

4C2 - Designing with Composites

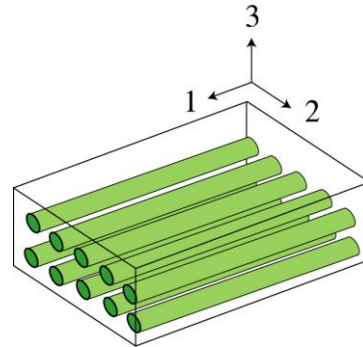
Cribs

Question 1

(a) 4 independent constants $E_1, E_2, G_{12}, \nu_{21}$ because of the following symmetry relationship

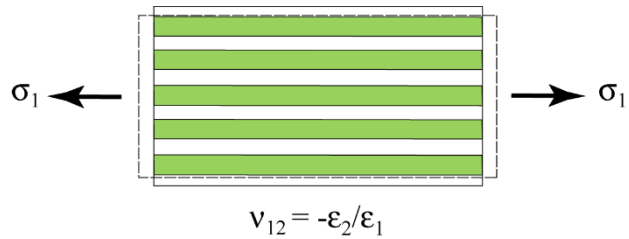
$$\frac{\nu_{12}}{E_1} = \frac{\nu_{21}}{E_2}$$

$$\begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{pmatrix} = \begin{bmatrix} 1/E_1 & -\nu_{12}/E_1 & 0 \\ -\nu_{12}/E_1 & 1/E_2 & 0 \\ 0 & 0 & 1/G_{12} \end{bmatrix} \begin{pmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_{12} \end{pmatrix}$$



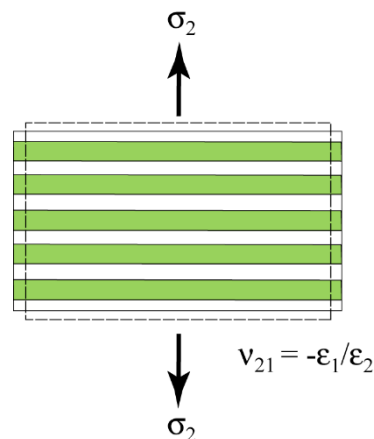
i. In-plane longitudinal tension (or compression)

Apply σ_1 Measure $\varepsilon_1 \Rightarrow E_1 \equiv \sigma_1/\varepsilon_1$
 Measure $\varepsilon_2 \Rightarrow \nu_{12} = -\varepsilon_2/\varepsilon_1$
 ($\gamma_{12} = 0$)



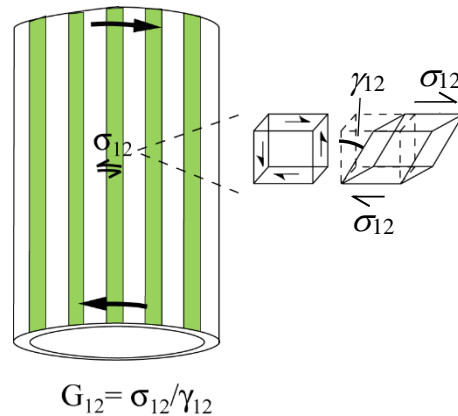
ii. In-plane transverse tension

Apply σ_2 Measure $\varepsilon_2 \Rightarrow E_2 \equiv \sigma_2/\varepsilon_2$
 Measure $\varepsilon_1 \Rightarrow \nu_{21} = -\varepsilon_1/\varepsilon_2$
 Check that $\nu_{12}/E_1 = \nu_{21}/E_2$!



iii. In-plane shear

Apply σ_{12} Measure $\gamma_{12} \Rightarrow G_{12} = \sigma_{12} / \gamma_{12}$
 ($\varepsilon_1 = \varepsilon_2 = 0$)



(b)

$$\frac{\nu_{12}}{E_1} = \frac{\nu_{21}}{E_2} \Rightarrow \nu_{21} \approx 0.02$$

Calculate $[Q]$ in principal material axes (1, 2)

$$Q_{11} = \frac{E_1}{1 - \nu_{12}\nu_{21}} = \frac{138}{1 - 0.3 \times 0.02} = 138.81 \text{ GPa}$$

$$Q_{22} = \frac{E_2}{1 - \nu_{12}\nu_{21}} = \frac{9}{1 - 0.3 \times 0.02} = 9.05 \text{ GPa}$$

$$Q_{12} = \frac{\nu_{12} E_2}{1 - \nu_{12}\nu_{21}} = \frac{0.3 \times 9}{1 - 0.3 \times 0.02} = 2.72 \text{ GPa}$$

$$Q_{66} = G_{12} = 6.9 \text{ GPa} \quad Q_{16} = Q_{26} = 0$$

$$[Q] = \begin{bmatrix} 138.81 & 2.72 & 0 \\ 2.72 & 9.05 & 0 \\ 0 & 0 & 6.9 \end{bmatrix} \text{ GPa}$$

Now, calculate the transformed stiffness matrix $[\bar{Q}]$ in the global x-y axes.

The transformed lamina stiffness matrix $[\bar{Q}]$ for the 0° plies is given by

$$[\bar{Q}] = \begin{bmatrix} 138.81 & 2.72 & 0 \\ 2.72 & 9.05 & 0 \\ 0 & 0 & 6.9 \end{bmatrix} \text{ GPa}$$

The transformed stiffness matrix for the $+60^\circ$ plies is given by

$$\left(\bar{Q}_{11}\right)_{60^\circ} = \left(\bar{Q}_{11}\right)_{-60^\circ} = 138.81 c^4 + 9.05 s^4 + 2(2.72 + 2 \times 6.9) s^2 c^2 = 19.96 \text{ GPa}$$

$$\left(\bar{Q}_{12}\right)_{60^\circ} = \left(\bar{Q}_{12}\right)_{-60^\circ} = (138.81 + 9.05 - 4 \times 6.9) s^2 c^2 + 2.72(c^4 + s^4) = 24.25 \text{ GPa}$$

$$\left(\bar{Q}_{22}\right)_{60^\circ} = \left(\bar{Q}_{22}\right)_{-60^\circ} = 138.81 s^4 + 9.05 c^4 + 2(2.72 + 2 \times 6.9) s^2 c^2 = 84.84 \text{ GPa}$$

$$\left(\bar{Q}_{66}\right)_{60^\circ} = (138.81 + 9.05 - 2 \times 2.72 - 2 \times 6.9) s^2 c^2 + 6.9(s^4 + c^4) = 28.43 \text{ GPa}$$

where $c = \cos 60$, $s = \sin 60$

The shear coupling terms (terms with subscripts 16 and 26) for $+60^\circ$ ply have the opposite sign for the corresponding terms for the -60° ply.

$$\left(\bar{Q}_{16}\right)_{60^\circ} = -\left(\bar{Q}_{16}\right)_{-60^\circ}$$

$$\left(\bar{Q}_{26}\right)_{60^\circ} = -\left(\bar{Q}_{26}\right)_{-60^\circ}$$

Set $t=0.1$ mm for lamina thickness

$$A_{11} = \left[\left(\bar{Q}_{11}\right)_{+60} + \left(\bar{Q}_{11}\right)_0 + \left(\bar{Q}_{11}\right)_{-60} \right] \cdot 8t = 142.99 \text{ MN m}^{-1}$$

$$A_{12} = \left[\left(\bar{Q}_{12}\right)_{+60} + \left(\bar{Q}_{12}\right)_0 + \left(\bar{Q}_{12}\right)_{-60} \right] \cdot 8t = 40.97 \text{ MN m}^{-1}$$

$$A_{22} = \left[\left(\bar{Q}_{22}\right)_{+60} + \left(\bar{Q}_{22}\right)_0 + \left(\bar{Q}_{22}\right)_{-60} \right] \cdot 8t = 142.99 \text{ MN m}^{-1}$$

$$A_{16} = \left[\left(\bar{Q}_{16}\right)_{+60} + \left(\bar{Q}_{16}\right)_0 + \left(\bar{Q}_{16}\right)_{-60} \right] \cdot 8t = 0$$

$$A_{26} = \left[\left(\bar{Q}_{26}\right)_{+60} + \left(\bar{Q}_{26}\right)_0 + \left(\bar{Q}_{26}\right)_{-60} \right] \cdot 8t = 0$$

$$A_{66} = \left[\left(\bar{Q}_{66}\right)_{+60} + \left(\bar{Q}_{66}\right)_0 + \left(\bar{Q}_{66}\right)_{-60} \right] \cdot 8t = 51.01 \text{ MN m}^{-1}$$

$$[A] = \begin{bmatrix} 142.99 & 40.97 & 0 \\ 40.97 & 142.99 & 0 \\ 0 & 0 & 51.01 \end{bmatrix} \text{ MNm}^{-1}$$

Since $A_{16}=A_{26}=0$, the laminate is balanced. This means that the laminate as whole does not exhibit any tensile-shear interactions. Tensile-shear interactions are tensile strains arising from applied shear stresses and visa versa and result in in-plane distortion of the laminate.

Furthermore, because the laminate is quasi-isotropic (the laminae are oriented at the same angle relative to adjacent laminae), $[A]$ has the following form

$$[A] = \begin{bmatrix} A_{11} & A_{12} & 0 \\ A_{12} & A_{11} & 0 \\ 0 & 0 & (A_{11} - A_{12}) / 2 \end{bmatrix}$$

(ii)

$$N_x = \frac{P_x}{2\pi r} = \frac{50 \text{ kN}}{2 \cdot \pi \cdot 45 \text{ mm}} = \frac{50 \cdot 10^3 \text{ N}}{2 \cdot \pi \cdot 45 \cdot 10^{-3} \text{ m}} = 0.1768 \text{ MN m}^{-1}$$

$$N_y = 0$$

$$N_{xy} = \frac{T_{xy}}{2\pi r^2} = \frac{10 \text{ kNm}}{2 \cdot \pi \cdot 45^2 \text{ mm}^2} = \frac{10 \cdot 10^3 \text{ Nm}}{2 \cdot \pi \cdot (45 \cdot 10^{-3} \text{ m})^2} = 0.786 \text{ MN m}^{-1}$$

$$[A] = \begin{bmatrix} 142.99 & 40.97 & 0 \\ 40.97 & 142.99 & 0 \\ 0 & 0 & 51.01 \end{bmatrix} \text{ MNm}^{-1}$$

$$\begin{pmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{pmatrix} = [A]^{-1} \begin{pmatrix} N_x \\ N_y \\ N_{xy} \end{pmatrix} = \begin{bmatrix} 0.0076 & -0.0022 & 0 \\ -0.0022 & 0.0076 & 0 \\ 0 & 0 & 0.0196 \end{bmatrix} \begin{pmatrix} 0.1768 \\ 0 \\ 0.7860 \end{pmatrix} = \begin{pmatrix} 1.35 \\ -0.39 \\ 15.41 \end{pmatrix} \times 10^{-3}$$

(iii) Relevant available manufacturing routes for such a thin tubes are pre-preg lay-up and filament winding. Filament winding is a process suited to automation and suitable to certain components shapes such as tubes. If filament winding is chosen, there are some limitations on the paths that the fibres take over the surface of the component. For example, 0° plies would be difficult to include, perhaps need to be replaced by other hybrid lay-ups (e.g. 10° or 20°). Worth noting that it may be difficult to ensure that fibres cover some parts of the surface or lie in certain orientations. However, filament winding is often used to produce high performance components and is obviously well suited to simple shapes such as tubes.

In terms of design, we need to estimate the thickness of the 0° plies needed to take the axial load due to axial loading and the thickness of the $\pm 60^\circ$ plies needed for the shear load associated with the torque and shear flow.

To maximise torsional stiffness of the tube, we could perhaps change the $\pm 60^\circ$ plies to $\pm 45^\circ$ to maximise G_{xy} .

To maximise E_x we need to include 0° plies but they are prone to splitting. Important to ensure there are $\pm 60^\circ$ or $\pm 45^\circ$ plies. Also it is worth considering having the latter outside to improve impact resistance (see also below). 90° plies will be hard to consolidate in a tube.

Impact/damage assessment: The shaft needs to be made to absorb impacts. A protective woven Kevlar or GFRP cloth should be added to protect against impact.

Prototyping: Testing of coupons and/or a small section of shaft is needed to confirm the axial and torsional stiffness of the tube but also fatigue behaviour, the impact of ageing and environmental conditions and features such as joints.

Costs analysis needs to be carried out.


A sophisticated failure analysis should be carried out.

Other considerations include: Whirling. To avoid this we need to ensure that the resonant frequency of the fundamental mode is above the operating frequency of the shaft.

2 (a)

- need to establish **baseline properties** using coupon tests

←  → stiffness and strength

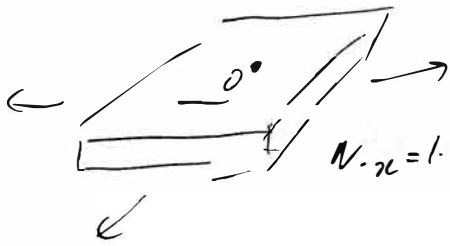
←  → off-axis for shear

 squat specimen for compression

- different ply layups will have different failure modes. For example this will be affected by ply blocking
- **manufacturing details** or environmental conditions (e.g. hot wet) may be critical
- application to design not straightforward for strength.
- need to use **tests** to fit appropriate failure modes, which may require appropriate bi-axial testing
- **failure models** can then be used in structural analysis to predict local failure in the structure
- **local features** need to be included, for example ply drops, edges, holds, which will cause a knockdown in strength

$$2(b) \quad Q_0 = \begin{pmatrix} 139 & 2.7 & 0 \\ 2.7 & 9 & 0 \\ 0 & 0 & 6.9 \end{pmatrix}, \quad Q_{90} = \begin{pmatrix} 9 & 2.7 & 0 \\ 2.7 & 139 & 0 \\ 0 & 0 & 6.9 \end{pmatrix} \text{ GPa}$$

$$\rightarrow N_y = 0.5 \text{ MN m}^{-1}$$



No shear, only consider direct stresses

$$N_x = 1.5 \text{ MN m}^{-1} \quad t = \text{ply thickness}$$

$$[A] = 2nt \begin{pmatrix} 2 \times 139 + 9 & 3 \times 2.7 \\ 3 \times 2.7 & 139 + 2 \times 9 \end{pmatrix} = 2nt \begin{pmatrix} 287 & 8.1 \\ 8.1 & 157 \end{pmatrix} \text{ GPa}$$

$$\epsilon = A^{-1} N = \frac{1}{2nt} \begin{pmatrix} 0.00348 & -0.000180 \\ -0.000180 & 0.00638 \end{pmatrix} \begin{pmatrix} 1.5 \\ 0.5 \end{pmatrix} = \frac{1}{2nt} \begin{pmatrix} 5140 \\ 2920 \end{pmatrix} \times 10^{-9} \text{ m}$$

$\times 10^{-9} \text{ N m}^{-2} \quad \times 10^6 \text{ N m}^{-1}$



$$\sigma = Q_0 \epsilon = \begin{pmatrix} 139 & 2.7 \\ 2.7 & 9 \end{pmatrix} \begin{pmatrix} 5140 \\ 2920 \end{pmatrix} \frac{1}{2nt} = \begin{pmatrix} 7.22 \times 10^5 \\ 0.60 \times 10^5 \end{pmatrix} \frac{\text{N m}^{-1}}{2nt}$$

$\times 10^9 \text{ N m}^{-2} \quad \times 10^{-9} \text{ m} \quad \leftarrow \epsilon_s \text{ reversed}$

* critical



$$\sigma = Q_{90} \epsilon = \begin{pmatrix} 9 & 2.7 \\ 2.7 & 139 \end{pmatrix} \begin{pmatrix} 2920 \\ 5140 \end{pmatrix} \frac{1}{2nt} = \begin{pmatrix} 4.2 \times 10^5 \\ 0.56 \times 10^5 \end{pmatrix} \frac{\text{N m}^{-1}}{2nt}$$

Critical case for axial failure in 0° ply ($\sigma_f = 1468 \text{ MPa}$)

$$n = 7.22 \times 10^5 / (2 \times 0.125 \times 10^{-3} \times 1468 \times 10^6) = 1.99$$

$t \uparrow$

Critical case for transverse failure in 90° ply ($\sigma_f = 48.3 \text{ MPa}$)

$$n = 0.56 \times 10^5 / (2 \times 0.125 \times 10^{-3} \times 48.3 \times 10^6) = 4.67$$

So choose bigger value of n $4.67 \Rightarrow n = 5$, $6n = 30$ plies

2(c) If we neglect the 90° ply stiffness, to get simultaneous failure in both directions at the same strain we would want 3:1 $0^\circ:90^\circ$ plies to match the 3:1 loading. (This also reflects poisson effects.)

So increasing the proportion of 0° plies from $\frac{2}{3}$ to $\frac{3}{4}$ will approximately reduce the stresses in the 90° plies by the same factor of $\frac{2/3}{3/4} = \frac{8}{9}$

So we would need approximately $\frac{8}{9} \times 6 \times 4.47 = 24$ plies.

This 24 ply laminate can be achieved with 18 0° s and 6 90° s to give the right mix.

[In fact laminate calculations show that this laminate fails with $N_x = 1.6 \text{ MN/m}$, $N_y = 0.47 \text{ MN/m}$.]

3(a)

Why is it important?

- matrix element in composite tends to be brittle
- fatigue failure is often important so the role of crack initiation and growth is important

Challenges

- the microstructure creates various mechanisms of failure and energy absorption depending on the details of the architecture and layup, including fibre bridging, pull-out and debonding
- large scale bridging means that linear elastic fracture mechanics is invalid for crack initiation and growth of small cracks

Modelling

- needs sophisticated material and geometric modelling making it difficult to implement in design
- simple models can capture laminate toughness, with the need to take into account ply layup and mode mixity

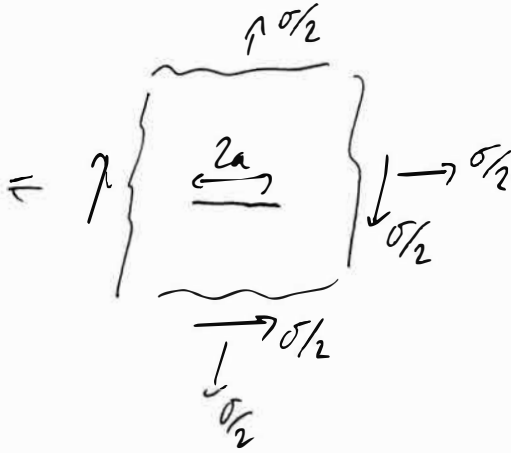
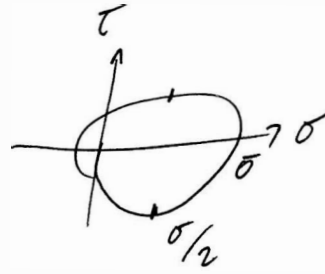
Testing

- a range of specimen geometries are needed to measure toughness, including double cantilever beam testing and impact testing

Design

- a lot depends on design rules established semi-empirically (e.g. knockdowns at features and ply drops)
- testing of features and sub-components with typical laminates is important

3(b)

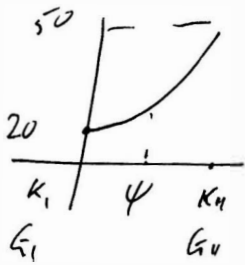


$$K_I = \frac{\sigma}{2} \sqrt{\pi a}$$

$$K_{II} = \frac{\sigma}{2} \sqrt{\pi a}$$

$$\left. \begin{aligned} G_I E_A' &= K_I^2 \\ G_{II} E_B' &= K_{II}^2 \end{aligned} \right\} \text{Crack running in 1 direction}$$

$$G = G_I + G_{II}$$



$$\psi = 65^\circ \text{ as } K_I = K_{II}$$

Assume quadratic variation of G with ψ

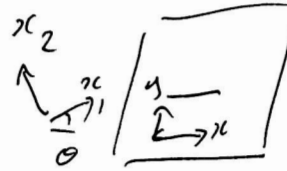
$$\text{so } G_c \approx 20 + \frac{30}{4} = 27.5 \text{ kJ/m}^2$$

$$\text{For failure } G = G_c = \frac{K_I^2}{E_A'} + \frac{K_{II}^2}{E_B'} = \left(\frac{1}{E_A'} + \frac{1}{E_B'} \right) \frac{\sigma^2}{4} \pi a$$

$$\sigma^2 = \frac{4 G_c}{\pi a} \frac{E_A' E_B'}{E_A' + E_B'}$$

3 (b) cont To find E'_A and E'_B we can use the data sheet formulae but first need to find the laminate compliance in the axis of the notch.

Following the datasheet we want to rotate from x_1, x_2 to x, y so here $\theta = 45^\circ$



$$c^2 = s^2 = \frac{1}{2} ; c^4 = s^4 = c^2 s^2 = \frac{1}{4}$$

$$S_{11} = 0.02, S_{12} = -0.01, S_{22} = 0.04, S_{66} = 0.05$$

$$\bar{S}_{11} = S_{11}c^4 + S_{22}s^4 + (2S_{12} + S_{66})c^2s^2 = \frac{1}{4}(0.02 + 0.04 - 0.02 + 0.05) = 0.0225 \text{ GPa}$$

$$\bar{S}_{22} = \bar{S}_{11}$$

$$\bar{S}_{12} = S_{12}(c^4 + s^4) + (S_{11} + S_{22} - S_{66})sc^2 = \frac{-0.01}{2} + (0.02 + 0.04 - 0.05)\frac{1}{4} = -0.0025 \text{ GPa}$$

$$\begin{aligned} \bar{S}_{66} &= (4S_{11} + 4S_{22} - 8S_{12} - 2S_{66})sc^2 + S_{66}(c^4 + s^4) \\ &= (0.08 + 0.16 + 0.08 - 0.1)\frac{1}{4} + 0.05 \times \frac{1}{2} = 0.08 \text{ GPa} \end{aligned}$$

$$\begin{aligned} \frac{1}{E'_A} &= \left(\frac{S_{11} S_{22}}{2} \right)^{\frac{1}{2}} \left(\left(\frac{S_{12}}{S_{11}} \right)^2 \left(1 + \frac{2S_{12} + S_{66}}{2\sqrt{S_{11} S_{22}}} \right) \right)^{\frac{1}{2}} \\ &= \frac{0.0225}{\sqrt{2}} \left(1 + \frac{2 \times (-0.0025 + 0.08)}{2 \times 0.0225} \right)^{\frac{1}{2}} = 0.026 \text{ GPa} \end{aligned}$$

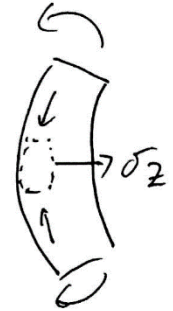
$$E'_A = E'_B = 38.5 \text{ GPa}$$

$$\sigma^2 = 27.5 \times 10^3 \frac{\text{Nm}^3}{\text{m}^2} \times \frac{4}{\pi \times 25 \times 10^{-3} \text{m}} \times \frac{38.5 \times 10^9}{2} \frac{\text{Nm}^{-2}}{\text{m}^{-2}}$$

$$\Rightarrow \underline{\underline{\sigma = 160 \text{ MPa}}}$$

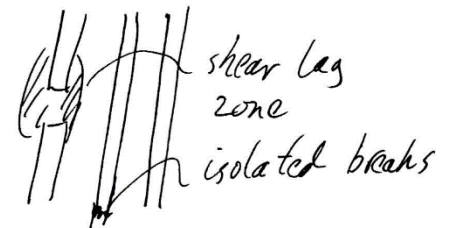
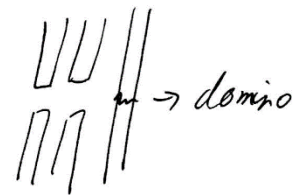
3(c)

(i) For thin laminates in-plane stresses due to bending are large because of the relatively small second moment of area of the laminate. For thicker laminates this bending stress can be resisted more easily but the through-thickness stress generated via the illustrated mechanism can still build up and becomes dominant.



(ii) In general fatigue is a problem for many structures. For composites, initiation of failure can occur particularly at joints due to stress concentrations, with complex three-dimensional geometries and stresses leading to potential for delamination between layers and through-thickness cracking.

(iii) The key here is the way that cracks propagate from one fibre break to the next. This can either be in a domino fashion or with isolated breaks. Stresses build up again away from a fibre break associated with the shear lag zone. With an increase in the variation in fibre strength, it is less likely that the adjacent fibre next to a given fibre break will fail, instead fibres will break in an isolated manner. This switch from domino to isolated failure can lead to an increase in tensile strength.



4 (a)

Material and layup

- for a moving application weight will be important
- toughness/robustness is likely to be an issue
- probably going to be a relatively thin walled structure
- >> woven GFRP or cheaper CFRP could be a good choice

Structural design

- like many lightweight structures some distribution of structural function will be helpful, for example having a space frame with lightweight panels
- alternatively perhaps the panels could be stiff and strong enough to be the load carrying members (e.g. sandwich structures)
- attachment points (e.g. wheels) will need reinforcing

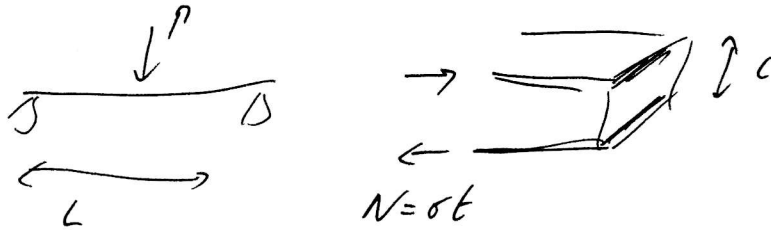
Manufacture

- relatively complex shape
- manufacturing costs will depend on production rate, probably relatively modest rate
- keeping cost down is probably more important than high mechanical performance
- >> perhaps a simple forming moulding process, e.g. vacuum injection moulding

Other

- aesthetics could be important, need to have a good finish
- impact loading - will need careful and realistic testing
- joining - this could be critical and will need testing and prototyping
- environmental - check weathering effects
- sustainability - repair, recycling

4 (b) (i) Due to alternate load cases need to include equal proportions of 0s and 90s. Assume we can manage without GFS, as splitting should be avoided with thin cross-ply layup.



$$\text{Max bending moment / unit length} = \frac{P}{L} \times \frac{L}{2} = \frac{P}{2}$$

$$\text{Bending moment due to stresses} = 2 \times N \times \frac{c}{2} = Nc$$

$$\Rightarrow N = P/2c$$

Use ϵ allowables. Assume only 0 plies carry load. t_0 = thickness of 0 plies

$$\epsilon = \frac{N/t_0}{E_1} \Rightarrow t_0 = \frac{N}{E_1 \epsilon}, \quad t_{90} = t_0, \quad t = 2t_0$$

$$\text{Mass}_{80} = \rho \times 2t \times L^2 = 4\rho t_0 L^2 = \frac{4\rho N L^2}{E_1 \epsilon} \quad \text{putting } \epsilon = \epsilon_f$$

CFRP GFRP Kevlar

Choose	E_1	140	45	80	\rightarrow choose CFRP. 50% of 90° plies, 50% of 0° plies $t = 2t_0 = \frac{P}{c E_1 \epsilon_f} = \frac{4 \times 10^3}{10^{-2} \cdot 140 \cdot 10^9 \cdot 0.004} = 0.72 \text{ mm}$
worst case	ϵ_f	0.6	0.3	0.1	
	ρ	1500	1900	1600	
	$\frac{E_1 \epsilon_f}{\rho}$	0.037	0.047	0.006	

(ii)	C+P	300	105	125
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	$E_1 \epsilon_f$	1.2×10^{-4}	2×10^{-5}	4.8×10^{-6}
	$\rho(C+P)$			

	$\frac{E_1}{\rho(C+P)}$	3.1×10^{-4}	2.2×10^{-4}	4.6×10^{-4}
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Cost profit = mass \times (C+P) \leftarrow premium/mass

Minimise cost profit \Rightarrow maximise $\frac{E_1 \epsilon_f}{\rho(C+P)}$

\rightarrow choose CFRP still

Same layup and thickness

4 (b) (iii)

The material performance index for bending deflection is $E_1 / \rho(L+P)$

Adding in an extra line shows that Kevlar is the best choice with this constraint.

Need to draw up a table to compare masses.

$$N = \frac{P}{2c}$$

$$Mass_{\sigma} = \frac{4PNL^2}{E_1 E_f} = \frac{P}{E_1 E_f} \times \frac{4 \times 10^3 \times 0.8^2}{2 \times 10^{-2}}$$

Deflection $\delta = \frac{WL^3}{6EI}$

Working per unit depth

$$W = \frac{P}{L}, \quad I = 2t\left(\frac{c}{2}\right)^2 = tc^2/2, \quad t = 2t_0, \quad E = E_1/2$$

laminated E and t

$$\Rightarrow \delta = \frac{PL^2}{12tc^2E_1}, \quad t = \frac{PL^2}{12c^2E_1\delta}, \quad mass_{\delta} = 2\rho tL^2 = \frac{PL^4}{6c^2E_1\delta}$$

$$= \frac{P}{E_1} \times \frac{4 \times 10^3 \times 0.8^4}{6 \times 10^{-2} \times 10^{-2} \times \delta}$$

convert E_f from %

CFRP GFRP Kevlar

$\frac{P}{E_1 E_f}$	2680	14100	17,500	$\times 10^{-9}$
m_{σ}	1370	7200	8900	$\times 10^{-3}$
$\frac{P}{E_1}$	10.7	42	17.5	$\times 10^{-9}$
m_{δ}	2920	11,600	4,800	$\times 10^{-3}$

4.6 108 71

→ CFRP is still the best choice. Same so: so lay up.

$$t = \frac{PL^2}{12c^2E_1\delta} = \frac{4 \times 10^3 \times 0.8^2}{12 \times 10^{-2} \times 160 \times 10 \times 10^{-2}} = 1.5 \text{ mm}$$

$\frac{m_{\delta}}{m_{\sigma}}$

Examiner's Comments

Question 1: Elastic Deformation

Part (a) was answered reasonably well, marks were lost mainly because of lack of details. Part b(i) was answered well, albeit not always using correct units for the extensional stiffness matrix. In part b(ii), several candidates made numerical errors in estimating the strains. Part b(iii) was answered poorly, the majority of candidates focused on manufacturing considerations, with only a few candidates discussing layup optimisation and other considerations.

Question 2: Laminate Strength

In part (a), candidates lost marks for failing to discuss the role of testing in design. In Part b(i), a lot of candidates made numerical errors in estimating the laminate stiffness matrix, strains and associated stresses. Note the advantage of substituting in values at the end. Part b(ii) wasn't answered well. Some candidates used strain allowables to estimate the laminate thickness and were appropriately credited with marks.

Question 3: Crack Growth

Part (a) wasn't answered well because several candidates focused on discussing testing methods and didn't address the other parts of the question. Parts b and c were answered reasonably well. In part b, several candidates assumed that 1 and 2 directions were aligned with the 45° direction and were appropriately credited with marks.

Question 4: Practical Design

Part (a) was answered reasonably well. Parts b(i-iii) were answered less well as candidates seemed to run out of time. Only a few candidates were able to complete the merit index calculations and only a few candidates commented on a possible layup.

Athina E. Markaki (Principal Assessor)