ENGINEERING TRIPOS PART IIB,
ADVANCED MACHINE DESIGN, 4C16
CRIB 2013/14 (M SUTCLIFFE)

Q1 (a) $$W^* = \frac{Wc^2}{\eta \omega L R^3}$$

So as c is reduced the value of W^* falls. But from table, if W^* falls so does eccentricity ratio ε, i.e. bearing runs more concentrically.

(b) Petrov's law assumes bearing runs concentrically and that the oil film is continuous

Then $\tau = \frac{\eta R \omega}{c}$, $M = 2\pi R \eta \frac{R\omega}{c} . L . R = \frac{2\pi \eta \omega L R^3}{c}$

i.e. $m^* = 2\pi$

Changes associated with variation of clearance around bearing tend to cancel out.

(c) Heat flow through bearing $= 2\pi \frac{RLk}{d}(\theta_{oil} - \theta_a)$

All work done by M flows this way

$$\Rightarrow \quad M\omega = 2\pi \frac{RL}{d} k \, \Delta\theta \qquad \text{where } \Delta\theta = \theta_{oil} - \theta_a$$

$$\text{i.e. } \Delta\theta = m^* \frac{\eta \omega L R^3}{c} \frac{d\,\omega}{2\pi R L k}$$

$$\Delta\theta = \frac{m^*}{2\pi} \eta \frac{(R\omega)^2}{k} \frac{d}{c} = \frac{m^*}{2\pi} \frac{\eta U^2}{k} \frac{d}{c}$$

(d) Shaft will be at uniform temperature, equal to that of oil film:

$\theta_{oil} = \theta_a + \Delta\theta$

Bearing thin so average temperature $= \frac{1}{2}(\theta_A + \theta_{oil}) = \theta_a + \frac{\Delta\theta}{2}$

So clearance will be reduced from initial value c_0

$$c = c_0 - \tfrac{1}{2}\beta \Delta\theta R$$

1 (d) (cont) or $\frac{c}{R} = \frac{c_0}{R} - \frac{1}{2}\beta\,\delta\theta$

where $\delta\theta = \frac{m^*}{2\pi}\,\frac{\eta u^2}{\kappa}\,\frac{d}{c} = \lambda\,\frac{R}{c}$ where $\lambda = \frac{m^*}{2\pi}\,\frac{\eta u^2}{\kappa}\,\frac{d}{R}$

then $\frac{c}{R} = \frac{c_0}{R} - \frac{1}{2}\beta\lambda\frac{R}{c}$

or $\left(\frac{c}{R}\right)^2 - \left(\frac{c}{R}\right)\left(\frac{c_0}{R}\right) + \frac{1}{2}\beta\lambda = 0$

$\frac{c}{R} = \frac{c_0}{2R} \pm \sqrt{\left(\frac{c_0}{2R}\right)^2 - \frac{1}{2}\beta\lambda}$

This part not done effectively - marks given for sensible attempts.

No solution if $\frac{c_0}{2R} < \sqrt{\frac{1}{2}\beta\lambda}$ or $\frac{c_0}{R} < \sqrt{2\beta\lambda}$

So critical case $\frac{c_0}{R} = \left(2\beta\,\frac{m^*}{2\pi}\,\frac{\eta u^2}{\kappa}\,\frac{d}{R}\right)^{\frac{1}{2}}$

But as c falls m^* approaches 5.1

$\frac{c_0}{R} \approx \sqrt{\frac{5.1}{\pi}}\left(\beta\,\frac{\eta u^2}{\kappa}\,\frac{d}{R}\right)^{\frac{1}{2}} = 1.27\left(\beta\,\frac{\eta u^2}{\kappa}\,\frac{d}{R}\right)^{\frac{1}{2}}$

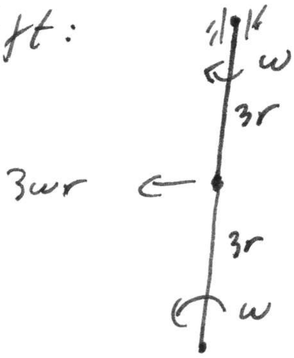
e) $\frac{c_0}{R} = 1.27\left(3\times10^{-5}\left(\frac{0.1\times25}{50}\right)\frac{0.02}{0.2}\right)^{\frac{1}{2}} = 4.9\times10^{-4}$

$\delta\theta = \frac{m^*}{2\pi}\,\frac{\eta u^2}{\kappa}\,\frac{d}{c} = \frac{5.1}{2\pi}\cdot\frac{0.1\times25}{50}\cdot\frac{0.02}{4.9\times10^{-4}\times0.2} = 8\,^\circ C$

As viscosity is rather sensitive to temperature this rise of 8°C may well be significant.

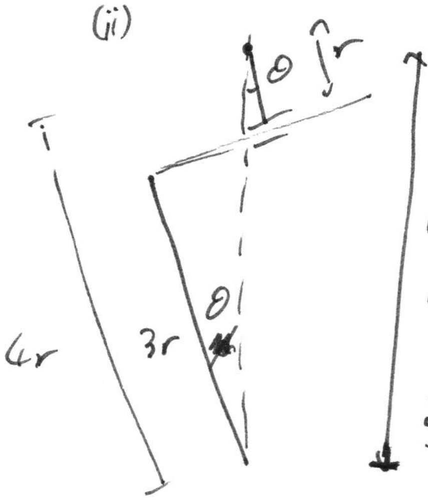
Q2 (i) Minimum lift : acceleration = zero (except as follower leaves base circle)

Maximum lift :



$$\text{accn} = 3r\omega^2 + 3r\omega^2 = 6r\omega^2 \downarrow$$

(ii)



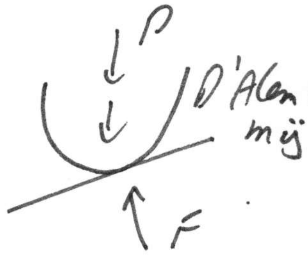
$$y = 4r/\cos\theta$$

$$\dot{y} = \frac{4r}{\cos^2\theta}\sin\theta\,\dot{\theta}$$

$$\ddot{y} = 4r\dot{\theta}^2\left(\frac{1}{\cos^3\theta} + \frac{\sin^2\theta}{\cos^3\theta}\right) \quad [\ddot{\theta} = 0]$$

(ii)

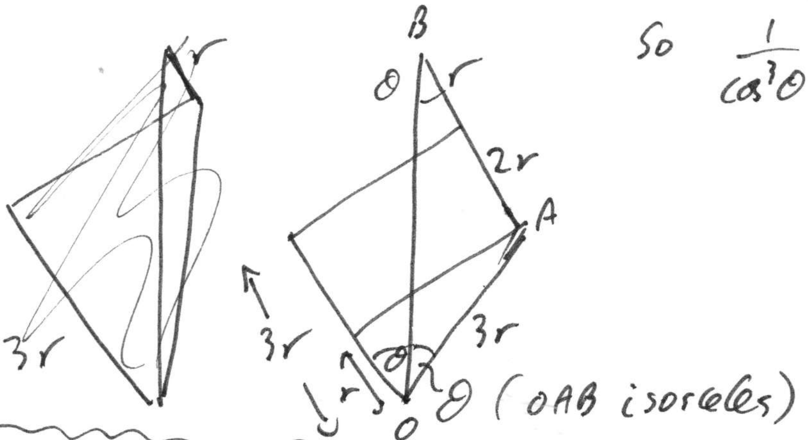
Neglect gravity force



Minimum force corresponds to maximum negative (downwards) acceleration.

For flank contact $\ddot{y}$ is positive for small $\pm\theta$

At limit of flank contact $\cos 2\theta = \frac{1}{3} \Rightarrow \theta = 35.3^\circ$



So $\dfrac{1}{\cos^3\theta} + \dfrac{\sin^2\theta}{\cos^3\theta} > 0$ on flanks.

So critical case is at maximum lift

$P = 6mr\omega^2$

$= 6 \times 0.1 \times 0.01 \times 200^2$

$P = 240\,N$

Most candidates correctly assumed that the critical condition was at the tip.

2 (b)(i) To avoid significant metal-to-metal contact choose $h_{min} = \sigma$

On base circle $\bar{u} = \omega \cdot \frac{3r}{2} = \frac{1}{2} \cdot 200 \times 0.3 = 3 \text{ m/s}$

$w = P = 260\text{ N}$

$\frac{1}{R} = \frac{1}{r} + \frac{1}{3r} \Rightarrow R = 7.5 \times 10^{-3} \text{ m}$ $[1.673 \times 10^{-7}]$

$$h_{min} = 10^{-6} = 2.65 \times 7.5 \times 10^{-3} \times \underbrace{\left(2 \times 2 \times 10^{-8} \times 115 \times 10^{9}\right)^{0.54}}_{[95.04]} \times \left(\frac{3 \times 0.1}{2 \times 115 \times 10^{9} \times 7.5 \times 10^{-3}}\right)^{0.7}$$

$$\times \left(\frac{260}{2 \times 115 \times 10^{9} \times 7.5 \times 10^{-3} \times L}\right)^{-0.13}$$

'x'

$10^{-6} = 2.78 \times 10^{-7} (x)^{-0.13}$

$x^{-0.13} = 3.59$

$\ln x = \ln(3.59)/0.13$

$x = 5.26 \times 10^{-5}$

$L = \frac{260}{2 \times 115 \times 10^{9} \times 7.5 \times 10^{-3} \times 5.26 \times 10^{-5}} = 2.65 \text{ mm}$

Method marks given even without right answer from a

(ii) Calculation not adequate. Need also to consider much more extreme conditions (potentially) with high accelerations on lift phase with smaller tip radius.

3 (a) $\omega_C r_C = \frac{1}{2}(\omega_A r_A + \omega_S r_S)$

$r_C = \frac{1}{2}(r_A + r_S)$

$\therefore \omega_C (r_A + r_S) = \omega_A r_A + \omega_S r_S$

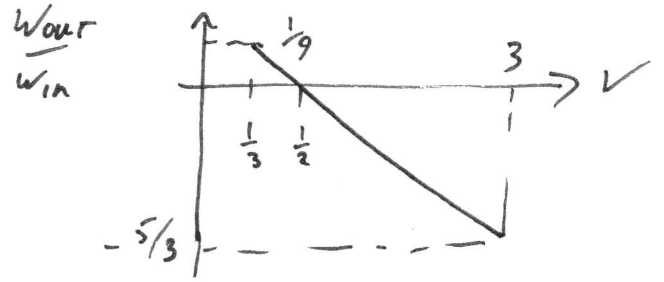
$\omega_C (B+1) = \omega_A B + \omega_S$

$\omega_S = \omega_{in}, \quad \omega_A = -V\omega_{in}, \quad \omega_C = \omega_{out}$

$\Rightarrow \omega_{out}(B+1) = -BV\omega_{in} + \omega_{in} \Rightarrow \frac{\omega_{out}}{\omega_{in}} = \frac{1-BV}{1+B}$

(b) $B = 2 \Rightarrow \frac{\omega_{out}}{\omega_{in}} = \frac{1-2V}{3}$. For $V = \frac{1}{3}$, $\frac{\omega_{out}}{\omega_{in}} = \frac{1}{9}$

For $V = 3$, $\frac{\omega_{out}}{\omega_{in}} = -\frac{5}{3}$



(c) If $BV = 1$, ie $V = \frac{1}{B} = \frac{1}{2}$ then $\frac{\omega_{out}}{\omega_{in}} = 0$

So this is effectively an infinite gear ratio.
A vehicle with this transmission can pull away from stationary without the need for a slipping clutch.

(d) (i) $T_{out} = -1\,Nm$, $\omega_{in} = 1\,rad/s$

$P_{out} = T_{out}\,\omega_{out} = -1 \times \left(\frac{1-2V}{3}\right) = \frac{2}{3}V - 1 = -\frac{\omega_{out}}{\omega_{in}}$

$= -\omega_{out}$ Watts

(for ω_{out} in rad/s)

(ii) Virtual work (Ts, ωs all +ve)

$T_A \omega_A' + T_C \omega_C' + T_S \omega_S' = 0$

Students tended to get bogged down in algebra in this question.

3 (d) (ii) (cont)

Put $\omega_s' = 0 \Rightarrow \omega_A' = \omega_c' \dfrac{B+1}{B} = \dfrac{3}{2}\omega_c'$

$\therefore\ T_A \cdot \dfrac{3}{2}\omega_c' + T_c\omega_c' + 0 = 0$

$T_A = -\dfrac{2}{3}T_c \quad$ but $T_c = -T_{OUT} = 1\ Nm$

$\therefore\ T_A = -\dfrac{2}{3}\ Nm$

Power into annulus $P_A = T_A\omega_A = -\dfrac{2}{3} \times (-V\omega_{in})$

$= \dfrac{2}{3}V$

Hence power into CVT $P_{CVT} = \dfrac{P_A}{\eta} = \dfrac{2V}{3 \times 0.8}$

But $2V = 1 - 3\omega_{OUT}$

$\Rightarrow P_{CVT} = \dfrac{1 - 3\omega_{out}}{2.4}$

(iii) Power into sun $P_s = T_s\omega_s \quad (\omega_s = \omega_{in})$

Let $\omega_A' = 0$, $T_c\omega_c' + T_s\omega_c'(B+1) = 0$

$\Rightarrow T_s = -T_c/3 = -1/3\ Nm$

$P_s = -\dfrac{1}{3}\omega_{in} = -\dfrac{1}{3}$ Watts (Always −ve $P_{in} < P_{CVT}$)

$P_{in} = P_{CVT} + P_s = \dfrac{1}{2.4} - \dfrac{\omega_{out}}{0.8} - \dfrac{1}{3} = \dfrac{1}{12} - 1.25\,\omega_{out}$ Watts

(e)