4C15 MEMS DESIGN

Q1 (a) $k = 100\,N/m$. $E^* = 87.9\,GPa$.

Radius of contact spot $= a = \left(\dfrac{3PR}{4E^*}\right)^{1/3}$ (databook)

$$a = \left(\frac{3\times10^{-6}\times10^{-5}}{4\times87.9\times10^{9}}\right)^{1/3} = 44\,nm.$$

mean contact pressure $= \dfrac{P}{\pi a^2} = 164\,MPa.$

(b) JKR (spherical contact with adhesion)

$$\frac{4E^* a^3}{3R} = P + 2\sqrt{2\pi w E^* a^3} \quad \text{(databook)}.$$

$$1.17\times10^{16} a^3 = 10^{-6} + 1.682\times10^{6} a^{3/2}$$

setting $x = a^{3/2}$ and solving:

$$x = \frac{1.682\times10^{6} + \sqrt{(1.682\times10^{6})^2 + 4\times10^{-6}\times1.17\times10^{16}}}{2.34\times10^{16}}$$

$\therefore\ a = 275\,nm$

(c) Reducing load to zero gives:

$$a_c = \sqrt[3]{\frac{9\pi w R^2}{2E^*}} = 274.1\,nm$$

pull-off force $= \dfrac{3}{2}\pi w R \quad = \dfrac{3\pi}{2}\times1.28\times10^{-5} = 60.3\,\mu N$

(d) with liquid drop added:

pull-off force $\simeq 2\pi R\gamma\,(\cos\alpha_1 + \cos\alpha_2)$

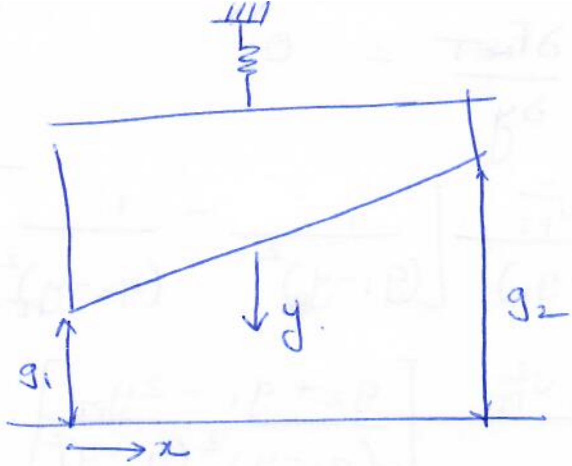
For $\alpha_1 = \alpha_2 = 15^\circ$ we have

$$P_M = 2\pi \times 10^{-5} \times 0.072 (\cos 15^\circ) \times 2$$
$$= 8.74\,\mu N.$$

(c) Failure due to permanent adhesion of the surfaces is possible due to the high forces of attraction involved in contact as calculated above. This can be mitigated in practice by high force density actuators, robust mechanical flexure arrangements for the robot legs and by surface coatings and surface texturing to reduce work of adhesion and increase surface roughness

Q2

(a)



nominal gap spacing at position x along the length of the beam $= g(x) = g_1 + \frac{(g_2 - g_1)x}{l}$

For the case of voltage control and gap closing response use parallel-plate approximation for a small region of incremental width dx at position x. Electrostatic force generated (dF) given by:

$$dF = \frac{\epsilon_0 w V^2 dx}{2(g(x)-y)^2}$$ for displacement of beam, y.

$$\text{Total force} = \int_0^l \frac{\epsilon_0 w V^2 dx}{2(g(x)-y)^2}$$

$$= \frac{\epsilon_0 w V^2}{2} \int_0^l \frac{dx}{(g(x)-y)^2}$$

$$= \frac{\epsilon_0 w V^2 (-l)}{2(g_2-g_1)} \left[\frac{1}{g_2 - y} - \frac{1}{g_1 - y} \right]$$

$$= \frac{\epsilon_0 w l V^2}{2(g_2-y)(g_1-y)}.$$

(b) Conditions for pull-in are $F_{NET} = 0$

i.e. $$\frac{\epsilon_0 w l V_{PI}^2}{2(g_2-y_{PI})(g_1-y_{PI})} = k y_{PI} \quad -(1)$$

and $\frac{\partial F_{NET}}{\partial y} = 0$

or $\frac{\epsilon_0 w l V_{PI}^2}{2(g_2 - g_1)}\left[\frac{1}{(g_1 - y_{PI})^2} - \frac{1}{(g_2 - y_{PI})^2}\right] = k$

or $\frac{\epsilon_0 w l V_{PI}^2}{2}\left[\frac{g_2 + g_1 - 2y_{PI}}{(g_2 - y_{PI})^2 (g_1 - y_{PI})^2}\right] = k$ — (2)

(c) Solving (1) and (2) simultaneously:

$$\left[\frac{g_2 + g_1 - 2y_{PI}}{(g_2 - y_{PI})(g_1 - y_{PI})}\right] \times k y_{PI} = k \quad \text{(substituting (1) in (2))}$$

$$\therefore (g_2 + g_1 - 2y_{PI})\, y_{PI} = (g_2 - y_{PI})(g_1 - y_{PI}).$$

$$3y_{PI}^2 - 2(g_1 + g_2) y_{PI} + g_1 g_2 = 0.$$

$$\text{or } y_{PI} = \frac{2(g_1 + g_2) \pm \sqrt{4(g_1 + g_2)^2 - 12 g_1 g_2}}{6}$$

The negative square root is chosen as physically meaningful

(d) V_{PI}^2 (pull-in voltage) $= \frac{2k\, y_{PI}(g_2 - y_{PI})(g_1 - y_{PI})}{\epsilon_0 w l}$

For $g_1 = g_2 = g \Rightarrow y_{PI} = g/3$

and $V_{PI}^2 = \frac{8 k g^3}{27 \epsilon_0 A}$

which is the standard expression for uniform gap parallel-plate actuation (datasheet)

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(a) $$\text{displacement} = \frac{ma}{k} = \frac{a}{\omega_x^2}$$

$$= \frac{1}{(2\pi \times 10^4)^2} = 0.253 \text{ nm}$$

(b) For the comb drive:

$$C(x) = \frac{\epsilon_0 (w - x) t}{g}.$$

$$\Delta C = -C(x) + C_0 = \frac{\epsilon_0 x t}{g}$$

$$\therefore \frac{\Delta C}{C_0} = \frac{x}{w}$$ where w = overlap length.

For input acceleration of 1 m/s^2 we have:

$$\frac{\Delta C}{C_0} = \frac{0.253 \times 10^{-9}}{300 \times 10^{-6}} = 0.844 \times 10^{-6}$$

(c) For the differential parallel-plate

$$\Delta C(x) = \frac{\epsilon_0 w t}{g - x} - \frac{\epsilon_0 w t}{g + x} \approx \frac{2\epsilon_0 w t}{g}\left(\frac{x}{g}\right)$$

$$C_0 = \frac{2\epsilon_0 w t}{g}$$

$$\therefore \frac{\Delta C}{C_0} = \frac{x}{g} = \frac{0.253 \times 10^{-9}}{10^{-6}} = 2.53 \times 10^{-4}$$

(d) Total damping constant (b)

$$= b_{\text{mass-substrate}} + b_{\text{comb-drive}} + b_{\text{parallel-plate}}$$

$$= \eta\left(\frac{2 \times 2 \times 10^{-6}}{2 \times 10^{-6}}\right) + \eta\left(\frac{50 \times 300 \times 10 \times 10^{-12}}{10^{-6}}\right) \times 2$$

$$+ \eta \times \frac{96 \times 100 \times 300 \times 10^{3} \times 10^{-6}}{\pi^4 \times 1^3}$$

$\therefore \; b = 1.8\times10^{-5}\left(2 + 0.15 * 2 + 29.56\right)$

$\therefore \; b = 5.735\times10^{-4}$

Thermo-mechanical noise limited resolution at room temperature $= \dfrac{\sqrt{4k_BTb}}{m}$

$$= \frac{\sqrt{4k_B\times300\times5.735\times10^{-4}}}{2330\times2\times2\times10\times10^{-12}} = 3.31\times10^{-5}\ m/s^2/\sqrt{Hz}$$

(e) Limitation of the current design include:

→ lower sensitivity detection using comb drives as opposed to differential parallel-plate configuration. Recommend reducing overlap length of comb drives.

→ However, more squeeze-film damping associated with parallel-plate structures resulting in greater thermo-mechanical damping as compared to comb drives. Recommend vacuum packaging to reduce damping if increased resolution is needed

4 (a) $$U_0 = -\frac{\epsilon \zeta E_x}{\eta}$$

$$= -80 \times 0.08 \times 8.85\times10^{-12} \times \frac{100}{8\times10^{-3}} \times \frac{1}{10^{-3}}$$

$$= -7.08 \times 10^{-4} \text{ m/s}$$

volumetric flow rate $= |AU_0| = 7.08\times10^{-12}\ \text{m}^3/\text{s}$.

(b) volumetric flow rate $= \frac{hw^3 k}{12\eta} = 7.08\times10^{-12}$

$$\therefore k = \frac{7.08\times10^{-12} \times 12\times 10^{-3}}{10^{-16}} = 849.6$$

$$\therefore \Delta P = k \cdot L = 849.6 \times 0.008 = 6.8 \text{ Pa}.$$

(c) flow velocity $= \frac{-\epsilon \zeta}{\eta} \times \frac{200}{0.005}$

$$= -\frac{80\times 8.85\times10^{-12}}{10^{-3}} \times 0.08 \times \frac{200}{0.005}$$

$$= 2.26\times10^{-3} \text{ m/s}.$$

(d) time taken for second plug $= \frac{\text{distance to port4}}{\text{plug velocity to port4}}$

$$= \frac{0.005\times10^{-3}\times0.005}{80\times8.85\times10^{-12}\times0.08\times300}$$

$$= 1.47 \text{ s}$$

time taken for first plug $= 5\times10^{-3}/2.26\times10^{-3} = 2.21\text{s}$.

Given the 10 s additional latency, the second plug would reach port 4 only after the first plug reaches port 3.

(e) separation distance $= (\Delta\mu) E t$

1st plug : $d_1 = 10^{-8} \times \frac{200}{0.005} \times \frac{3\times10^{-3}}{2.26\times10^{-3}} = 530\,\mu m$.

2nd plug : $d_2 = 10^{-8} \times \frac{300}{0.005} \times \frac{3\times10^{-3}}{u_0} = \frac{10^{-8}\times300}{0.005}\times\frac{3}{5}\times1.47 = 529\,\mu m$

(f) The species diffuse in solution such that the band size $\propto \sqrt{Dt}$, $t \approx L/u_0$.

$\therefore$ use short columns, large electric fields and low ionic strength buffers to achieve large u_0 and good separation while ensuring plug flow behaviour for transporting electrolyte solution.