Version AEM/3

EGT3
ENGINEERING TRIPOS PART IIB

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Monday 12 May 2025    14:00 to 15:40

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**Module 4C2**

**DESIGNING WITH COMPOSITES**

*Answer not more than **three** questions.*

*All questions carry the same number of marks.*

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

*Write your candidate number **not** your name on the cover sheet.*

**STATIONERY REQUIREMENTS**
Single-sided script paper

**SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM**
CUED approved calculator allowed
Attachment: 4C2 Designing with Composites data sheet (6 pages).
Engineering Data Book

**10 minutes reading time is allowed for this paper at the start of the exam.**

**You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.**

**You may not remove any stationery from the Examination Room.**

1 (a) A unidirectional lamina consisting of 65 vol% high-modulus carbon fibres ($E_f$ = 350 GPa) in an epoxy matrix ($E_m$ = 4 GPa) is subject to uniaxial tension. The fibres are oriented at either 0°, 30°, 60° or 90° to the loading direction. Show which fibre inclination angle produces the largest shear distortion using one of the tensile-shear interaction terms. Use a plot to demonstrate your answer. Additional lamina material properties: $G_{12}$ = 3.5 GPa, $\nu_{12}$ = 0.3. [25%]

(b) The wall of a spherical vessel is made of the lamina described above, and the fibres have been wound so that at every point it approximates to a laminate stacked in the sequence $[(0/\pm 60)_{12}]_s$. The orthogonal local reference axes (1, 2) are defined such that the 1-direction aligns with the 0° fibres and the 2-direction is perpendicular to these fibres. Each lamina has a thickness of 0.25 mm, and the vessel outer diameter is 3 m.

(i) Find the stiffness $[Q]$ of the lamina in the principal material axes (1, 2). [15%]

(ii) Find the laminate extensional stiffness $[A]$. Comment on its form. [30%]

(iii) Calculate the hoop strain of the vessel when it is internally pressurised to 10 MPa. Predict the change in the diameter of the vessel in service. [30%]

2 (a) Explain the following observations:

(i) The toughness of long fibre composites, for cracking transverse to the fibre direction, has an inverse relationship with matrix strength. [10%]

(ii) Models of axial compressive strength of unidirectional CFRP predict that laminate strength is proportional to matrix strength. [10%]

(iii) Pultruded beam sections, made of GFRP, can be a good choice for bridge structural applications. [10%]

(iv) It is difficult to make continual incremental improvements in composite aerospace products, while these are common in the sports sector. [10%]

(b) A unidirectional laminate of Scotchply/1002 GFRP (properties on the datasheet) of thickness 2 mm contains a 100 mm long sharp through-thickness slit parallel to the fibre direction, as illustrated in Fig. 1. The panel is loaded in tension with a remote stress $\sigma$ at an orientation of 45° to the fibre direction. The strain energy release rate $G_C$ of the laminate depends on the mode mixity $\psi = \tan^{-1}(K_I/K_{II})$ according to the relationship

$$G_C = G_{IC} + (G_{IIC} - G_{IC})\left(\frac{2\psi}{\pi}\right)^2$$

where $G_{IC} = 5$ kJ m$^{-2}$ and $G_{IIC} = 10$ kJ m$^{-2}$. Calculate the stress $\sigma$ for crack propagation at the end of the slit. Use stress intensity values for an infinite isotropic plate with a through-thickness crack. Assume that linear elastic fracture mechanics applies. [60%]

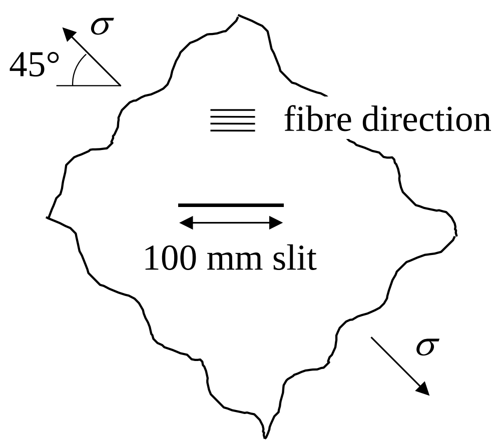


Fig. 1

3 A composite I beam of length $L$, width $W$ and depth $D$ is built in at one end and loaded as a cantilever at its tip by a force $F$, as illustrated in Fig. 2. The flanges and web of the beam both have a uniform thickness $t$, which can be assumed to be much less than the other dimensions of the beam. End effects in the beam can be neglected.

(a) Derive approximate expressions for each of the following:

(i) The maximum line load in the flange acting along the beam. [10%]

(ii) The maximum shear line load in the web. [10%]

(b) The beam has length $L$ = 3 m, $W$ = 0.2 m and $D$ = 0.4 m and is made from a CFRP composite laminate containing a total of twenty plies, with a mix of 0°, 90° and ±45° plies. Each ply is of thickness 0.25 mm. The lay-up is the same throughout the beam. Use the carpet plots provided in Fig. 3 to choose a laminate which maximises the load $F$ that the beam can support without failing either in the flanges or the web, and calculate the corresponding load. [60%]

(c) Identify two possible manufacturing routes for the beam, and discuss the merits of these different routes, depending on the application case, using example products to illustrate your answer. [20%]

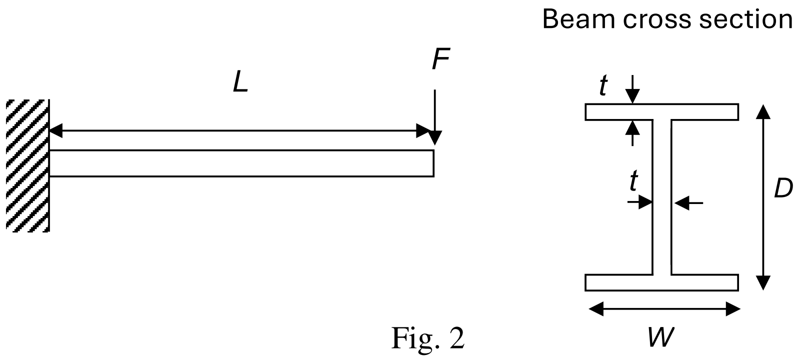


Fig. 2

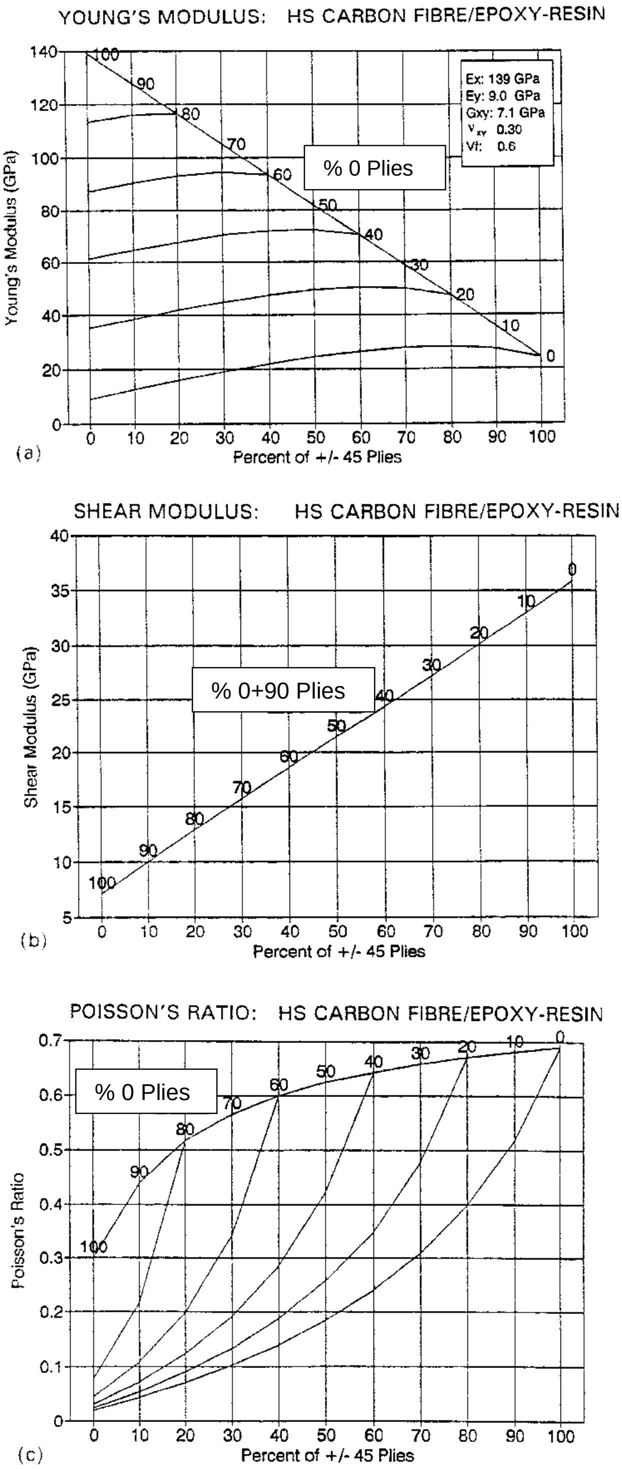


Fig. 3



(TURN OVER

4 A tubular drive shaft is required for an off-road quad-bike. The radius of the tube is 30 mm and the wall thickness is 3 mm. The tube is a balanced symmetric laminate made from $\pm 45^\circ$ Scotchply/1002 GFRP composite (properties on the datasheet).

(a) Discuss why GFRP tube using $\pm 45^\circ$ plies might be a good choice for this application. [20%]

(b) Estimate the torque that the tube could transmit without failing for the following two cases.

(i) Using the data in Table 1 of the datasheet. State any assumptions made. [20%]

(ii) Using laminate plate theory with a maximum stress failure criterion. [60%]

**END OF PAPER**

**Answers**

1 (a) 30°

b(i) $[Q] = \begin{bmatrix} 229.91 & 3.37 & 0 \\ 3.37 & 11.24 & 0 \\ 0 & 0 & 3.50 \end{bmatrix}$ GPa

b(ii) $[A] = \begin{bmatrix} 1674.45 & 556.62 & 0 \\ 556.62 & 1674.45 & 0 \\ 0 & 0 & 558.92 \end{bmatrix}$ $\mathrm{MNm^{-1}}$

b(iii) ~10.8 mm

2 (b) $\sigma$ = 33 MPa

4(b) (i) 933 Nm

(ii) 1.64 kNm