EGT3, ENGINEERING TRIPOS IIB, 4C2 DESIGNING WITH COMPOSITES

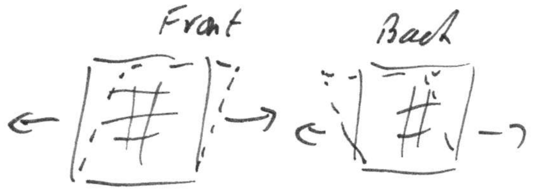
CRIB 2013/14, (M SUTCLIFFE)

Q1. (a)(i) Shear loading induces direct strains, direct loading induces shear strains, eg [0,45] laminate.

(ii) No shear-tensile interaction, for example laminates with same number of $+\theta$ as $-\theta$ plies.

(iii) Ply orientations are symmetric about the midplane.

(b) This is balanced as no plies at θ not equal to 0 or 90°.



From symmetry the laminate looks the same from front and back so no shear possible.

Alternatively write down generic Q matrices & show that $A_{16} = A_{26} = 0$.

(c)(i) Balanced symmetric so $[B]=0$, $A_{16} = A_{26} = 0$ as balanced.

$$\bar{Q}_{11} = \frac{Q_{11}}{4} + \frac{Q_{22}}{4} + 2(Q_{12} + 2Q_{66})\frac{1}{4} = 11.75\text{ GPa} \quad \text{since } s^2 = c^2 = \frac{1}{2}, \; s^4 = c^4 = 1/4$$

$$\bar{Q}_{12} = (Q_{11} + Q_{22} - 4Q_{66})\frac{1}{4} + Q_{12}\cdot\frac{1}{2} = 1.75\text{ GPa}$$

$$\bar{Q}_{22} = \bar{Q}_{11}, \quad \bar{Q}_{66} = (Q_{11} + Q_{22} - 2Q_{12} - 2Q_{66})\frac{1}{4} + Q_{66}\frac{1}{2} = 5.75\text{ GPa}$$

$$A = 4t\bar{Q} = \begin{bmatrix} 9.4 & 1.4 & 0 \\ 1.4 & 9.4 & 0 \\ 0 & 0 & 4.6 \end{bmatrix} \frac{MN}{m}$$

NB include units

This question had a high average, reflecting a good understanding of laminate plate theory.

Q 1 (c)(ii)

$$F = N_x \pi D \Rightarrow N_x = \frac{100}{\pi\, 100} \frac{kN}{mm} = \frac{1}{\pi} \frac{MN}{m}$$

These calcs not well done.

$$N_y = 0$$

$$Q = N_{xy} . R . \pi D \Rightarrow N_{xy} = \frac{2Q}{\pi D^2} = \frac{2.5}{\pi . 100 . 0.1} \frac{kNm}{m . mm} = \frac{1}{\pi} \frac{MN}{m}$$

$$\begin{pmatrix} N_x \\ N_y \\ N_{xy} \end{pmatrix} = [A] \begin{pmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{pmatrix}$$

$$\gamma_{xy} = N_{xy} / A_{66} = \frac{1}{4.6} \cdot \frac{1}{\pi} \frac{MN/m}{MN/m} = 0.069$$

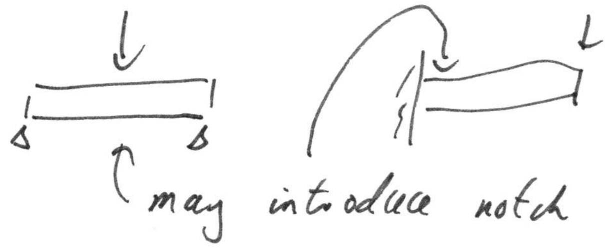
$$\begin{pmatrix} \varepsilon_x \\ \varepsilon_y \end{pmatrix} = \begin{pmatrix} 9.4 & 1.4 \\ 1.4 & 9.4 \end{pmatrix}^{-1} \begin{pmatrix} 1/\pi \\ 0 \end{pmatrix} \frac{MN/m}{MN/m}$$

$$= \frac{1}{9.4^2 - 1.4^2} \begin{pmatrix} 9.4 & -1.4 \\ -1.4 & 9.4 \end{pmatrix} \begin{pmatrix} 1/\pi \\ 0 \end{pmatrix}$$

$$\left.\begin{aligned} \varepsilon_x &= 0.035 \\ \varepsilon_y &= -0.0052 \\ \gamma_{xy} &= 0.069 \end{aligned}\right\}$$

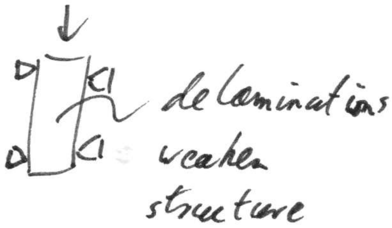
[These strains rather large - the tube will probably fail at these loads.]

Q2 (a)



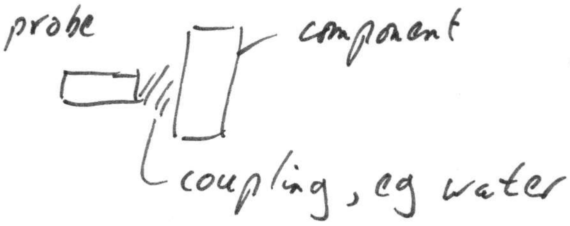
Controlled impact tests with varying energies to assess low energy (tool drop), medium energy (bird strike) large energy (ballistic impact). Can use birds to check fan blade integrity.

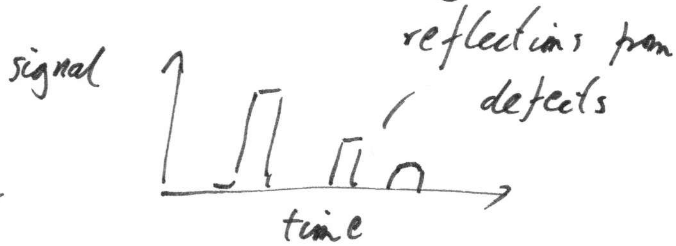
Compression after impact measured using static tests.



(b) Porosity and flaws act as starters for cracks (eg in fatigue) which are particularly damaging given the low fracture toughness of the matrix.

Ultrasound is an effective way of measuring such cracks.





(c) See notes.

2 (d)

→ full scale

→ subcomponent

→ generic tests

→ many coupon tests
- material properties

Many coupon tests establish material properties (stiffness, strength, toughness) for a wide range of conditions.

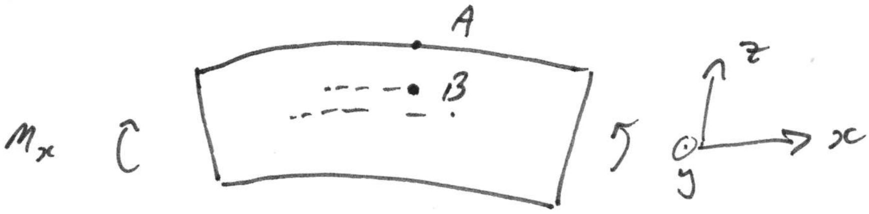
Fewer generic tests and even fewer sub-component tests check stiffness and strength of typical features with given material combination and manufacturing route. Use modelling to limit full-scale tests.

Pros: very thorough so gives confidence in design. Particularly for crack-based failure or complex load paths failure can be difficult to predict so testing is essential. For aircraft, failure would be very damaging commercially so this testing is needed, also to comply with regulations.

Cons: very expensive and limits the ability to modify or upgrade. E.g. a new material will need to go through the complete testing cycle. More conservative designs which pass the final stages are preferred.

(e) The racquet will be designed to hold the corresponding compressive loads and knocks at room temperature for long periods of time. What is different is the liklihood of high temperatures in the container, leading to matrix softening and premature micro buckling ($\sigma_c \approx \tau_y/\phi$) (c.f. the demo in lectures).

3 (a)



From symmetry $\kappa_{xy} = \gamma_{xy} = 0$.
$[B] = 0$ as balanced symmetric.

$$\begin{pmatrix} 0 \\ 0 \\ 0 \\ M_x \\ M_y \\ 0 \end{pmatrix} = \begin{bmatrix} A & 0 \\ 0 & D \end{bmatrix} \begin{pmatrix} \varepsilon_x^\circ \\ \varepsilon_y^\circ \\ 0 \\ \kappa_x \\ \kappa_y \\ 0 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} M_x \\ M_y \end{pmatrix} = (Nt)^3 \begin{bmatrix} 85.6 & 4.9 \\ 4.9 & 21.4 \end{bmatrix} \begin{pmatrix} \kappa_x \\ \kappa_y \end{pmatrix} \text{GPa}$$

$$\begin{pmatrix} \kappa_x \\ \kappa_y \end{pmatrix} = \begin{pmatrix} 85.6 & 4.9 \\ 4.9 & 21.4 \end{pmatrix}^{-1} \frac{1}{(Nt)^3} \begin{pmatrix} M_x \\ M_y \end{pmatrix} = \begin{pmatrix} 21.4 & -4.9 \\ -4.9 & 85.6 \end{pmatrix} \frac{1}{\Delta (Nt)^3} \begin{pmatrix} M_x \\ M_y \end{pmatrix} (\text{GPa})^{-1}$$

$$\Delta = \cancel{433.95}\ 1808\ \text{GPa}^2$$

$$= \begin{pmatrix} 21.4 & -4.9 \\ -4.9 & 85.6 \end{pmatrix} \frac{M_x}{\cancel{433.95}\ 1808\ (Nt)^3} \begin{pmatrix} 1 \\ 0.0572 \end{pmatrix}$$

$$= \begin{pmatrix} 0.0117 \\ -2 \times 10^{-6} \end{pmatrix} \frac{M_x}{(Nt)^3} \frac{1}{\text{GPa}}$$

κ_y is effectively zero, compared to κ_x

To find in-plane εs: $\begin{pmatrix} 0 \\ 0 \end{pmatrix} = [A] \begin{pmatrix} \varepsilon_x^\circ \\ \varepsilon_y^\circ \end{pmatrix} \Rightarrow \varepsilon_x^\circ = \varepsilon_y^\circ = 0$

At (A) and (B) $\varepsilon_y = 0$

At (A) $\varepsilon_x = \varepsilon_x^\circ - \frac{Nt}{2} \kappa_x = -5.85 \times 10^{-3} \frac{M_x}{(Nt)^2} \frac{1}{\text{GPa}}$

At (B) $\varepsilon_x = \varepsilon_x^\circ - \frac{3}{2} Nt \kappa_x = -0.0175 \frac{M_x}{(Nt)^2} \frac{1}{\text{GPa}}$

3 (b) Failure will ~~occa~~ occur at top (A) or bottom of beam.

$$\begin{pmatrix}\sigma_1\\ \sigma_2\end{pmatrix} = [Q]\begin{pmatrix}\varepsilon_x\\ \varepsilon_y\end{pmatrix} = \begin{pmatrix}39.2 & 2.2\\ 2.2 & 8.44\end{pmatrix}\begin{pmatrix}\pm 0.0175\\ 0\end{pmatrix}\frac{M_x}{(Nt)^2}\ \text{GPa}\ \text{/GPa}$$

$$= \pm \frac{M_x}{(Nt)^2}\begin{pmatrix}0.687\\ 0.0386\end{pmatrix}\text{GPa}\ \text{/GPa},$$ +ve for bottom face, −ve for top face

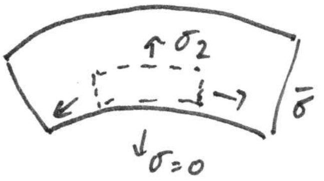
For Scotchply, along fibre compression strength (621 MPa) is critical, transverse tensile strength (27.6) is critical.

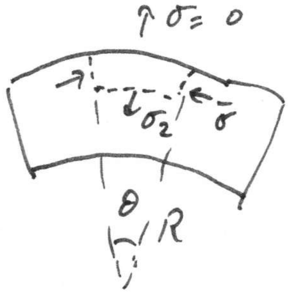
Since $\frac{0.687}{0.0386} < \frac{621}{27.6}$ transverse tension is then critical failure mechanism

$$\Rightarrow \sigma_2 = 27.6\ \text{MPa} = 2.2 \times 0.0175 \frac{M_x}{(Nt)^2}\ \text{GPa}\ \text{/GPa at failure}$$

$$M_x = 717 (Nt)^2\ \text{MPa at failure.}$$

(c)





In both cases σ_2 is the same, assuming $Nt \ll R$, since $\bar\sigma$ is equal and opposite.

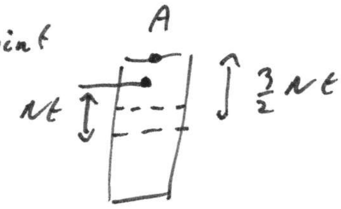
$R \downarrow$ (for right hand figure)

$$R\theta\sigma_z - 2.Nt\bar\sigma.\sin\frac{\theta}{2} = 0$$

$$\Rightarrow \sigma_z = \bar\sigma\frac{Nt}{R}$$

But $\bar\sigma = \frac{M_x}{(Nt)^2} \cdot \frac{2}{3} \cdot 0.687\ \text{GPa}$

midpoint



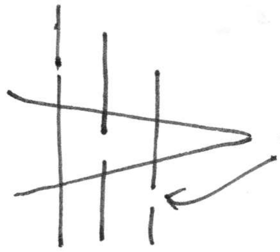
scaling from (b) to find σ_1 in middle of top layer

$$\bar\sigma = 0.458\frac{M_x}{(Nt)^2}\ \text{GPa}$$

$$\sigma_2 = 0.458\frac{M_x}{NtR}\ \text{GPa}$$

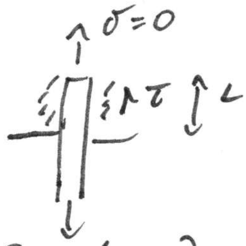
This part not well done - perhaps out-of-time or lacking confidence on previous parts, though method marks were awarded, not dependent on previous parts.

4 (a)

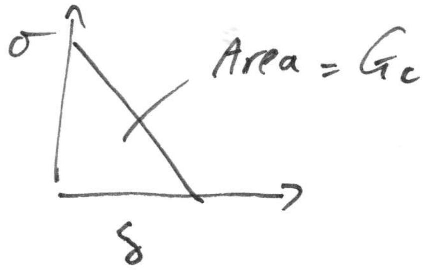


- fibre breaks don't happen along a single crack line, due to fibre strength randomness.

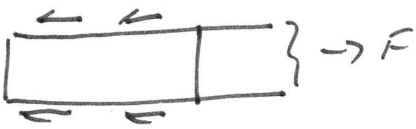
- fibre pullout is modelled using shear lag theory, expressing the pull-out force as a function of embedded length.



- work done in toughening zone depends on the bridging law, with σ-δ given by shear lag theory as above.



(b) (i)



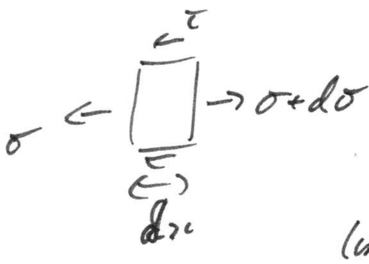
At failure τ acts along whole bond surface

$\Rightarrow F = 2\tau L D$

Need to consider slip and no-slip cases.

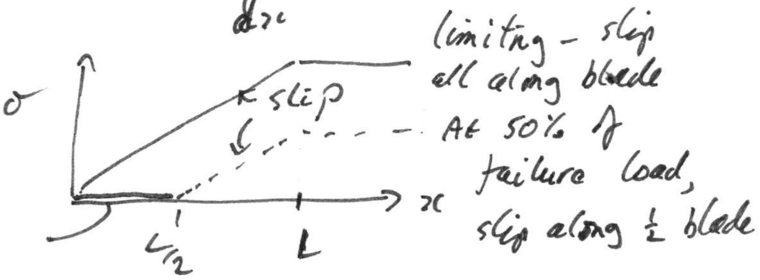
No-slip $\Rightarrow$ no strain in blade (since hub is rigid) $\Rightarrow$ no stress in blade.

Slip: $\dfrac{d\sigma}{dx} = \dfrac{2\tau}{t}$ from equilibrium



$$\frac{d\varepsilon}{dx} = \frac{2\tau}{Et}$$

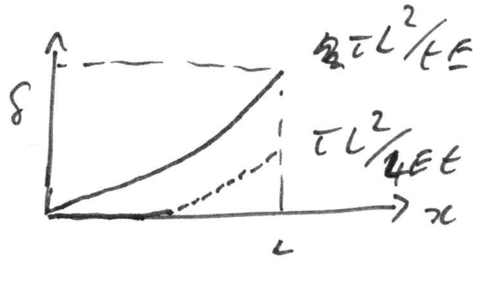
$$\delta = \int \varepsilon \, dx$$



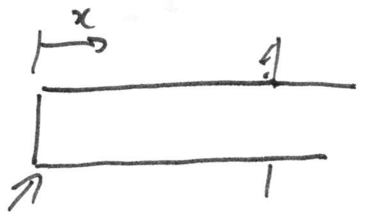
slip along whole length:

$$\delta_L = \int_0^L \frac{2\tau}{Et} x \, dx = \frac{2\tau}{Et}\frac{L^2}{2} = \frac{\tau L^2}{Et}$$

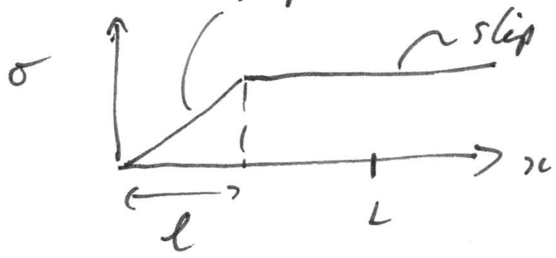
At 50% $\delta = \displaystyle\int_0^{L/2} \frac{2\tau}{Et} x \, dx = \tau L^2 / 4Et$



4 (b)(ii)



At failure this point starts to slip



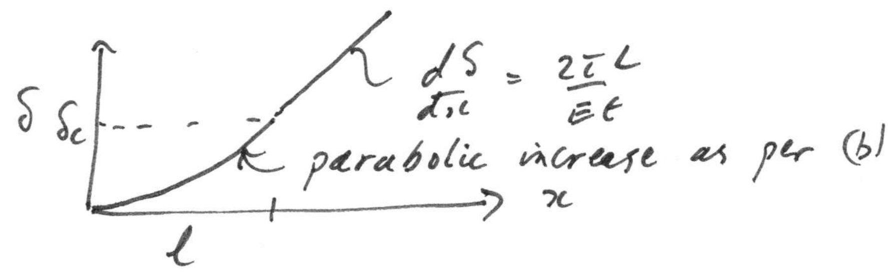
To find l where relative displacement $= \delta_c$

For $0 < x < l$ analysis as before: $\delta_c = \dfrac{\tau l^2}{Et}$

$\Rightarrow l = \sqrt{\dfrac{\delta_c E t}{\tau}}$

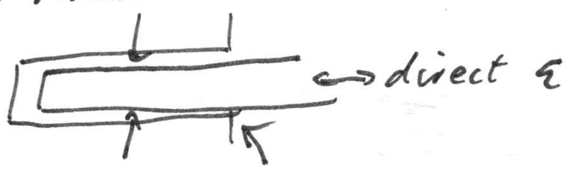
~~for~~ For $x > l_c$, σ is constant $\left(= \dfrac{2\tau l}{t}\right)$

$\Rightarrow \varepsilon = \sigma / E = \dfrac{2\tau l}{Et}$, $\dfrac{d\delta}{dx} = \varepsilon = \dfrac{2\tau l}{Et}$



(iii) - High shear stress - may lead to combined shear plus tensile failure in blade inside joint section.

- Danger of delamination for UD material, especially as Poisson's ratio strain tends to contract blade



- stress concentration at end of joint is likely to be a problem