4C2 crib 2024

Question 1

(a) To estimate the tensile-shear distortions we can use one of the tensile-shear interaction terms, $\bar{S}_{16}$ or $\bar{S}_{26}$. First we need to calculate the axial and transverse stiffness of the lamina:

$$E_1 = \left[fE_f + (1-f)E_m\right] = 0.65(350) + 0.35(4) = 228.9 \text{ GPa}$$

$$E_2 = \left[\frac{f}{E_f} + \frac{(1-f)}{E_m}\right]^{-1} = \left[\frac{0.65}{350} + \frac{0.35}{4}\right]^{-1} \approx 11.19 \text{ GPa}$$

From Datasheet: $\bar{S}_{16} = (2S_{11} - 2S_{12} - S_{66})c^3s - (2S_{22} - 2S_{12} - S_{66})cs^3$

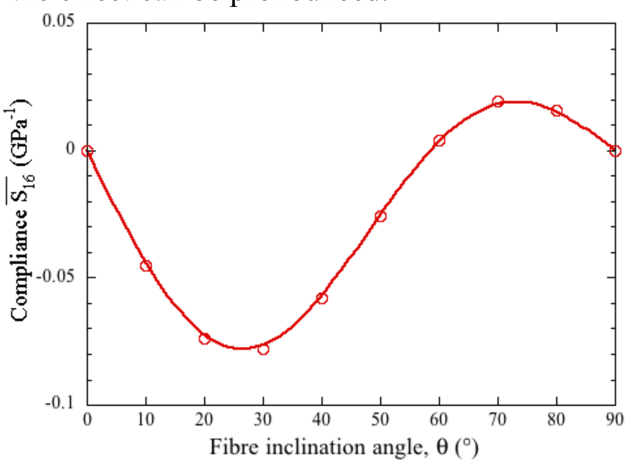
$$S_{11} = \frac{1}{E_1} = \frac{1}{228.9} \text{ GPa}^{-1}$$

$$S_{22} = \frac{1}{E_2} = \frac{1}{11.19} \text{ GPa}^{-1}$$

$$S_{12} = -\frac{\nu_{12}}{E_1} = -\frac{0.3}{228.9} \text{GPa}^{-1}$$

$$S_{66} = \frac{1}{G_{12}} = \frac{1}{3.5} \text{ GPa}^{-1}$$

The figure below shows $\bar{S}_{16}$ as a function of the fibre inclination angle. It can be seen that a large shear strain is predicted at 30°. $\bar{S}_{16}$ is zero at 0° and 90° as expected, but at intermediate angles the effect can be pronounced.



[5 points]

(b) (i)

$$\frac{\nu_{12}}{E_1} = \frac{\nu_{21}}{E_2} \Rightarrow \frac{0.3}{228.9} = \frac{\nu_{21}}{11.19} \Rightarrow \nu_{21} \approx 0.015$$

$$1 - \nu_{12}\nu_{21} = 1 - (0.015 \times 0.3) \approx 0.9955$$

Calculate $[Q]$ in principal material axes (1, 2)

$$Q_{11} = \frac{E_1}{1 - \nu_{12}\nu_{21}} = \frac{228.9}{0.9955} = 229.91 \text{ GPa}$$

$$Q_{22} = \frac{E_2}{1 - \nu_{12}\nu_{21}} = \frac{11.19}{0.9955} = 11.24 \text{ GPa}$$

$$Q_{12} = \frac{\nu_{12}E_2}{1-\nu_{12}\nu_{21}} = \frac{0.3 \times 11.2}{0.9955} = 3.37 \text{ GPa}$$
$$Q_{66} = G_{12} = 3.5 \text{ GPa} \qquad Q_{16} = Q_{26} = 0$$
$$[Q] = \begin{bmatrix} 229.91 & 3.37 & 0 \\ 3.37 & 11.24 & 0 \\ 0 & 0 & 3.50 \end{bmatrix} \text{GPa}$$

[3 points]

(ii) To estimate the laminate extensional stiffness [A], first we need to calculate the transformed stiffness matrix $[\overline{Q}]$ of the three plies in the global x-y axes.

The transformed lamina stiffness matrix $[\overline{Q}]$ for the 0° plies is given by

$$[Q] = \begin{bmatrix} 229.91 & 3.37 & 0 \\ 3.37 & 11.24 & 0 \\ 0 & 0 & 3.50 \end{bmatrix} \text{GPa}$$

The transformed stiffness matrix for the +60° plies is given by

$$(\overline{Q}_{11})_{60^\circ} = 229.91\, c^4 + 11.24\, s^4 + 2(3.37 + 2 \times 3.50)s^2c^2 = 24.58 \text{ GPa}$$
$$(\overline{Q}_{12})_{60^\circ} = (229.91 + 11.24 - 4 \times 3.50)s^2c^2 + 3.37(c^4 + s^4) = 44.70 \text{ GPa}$$
$$(\overline{Q}_{22})_{60^\circ} = 229.91\, s^4 + 11.24\, c^4 + 2(3.37 + 2 \times 3.50)s^2c^2 = 133.92 \text{ GPa}$$
$$(\overline{Q}_{16})_{60^\circ} = (229.91 - 3.37 - 2 \times 3.50)c^3s - (11.24 - 3.37 - 2 \times 3.50)cs^3$$
$$= 23.48 \text{ GPa}$$
$$(\overline{Q}_{26})_{60^\circ} = (229.91 - 3.37 - 2 \times 3.50)cs^3 - (11.24 - 3.37 - 2 \times 3.50)c^3s$$
$$= 71.20 \text{ GPa}$$
$$(\overline{Q}_{66})_{60^\circ} = (229.91 + 11.24 - 2 \times 3.37 - 2 \times 3.50)s^2c^2 + 3.50(s^4 + c^4)$$
$$= 44.83 \text{ GPa}$$

where $c = cos\, 60^\circ = {}^1\!/_2, s = sin\, 60^\circ = {}^{\sqrt{3}}\!/_2$

$$[\overline{Q}]_{60^\circ} = \begin{bmatrix} 24.58 & 44.70 & 23.48 \\ 44.70 & 133.92 & 71.20 \\ 23.48 & 71.20 & 44.83 \end{bmatrix} \text{GPa}$$

The transformed lamina stiffness matrix $[\overline{Q}]$ for the -60° plies is given by

$$[\overline{Q}]_{-60^\circ} = \begin{bmatrix} 24.58 & 44.70 & -23.48 \\ 44.70 & 133.92 & -71.20 \\ -23.48 & -71.20 & 44.83 \end{bmatrix} \text{GPa}$$

The shear coupling terms (terms with subscripts 16 and 26) for the +60° plies have the opposite sign for the corresponding terms for the -60° plies.

$$(\overline{Q}_{16})_{60^\circ} = -(\overline{Q}_{16})_{-60^\circ}$$
$$(\overline{Q}_{26})_{60^\circ} = -(\overline{Q}_{26})_{-60^\circ}$$

Set t (=0.25 $\times 10^{-3}$ m) for lamina thickness

$$A_{11} = \left[(\overline{Q}_{11})_0 + (\overline{Q}_{11})_{+60} + (\overline{Q}_{11})_{-60}\right] \cdot 2 \cdot 12 \cdot t = [229.91 + 24.58 + 24.58] \cdot 24t$$
$$= 1674.45 \text{ MN m}^{-1}$$

$$A_{12} = \left[(\overline{Q}_{12})_0 + (\overline{Q}_{12})_{+60} + (\overline{Q}_{12})_{-60}\right] \cdot 2 \cdot 12 \cdot t = [3.37 + 44.70 + 44.70] \cdot 24t$$
$$= 556.62 \text{ MN m}^{-1}$$

$$A_{22} = \left[(\overline{Q}_{22})_0 + (\overline{Q}_{22})_{+60} + (\overline{Q}_{22})_{-60}\right] \cdot 2 \cdot 12 \cdot t$$
$$= [11.24 + 133.92 + 133.92] \cdot 24t = 1674.45 \text{ MN m}^{-1} = A11$$

$$A_{16} = \left[(\overline{Q}_{16})_0 + (\overline{Q}_{16})_{+60} + (\overline{Q}_{16})_{-60}\right] \cdot 2 \cdot 12 \cdot t = [0 + 23.48 - 23.48] \cdot 24t = 0$$

$$A_{26} = \left[(\overline{Q}_{26})_0 + (\overline{Q}_{26})_{+60} + (\overline{Q}_{26})_{-60}\right] \cdot 2 \cdot 12 \cdot t = [0 + 71.20 - 71.20] \cdot 24t = 0$$

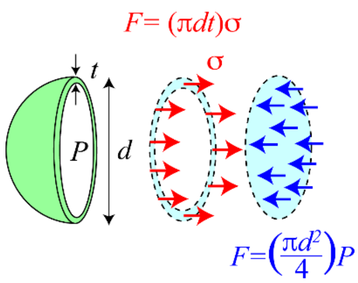
$$A_{66} = \left[(\overline{Q}_{66})_0 + (\overline{Q}_{66})_{+60} + (\overline{Q}_{66})_{-60}\right] \cdot 2 \cdot 12 \cdot t = [3.50 + 44.83 + 44.83] \cdot 24t$$
$$= 558.92 \text{ MN m}^{-1}$$

$$[A] = \begin{bmatrix} 1674.45 & 556.62 & 0 \\ 556.62 & 1674.45 & 0 \\ 0 & 0 & 558.92 \end{bmatrix} \text{MNm}^{-1}$$

Since $A_{16} = A_{26} = 0$, the laminate is balanced. This means that the laminate as whole does not exhibit any tensile-shear interactions which result in in-plane distortion of the laminate, i.e. a normal strain does not induce a shear stress resultant. Similarly a shear strain does not induce a direct stress resultant. **[6 points]**

(iii)

$$\begin{pmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{pmatrix} = [A]^{-1} \begin{pmatrix} N_x \\ N_y \\ N_{xy} \end{pmatrix}$$



$$\frac{P\pi d^2}{4} = (\pi d t)\sigma = \pi d N \Rightarrow N_x = N_y = \frac{Pd}{4}$$

$$\therefore N_x = N_y = 7.5 \text{ MN m}^{-1} \; (d = 3 \text{ m}, P = 10 \text{ MPa})$$

$$N_{xy} = 0$$

$$\begin{pmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{pmatrix} = [A]^{-1} \begin{pmatrix} N_x \\ N_y \\ 0 \end{pmatrix}$$

$$[A] = \begin{bmatrix} 1674.45 & 556.62 & 0 \\ 556.62 & 1674.45 & 0 \\ 0 & 0 & 558.92 \end{bmatrix} \text{MNm}^{-1}$$

$$\begin{pmatrix} \varepsilon_x \\ \varepsilon_y \end{pmatrix} = [A]^{-1} \begin{pmatrix} N_x \\ N_y \end{pmatrix} = \frac{1}{A_{11}A_{22} - A_{12}A_{21}} \begin{bmatrix} A_{22} & -A_{12} \\ -A_{21} & A_{11} \end{bmatrix} \begin{pmatrix} N_x \\ N_y \end{pmatrix}$$

$$= \frac{1}{(1674.45)^2 - (556.62)^2} \begin{bmatrix} 1674.45 & -556.62 \\ -556.62 & 1674.45 \end{bmatrix} \begin{pmatrix} 7.5 \\ 7.5 \end{pmatrix} =$$

$$\varepsilon_x = \varepsilon_y = \frac{1}{(1674.45)^2 - (556.62)^2} [1674.45 - 556.62] \times 7.5 \approx 3.36 \times 10^{-3}$$

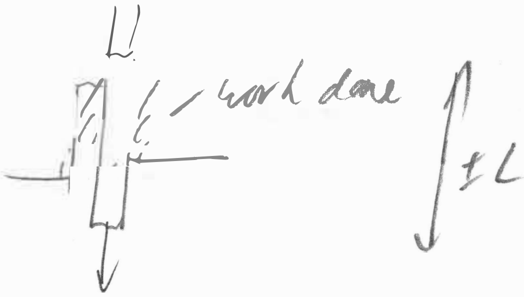
Vessel expands equally in all directions (i.e. remains a sphere).

$$3.36 \times 10^{-3} = \frac{\Delta d}{d_o} = \frac{\Delta d}{3} \quad \therefore \Delta d \approx 0.0108 \text{ m} = 10.8 \text{ mm}$$

[6 points]

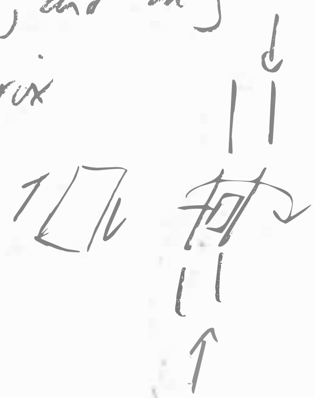
2 (a)(i) The key here is that the principal toughening mode is fibre pull out.

Work done is given by the pull-out force. But shear lag theory dictates that fibres tend to break at random locations around $\pm L$ from the nominal crack line. This distance L increases with a fall in matrix shear stress, giving the increase in toughness



[2 points]

(ii) This is due to the mechanism of plastic microbuckling. Waviness in the fibres causes an induced shear stress in the matrix, and only when the shear stress exceeds the matrix shear strength do the fibres rotate



[2 points]

2(iii) Pultruded sections of GFRP can be made relatively cheaply.

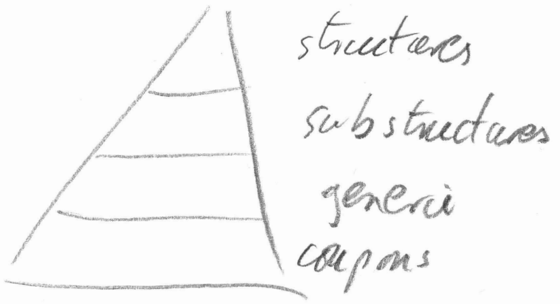
Variations in ply lay-up can give different properties for example in the flange and web to give good structural properties.

Light weighting could be useful, for example in transport, construction or to reduce support structure costs

Good weathering properties could be useful.

[2 points]

(iv) The aerospace industry is dominated by regulation. Especially the testing pyramid is a key requirement

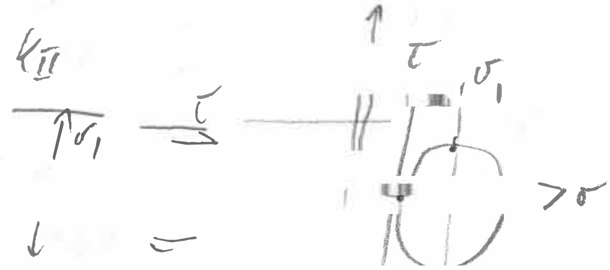


Small changes at the bottom in terms of say materials need to be fed right up; very costly and time consuming.

Sports goods are not regulated in the same way so the level of testing is much reduced.

[2 points]

2(b) First find K_I, K_{II}



$K_I = \sigma_1 \sqrt{\pi a}$

$K_{II} = \tau \sqrt{\pi a}$

where 2a is crack length

where $\sigma_1 = \sigma/2$, $\tau = \sigma/2$

Note that $K_I = K_{II}$, $\psi = \tan^{-1}(1) = \pi/4$

$G_c = G_{Ic} + (G_{IIc} - G_{Ic})\frac{1}{4} = 5 + \frac{1}{4}5 = 6.25\ kJ/m^2$

Now need to find $G = G_I + G_{II}$

Use expressions $G_I = K_I^2 / E_A'$

$G_{II} = K_{II}^2 / E_B'$

Note $\frac{E_B'}{E_A'} = \left(\frac{S_{22}}{S_{11}}\right)^{\frac{1}{2}} = \left(\frac{E_1}{E_2}\right)^{\frac{1}{2}}$

$E_1 = 39$ GPa
$E_2 = 8.3$ GPa
$\nu_{12} = 0.26$
$G_{12} = 4.1$ GPa

$S_{11} = 1/E_1$

$S_{12} = -\nu_{12}/E_1 = -\nu_{21}/E_2$

$S_{22} = 1/E_2$

$S_{66} = 1/G_{12}$

$$\frac{1}{E_A'} = \left(\frac{S_{11}S_{22}}{2}\right)^{\frac{1}{2}}\left(\left(\frac{S_{22}}{S_{11}}\right)^{\frac{1}{2}}\left(1 + \frac{2S_{12} + S_{66}}{2\sqrt{S_{11}S_{22}}}\right)\right)^{\frac{1}{2}}$$

In GPa

$$= \underbrace{\left(\frac{1}{2.39.8.3}\right)^{\frac{1}{2}}}_{0.0393}\left(\underbrace{\left(\frac{39}{8.3}\right)^{\frac{1}{2}}}_{2.17}\underbrace{\left(1 + \frac{2\left(\frac{-0.26}{39}\right) + \frac{1}{4.1}}{2\sqrt{\frac{1}{39\times 8.3}}}\right)}_{3.07}\right)^{\frac{1}{2}} = 0.101 \frac{1}{GPa}$$

$E_A' = 9.86$ GPa $\quad E_B' = 21.4$ GPa

2 (b) So $G = \frac{K_I^2}{E_A'} + \frac{K_{II}^2}{E_B'} = \left[\frac{\sigma}{2}\sqrt{\pi\left(\frac{0.1}{2}\right)}\right]^2 \left(\frac{1}{98.6} + \frac{1}{21.4}\right) \frac{m}{GPa}$

$a = 100\,mm/2$

Put $G = G_C = 6250 \frac{Nm}{m^2} = \frac{\sigma^2}{4} \frac{0.1\pi}{2} (0.148) \frac{m}{GPa}$

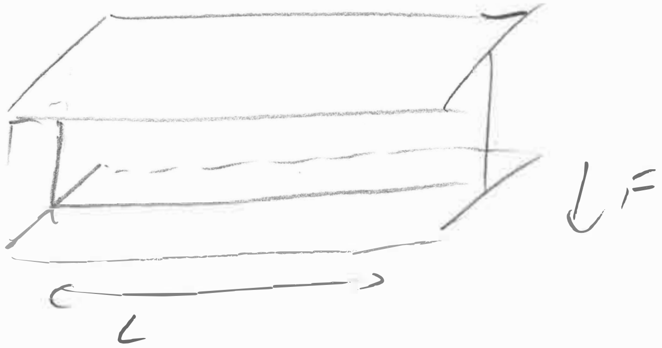
$\sigma^2 = \frac{4}{0.05\pi \cdot 0.148} \cdot 6250 \frac{N}{m} \cdot \times 10^9 \frac{Nm^{-2}}{m}$

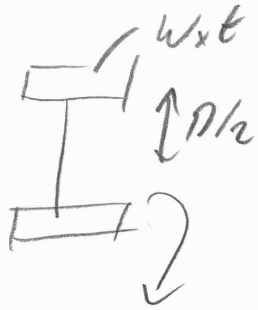
$= 1.08 \times 10^{15}\, N^2 m^{-4}$

$\sigma = 33\ MPa$

[12 points]

3 (a)





Neglecting the contribution of the web, $I = (Wt)\left(\frac{D}{2}\right)^2 \times 2$

$= \frac{WtD^2}{2}$ **

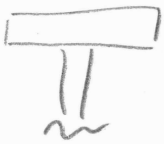
max at root

(i) $\frac{\sigma}{y} = \frac{M}{I} \Rightarrow \quad \sigma = \frac{FL \times D/2}{WtD^2/2} = \frac{FL}{WtD}$

$N_x = \sigma t = \frac{FL}{WD}$ (Nm^{-1}) ** $\frac{FL}{WD + D^2/6}$ including web

[2 points]

(ii)



$$q = \frac{S A_c \bar{y}}{I}$$

Assume $\bar{y} = D/2$, again neglecting web (could be a bit inaccurate)

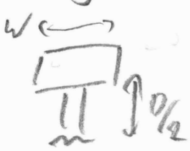
$A_c = Wt + D/2\, t$

$S = F$

$I = \frac{WtD^2}{2}$

$$N_{xy} = q = \frac{F\cancel{t}\left(W + \frac{D}{2}\right)D/\cancel{2}}{W\cancel{t}D^{\cancel{2}}/\cancel{2}} \approx F\left(\frac{1}{D} + \frac{1}{2W}\right) \; (Nm^{-1})$$

** Including web, but assuming $t \ll D, L$ $\quad I = \frac{WtD^2}{2} + \frac{1}{12}tD^3$



$$\bar{y} = \frac{WtD/2 + \frac{Dt}{2}\frac{D}{4}}{(W + D/2)t} = \frac{4WD + D^2}{4(2W+D)}, \quad q = \frac{F\cancel{t}\left(W + \frac{D}{2}\right)\left(\frac{4WD + D^2}{4(2W+D)}\right)}{\frac{W\cancel{t}D^2}{2} + \frac{1}{12}\cancel{t}D^3}$$

$$= \frac{3F\left(W + \frac{D}{2}\right)\left(\frac{4W + D}{2W + D}\right)}{6WD + D^2}$$

[2 points]

3(b)

$$N_x = \frac{FL}{WD} = F\frac{3}{0.2 \cdot 0.4} = 37.5F\ m^{-1}$$ ＊(28.1F with web)

$$N_{xy} = F\left(\frac{1}{D} + \frac{1}{2W}\right) = F\left(\frac{1}{0.4} + \frac{1}{0.4}\right) = 5\ F\ m^{-1}$$

＊2.8 $F\ m^{-1}$

Using carpet with strain allowables.

Will probably try to retain at least 10% of 0, 90 and 45 plies.

May not be critical.

$$\varepsilon_x = \frac{N_x}{E_x t} \qquad e^+ \text{ is critical } = 0.4\% = 0.004$$

$$\gamma_{xy} = \frac{N_{xy}}{G_{xy} t} \qquad e_{LT} = 0.005$$

Conceptual calcs: 0° plies $E_x \simeq 140\,GPa$

±45° plies $G_{xy} \simeq 35\,GPa$

So $$\frac{t_0}{t_{45}} = \frac{N_x}{N_{xy}} \cdot \frac{G_{xy}}{E_x} \frac{e_{LT}}{e^+} = \frac{37.5}{5} \cdot \frac{35}{140} \cdot \frac{5}{4} = 2.3$$

As a first approximation use $0:45:90 = 12:6:2$ (A)

60% 30% 10%

$E_x = 94\,GPa$, $G_{xy} = 16\,GPa$

$$\frac{t_x}{t_{xy}} = \frac{N_x}{N_{xy}} \frac{G_{xy}}{E_x} \frac{e_{LT}}{e^+} = \frac{37.5}{5} \times \frac{16}{94} \times \frac{5}{4} = 1.6$$ aiming for 1

This is somewhat above the ideal value, N_x is critical, thickness

$$N_x = e^+ E_x t = 0.004 \times 94\times10^9\,Nm^{-2} \times 4\times10^{-3}\,m = 1.5\times10^6\,N/m$$

$$F = 1.5\times10^6 / 37.5 = \underbrace{40\,kN}$$

N.B. This ply isn't symmetric

3(b) As $t_x > t_{xy}$, t_x is critical

Try increasing number of 0s

16 : 4 : 2

70% 20% 10% (B)

E_x = 108 GPa

G_{xy} = 13 GPa

$$\frac{t_x}{t_{xy}} = \frac{37.5}{5} \cdot \frac{13}{108} \cdot \frac{5}{4} = 1.1$$

Much nearer target value of 1, plus balanced symmetric

t_x is still critical.

$$N_x = 0.004 \times 108 \times 10^9 \times 4 \times 10^{-3} = 1.7 \text{ MN/m}$$

F = 46 kN ← choose laminate (A) with this strength

** Considering calcs with including web effects more

For (A) $\frac{t_x}{t_{xy}} = \frac{28.1}{2.8} \times \frac{16}{94} \times \frac{5}{4} = 2.1$

For (B) $\frac{t_x}{t_{xy}} = \frac{28.1}{2.8} \times \frac{13}{108} \times \frac{5}{4} = 1.5$ (t_x critical)

$$\Rightarrow F = 0.004 \times 108 \times 10^9 \times 4 \times 10^{-3} / 28.1 = 61 \text{ kN}$$

Try (C)

90% 10% 10%

16 : 2 : 2 — no longer balanced symmetric

0 : 45 : 90 so may not be optimal

E_x = 115 GPa

G_{xy} = 10 GPa

$\frac{t_x}{t_{xy}} = 1.1$ (t_x critical)

$$F = 0.004 \times 115 \times 4 \times 10^3 / 28.1 = 65 \text{ kN}$$

Probably go for (B) which has a slightly lower strength but is balanced symmetric.

[12 points]

3 (c) Pultrusion: this is a cheap option and feasible. Could improve the design by changing the lay up in the flange and web. Structural members could be made this way, though CFRP would be expensive. Perhaps a turbine spar component would be worth it.

Hand lay up would be more expensive per part but require cheaper tooling. Perhaps for one-off components this would be worth it. It could be difficult to get good quality with a large part. Perhaps using an autoclave could get better consolidation, eg for an aerospace component.

[4 points]

4 (a) As this is a drive shaft it is loaded in shear so ±45 plies are taking the load. (Strictly only one direction, could this be used if driving only in one direction?)

GFRP is relatively light which is always good for transport applications. Also relatively cheap which is likely to be relevant.

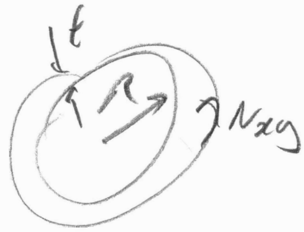
Impact loading may come into play where GFRP fares quite well.

Also corrosion won't be a problem.

[4 points]

4 (b)

(i) Table 1 gives us strain allowables. This assumes 0, 90, 45 while we only have 45s, but this may still be OK



$$\varepsilon_{LT} = \frac{N_{xy}}{t G_{xy}}$$

$$Q = R \cdot 2\pi R \cdot N_{xy}$$

Torque $\Rightarrow N_{xy} = \frac{Q}{2\pi R^2}$

Here $G_{xy} \approx 11$ GPa

$t = 3$mm

$R = 30$mm

$\varepsilon_{LT} = e_{LT}$ at failure $= 0.005$

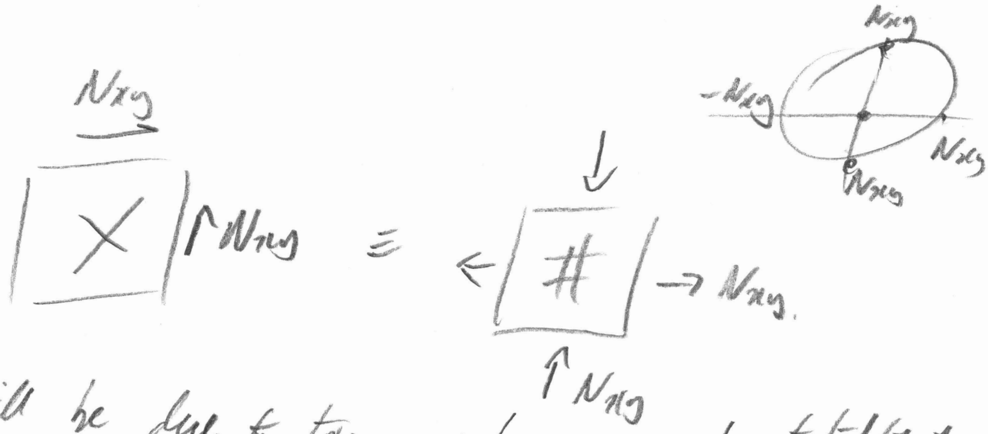
$$\Rightarrow 0.005 = \frac{Q / 2\pi \times (30 \times 10^{-3})^2 \quad \frac{1}{m^2}}{3 \times 10^{-3} \cdot 11 \times 10^9 \quad m \cdot Nm^{-2}}$$

$$Q = 0.005 \times 33 \times \cancel{10^{6}} \times 2\pi \times 900 \times \cancel{10^{-6}} \quad Nm^{-1} \cdot m^2$$

$$Q = 933 \text{ Nm}$$

[4 points]

4 (b) (ii)



Failure will be due to transverse tension.

t = total thickness = 3mm

- Find Q_0 and Q_{90}
- Find A
- Find laminate ε = lamina ε
- Find lamina σ

$E_1 = 39$ GPa

$E_2 = 8.3$ GPa

$\nu_{12} = 0.26$

$G_{12} = 4.1$ GPa

$\nu_{21} = 0.055$

$$Q_{11} = \frac{39}{1-0.26\times 0.055} = 39.6 \text{ GPa}$$

$$Q_{22} = \frac{8.3}{''} = 8.42$$

$$Q_{12} = \frac{0.26 .\ 8.3}{''} = 2.19$$

$$A = \frac{t}{2}(Q_0 + Q_{90}) = \frac{3\times 10^{-3}}{2}\times\begin{bmatrix} 39.6+8.42 & 2\times 2.19 & - \\ 2.2.19 & 8.42+39.6 & - \\ - & - & - \end{bmatrix} \text{GPa}$$

$$= \begin{bmatrix} 72.0 & 6.57 \\ 6.57 & 72.0 \end{bmatrix} \times 10^6 \text{ Nm}^{-1}$$

$$\begin{pmatrix} \varepsilon_x \\ \varepsilon_y \end{pmatrix} = A^{-1}\begin{pmatrix} N_x \\ N_y \end{pmatrix} = \begin{pmatrix} 72.0 & -6.57 \\ -6.57 & 72.0 \end{pmatrix} \frac{1}{\Delta}\frac{1}{10^6} \text{ m/N} \begin{pmatrix} N_{xy} \\ -N_{xy} \end{pmatrix}$$

$$= \begin{pmatrix} 15.3 \\ -15.3 \end{pmatrix} 10^{-9}\, N_{xy} \text{ m/N} \qquad \Delta = 5141$$

4(b)(ii)

Switched for 90°

In 90° ply $\begin{pmatrix}\sigma_1\\ \sigma_2\end{pmatrix} = Q_{90}\begin{pmatrix}\varepsilon_1\\ \varepsilon_2\end{pmatrix} = \begin{pmatrix}39.6 & 2.19\\ 2.19 & 8.42\end{pmatrix}\begin{pmatrix}-15.3\\ 15.3\end{pmatrix}$ $\times 10^{9}\ \mathrm{Nm^{-2}}$ $\times 10^{-9}\ \mathrm{mN^{-1}}$ $\times N_{xy}$

$= \begin{pmatrix}-572\\ 95.3\end{pmatrix} N_{xy}\ m^{-1}$ ← this is critical

Transverse failure when $\sigma_2 = S_T^{+} = 27.6$ MPa

$\Rightarrow N_{xy} = \dfrac{27.6\times10^{6}}{95.3} \dfrac{\mathrm{Nm^{-2}}}{\mathrm{m^{-1}}}$

$= 290$ kN/m ← critical case as expected

(CCSM → 288 kN/m)

Compare 0° ply

$\begin{pmatrix}\sigma_1\\ \sigma_2\end{pmatrix} = Q_0\begin{pmatrix}\varepsilon_1\\ \varepsilon_2\end{pmatrix} = \begin{pmatrix}39.6 & 2.19\\ 2.19 & 8.42\end{pmatrix}\begin{pmatrix}15.3\\ -15.3\end{pmatrix} m^{-1}$

$= \begin{pmatrix}572\\ -95.3\end{pmatrix} N_{xy}\ m^{-1}$

$\sigma_2 = S_T^{-} \Rightarrow N_{xy} = \dfrac{138\times10^{6}}{95.3} = 1.45$ MN/m

critical compared with σ_1

(not critical as expected)

So critical case has $N_{xy} = 290$ kN/m

$\Rightarrow Q = 2\pi R^2 N_{xy}$

$= 1.64$ kNm

[12 points]

Examiner's comments on questions

Question 1: Elastic Deformation

Part (a) appeared to be the most challenging section of this question. Several candidates did not use the tensile-shear interaction terms and failed to use a plot to support their answers. Part (b)(i) was generally answered very well, as expected. Part (b)(ii) was also answered well overall, though a significant number of students made numerical errors when calculating the elements of the laminate extensional stiffness with a common mistake the use of incorrect total laminate thickness. Several candidates stated wrong units, and provided insufficient detail when commenting on the form of the stiffness matrix. In Part (b)(iii), the main issue was calculating correctly the direct stress resultant N for the spherical vessel. Some candidates incorrectly used different values for N_x and N_y, while others mistakenly used the stress tensor instead.

Question 2: Crack Growth

Part (a) generally lacked full or clearly explained answers. Part (b) was answered reasonably well, though the main issues again stemmed from numerical errors. Common mistakes included incorrect estimation of the effective elastic moduli, with many candidates mistakenly concluding that they are equal, and errors in calculating the mode mixity ψ. A few candidates didn't use half the crack length in their calculations. Additionally, some incorrectly applied the total strain energy release rate G_c value separately to K_I and K_{II}, and estimated the stress for each mode individually rather than using the combined value.

Question 3: Practical Design

Almost all candidates struggled with this question. The main difficulty was with part (a), which impacted their ability to solve part (b). Candidates had trouble determining the longitudinal shear force per unit length of the web, q, and consequently the shear line load ((a)(ii)). Those who managed to complete part (a) were generally able to make progress in part (b), although not all had time to estimate the maximum load the beam could support without failing. Part (c) was answered well. For many candidates, it was evident that this was their final question and that they were running out of time.

Question 4: Laminate Loading

Part (a) was answered well. In part (b)(i), surprisingly, several candidates did not use the correct formula for the torque in a tube, despite it being included in the practicalities handout and used in the coursework. Part (b)(ii) was generally well attempted, however, a number of candidates did not complete the question, likely due to time constraints.