive an expression for the fractional change in capacitance as a function of input acceleration along the sensitive axis. [20%]

(b) Write down an expression for the minimum detectable acceleration signal at room temperature due to the Brownian motion of the mass. [30%]

(c) If the natural frequency of the system is 1 kHz, $m = 10^{-6}$ kg and the associated Quality factor is 10, estimate the minimum detectable acceleration signal at room temperature (300 K). [15%]

(d) Estimate the minimum detectable acceleration signal if the noise floor of the electronics limits acceleration measurement instead, such that a minimum capacitance change of 10^{-18} F can be resolved by the electronics in a 1 Hz bandwidth. The value of C_s is equal to 10^{-12} F. [15%]

(e) Describe design approaches to reduce the minimum detectable noise floor. [20%]

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