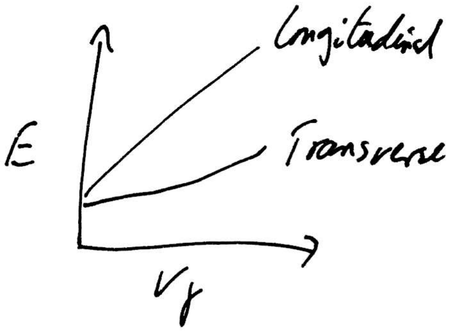
4C2 Designing with Composites 2020/21 (M Sutcliffe)

Q1 (a) Key factors - constituents
- volume fraction
- form / architecture

Constituents - stiff elements will give higher stiffness eg ceramic composites of boron carbide and polymers. Similarly carbon fibres, compared with lower modulus glass

Volume fraction

- increased volume fraction gives higher modulus, but most relevant to longitudinal modulus



Form - a key factor.
- eg long fibre composites give good axial properties compared with particulates
- meso scale architecture also important, eg woven or non-crimp fabrics have different fibre alignment while chopped strand has isotropic but lower stiffness

- also at a lamina scale, changes in laminate lay up change stiffness, so unidirectional material has higher stiffness in one directional, while quasi-isotropic laminates have the same stiffness in all in plane directions.

Comment: answers tended to lack a structure to group comments and work through relevant issues. Examples missing.

Q 1(b) (i) $E_1 \approx E_f v_f + E_m (1 - v_f) = 76 \times 0.65 + 2.4 \times 0.35 = 50.2\,\text{GPa}$ ✓

$$E_2 \approx \left(\frac{0.65}{76} + \frac{0.35}{2.4}\right)^{-1} = 6.48\,\text{GPa} \quad ✓$$

$$G = \frac{E}{2(1+\nu)} \Rightarrow G_f = 31.15\,\text{GPa}, \quad G_m = 0.876\,\text{GPa}$$

$$G_{12} = \left(\frac{0.65}{G_f} + \frac{0.35}{G_m}\right)^{-1} = 2.63\,\text{GPa}$$

This estimate, analogous to E_2, often missed

Comments on accuracy omitted commonly

E_1 is likely to be a good estimate. E_2 assumes equal stress and not likely to be very good as the corresponding plate geometric model is not so good. Similarly G assumes this simplified form, along with isotropic fibre, so rather approximate

(ii) ~~Balanced so don't need to calculate $A_{16}, A_{26} = 0$~~

$Q_{11} = E_1/((1 - \nu_{12}\nu_{21}) = 50.7\,\text{GPa}$ $\qquad \nu_{21} = \nu_{12} \times \frac{E_2}{E_1} = 0.0338$

$Q_{22} = E_2/'' = 6.54\,\text{GPa}$

$Q_{12} = \nu_{12} E_2/'' = 1.71\,\text{GPa}$ $\qquad Q = \begin{bmatrix} 50.7 & 1.71 & 0 \\ 1.71 & 6.54 & 0 \\ 0 & 0 & 2.63 \end{bmatrix} \text{GPa}$

$Q_{66} = G_{12} = 2.63\,\text{GPa}$

(c) (i) Balanced so no need to calculate $\bar{Q}_{16}, \bar{Q}_{26}$ as $A_{16}, A_{26} = 0$

$45s \rightarrow c^2 = s^2 = \frac{1}{2}$

$\bar{Q}_{11} = 55 \cdot \frac{1}{4} + 6.0 \cdot \frac{1}{4} + \frac{2}{4}(1.5 + 2 + 2.5) = 18.5\,\text{GPa}$

$\bar{Q}_{22} = \bar{Q}_{11}$

$\bar{Q}_{12} = (55 + 6.0 - 4 \times 2.5)\frac{1}{4} + 1.5 \cdot \frac{1}{2} = 13.5\,\text{GPa}$

$\bar{Q}_{66} = 14.5\,\text{GPa}$

$\textcircled{E}$

$$A = \underset{\text{m. GPa.}}{4 \times 0.5 \times 10^{-3}} \begin{bmatrix} 18.5 & 13.5 & - \\ 13.5 & 18.5 & - \\ - & - & 14.5 \end{bmatrix} + \underset{\text{GPa.m}}{2 \times 0.5 \times 10^{-3}} \begin{bmatrix} 60 & 1.5 & - \\ 1.5 & 55 & - \\ - & - & 2.5 \end{bmatrix}$$

$$= \begin{bmatrix} 43 & 28.5 & 0 \\ 28.5 & 92 & 0 \\ 0 & 0 & 31.5 \end{bmatrix} \times 10^6\,\text{Nm}^{-1}$$

check units carefully

Comment: The analysis parts were mostly well done.

Q 2 (a) Factors of importance

- cost of investment and per part cost
- rate required
- size of part, shape
- quality, tolerance
- cost
- processibility of material

Hand layup and open mould processes are relatively low investment and can extend to more complex shapes and large parts. So small or large boats would be relevant. Can increase speed with spray moulding. Also high quality parts can be produced using an autoclave (eg F1)

Automated tape lay-up on an open mould can increase rate. (eg aerospace)

Closed mould processes such as RTM and compression moulding can give intermediate quality and higher rate, eg for MRI cases or propellor blades.

Filament winding can produce good quality but limited in shapes, and challenges with 0° and 90° fibres not possible, eg fertiliser tanks, with GFRP.

Pultrusion is a high rate, low cost part (eg bridge sections) but limited to prismatic shapes.

Injection moulding has the highest rate, eg automotive parts, but worse mechanical performance with only short fibres surviving injection.

Comment: some answers lacked a structure and examples

Q2. (b) (i)

$\sigma_1 = \sigma_2 = -\sigma$

$\sigma_{12} = 0$

no need for any laminate analysis yet!

T-H: $\cancel{\left(\frac{\sigma_1}{S_L^-}\right)^2 - \frac{\sigma_1\sigma_2}{S_L^{-2}}} + \left(\frac{\sigma_2}{S_T^-}\right)^2 = 0$

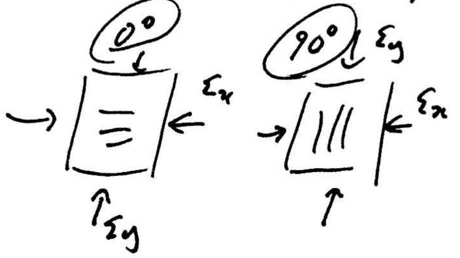
$\Rightarrow \sigma = S_T^- = 268$ MPa

(ii) Need to calculate laminate ε → lamina stresses.

$$A = \underset{\text{ply thickness}}{t}\begin{bmatrix} 139\times2+9 & 2.7\times3 & \overset{\text{not needed}}{-} \\ 2.7\times3 & 139+2\times9 & - \\ - & - & - \end{bmatrix} = t\begin{bmatrix} 287 & 8.1 & - \\ 8.1 & 157 & - \\ - & - & - \end{bmatrix} \text{GPa}$$

keep track of units

$$\begin{pmatrix}\varepsilon_x \\ \varepsilon_y\end{pmatrix} = A^{-1}\underset{N = 3\sigma t}{\begin{pmatrix}-3t\sigma \\ -3t\sigma\end{pmatrix}} = -\sigma\begin{pmatrix}0.0099 \\ 0.0186\end{pmatrix}\frac{1}{\text{GPa}} \qquad \left[A^{-1} = \begin{bmatrix} 0.00369 & -0.00018 & - \\ -0.00018 & 0.00638 & - \\ - & - & - \end{bmatrix}\frac{1}{\text{GPa}}\right]$$



Difficult to see what the critical ply/mechanism will be - check both

For 0° ply : $\begin{pmatrix}\sigma_1 \\ \sigma_2\end{pmatrix} = Q\begin{pmatrix}\varepsilon_1 \\ \varepsilon_2\end{pmatrix} = Q\begin{pmatrix}\varepsilon_x \\ \varepsilon_y\end{pmatrix} = -\sigma\begin{pmatrix}139 & 2.7 \\ 2.7 & 9\end{pmatrix}\begin{pmatrix}0.0099 \\ 0.0186\end{pmatrix} = \begin{pmatrix}-1.426 \\ -0.196\end{pmatrix}\sigma$

T-H: $\frac{1.436^2}{1172^2} - \frac{0.196 \cdot 1.436}{1172^2} + \frac{0.196^2}{268^2} = \frac{1}{\sigma^2} \Rightarrow \sigma = 724$ MPa

For 90° ply $\begin{pmatrix}\sigma_1 \\ \sigma_2\end{pmatrix} = Q\underset{\text{N.B}}{\begin{pmatrix}\varepsilon_y \\ \varepsilon_x\end{pmatrix}} = -\sigma\begin{pmatrix}139 & 2.7 \\ 2.7 & 9\end{pmatrix}\begin{pmatrix}0.0186 \\ 0.0099\end{pmatrix} = \begin{pmatrix}-2.611 \\ -0.160\end{pmatrix}\sigma$

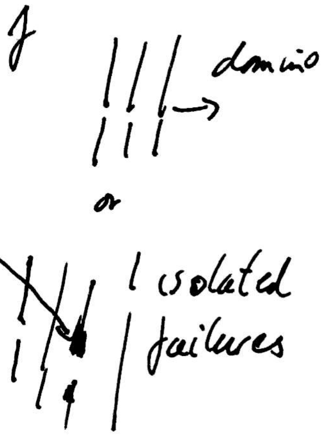
T-H: $\underset{S_L^-}{\frac{2.611^2}{1172^2}} - \underset{S_L^-}{\frac{0.160\times 2.611}{1172}} + \underset{S_T^-}{\frac{0.160^2}{268^2}} = \frac{1}{\sigma^2} \Rightarrow \sigma = 667$ MPa

Critical ply is 90° - failure stress = 667 MPa

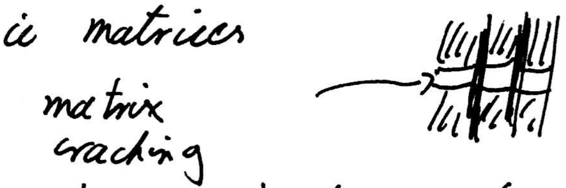
Comment: Reasonable answers, but some confusion over relating the εs in the different plies and not checking both ply orientations.

3 (a) Longitudinal failure

- strength of fibres, but also statistics of fibre strength and matrix failure eg as expressed in shear lag theory



- strength of matrix, eg for brittle ceramic matrices



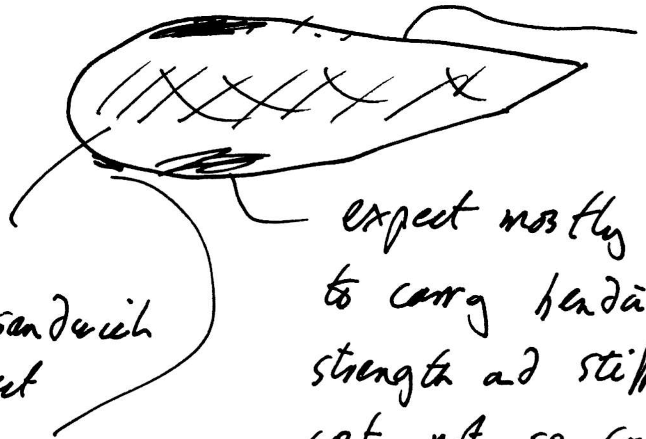
- fibre volume fraction - low v_f means less strong fibres

Transverse

Matrix failure and ductility affects how the matrix cracks

Sizing can prevent debonding

(b)



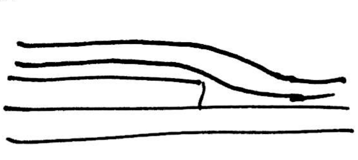
shell not so load bearing but needs to resist impact - braided or woven material

foam to provide sandwich panel effect

expect mostly UD CFPP in spar to carry bending loads. Good specific strength and stiffness is paramount, cost not so critical

metal strip for lightening protection

(c)



Ply drops occur where a lamina ends, or where a change in thickness is required, eg for structural reasons. The geometry with a local resin pocket is a stress concentrator, especially leading to starter fatigue cracks.

3 (d) High temperatures in a container will lead to reduction in matrix strength.
Where the arch is held under load this will give a big drop in compressive strength, since this is controlled by matrix strength, hence premature failure.
Could avoid transporting arches while bent under load, or monitor container temperature.

(e) Mechanical performance: needs good bending stiffness and strength indicates a good proportion of UD CFRP or GFRP
Weight is important, so again CFRP, depending on market
- impact damage - likely to occur so consider a protective outer layer, ±45 or woven?
- joints - could be needed to transport larger masts, but will be tricky to make efficient
- manufacturing - depending on whether a taper is justified in the design, perhaps pultrusion, or hand lay-up (for a taper) - could use automated roll wrapping.
Likely to be hollow so wrap around a mandrel
- waterproofing - composites good here, stainless steel fittings?
- aethetics - could be important

Comment: in general answers sometimes failed to address properly the points raised in the question, just writing down what was known.

4 (a)

Vibration due to whirling motion

Vibration depends on $f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$ approach

Here $k \sim EI$ which depends on t

m - mass per unit length also depends on t,

so t, cancel out.

E_x is an axial stiffness controlling tube bending.

(b) $\frac{E}{\rho} > 2\left(\frac{2fL^2}{\pi R}\right)^2 \Rightarrow E > 47.8$ GPa, using $\rho = 1500$ kg/m^3

Not quite clear from question if 90s are required.

For a tube with lots of ±45 probably leave them out.

To maximise Q we want to maximise the proportion of ±45s while keeping E > 47.8 GPa

sensible answers accepted

From carpet plots → ≈ 23% 0°, 77% ±45

$\Rightarrow 0:90:45 = 3.7:0:12.3 \rightarrow 4:0:12$, say $(\pm 45_3, 0_2]_s$

If we retain 10% 90s ⇒ 20% 0s

$0:90:45 = 20:10:70 \rightarrow 3.2:1.6:11.2 \rightarrow 4:2:10$

(c)(i) Only shear applied $\Rightarrow e_{LT} = 0.005$

Using laminate $[(\pm 45)_3, 0_2]_s \rightarrow G = 28.5$ GPa

$$\frac{N_{xy}}{t} = \sigma_{xy} = \gamma G = 0.005 \times 28.5 \text{ GPa}$$

This part well done

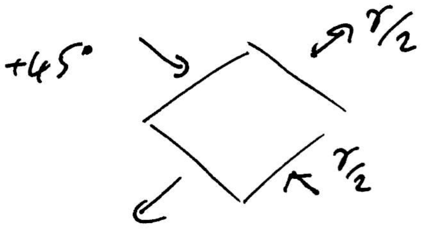
$$Q = N_{xy} . 2\pi R . R = 2\pi R^2 e_{LT} G t$$

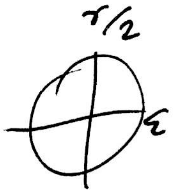
$$= 2.\pi.(0.1)^2.0.005.28.5\times10^9.2\times10^{-3}$$

$$= \underline{17.9 \text{ kNm}}$$

Q4 (c)(ii) Laminate shear strain given by laminate G.

Need to consider ε_s in individual plies.





~~From~~ As before $\gamma = \frac{N_{xy}}{tG} = \cancel{N_{xs}} \frac{Q}{2\pi R^2 . t G}$

For 0° ply $\gamma < \varepsilon_{LT} = 2\%$ ← (this step/logic mostly missed)

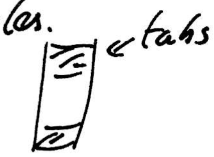
For 45 plies $\frac{\gamma}{2} < \min\{\varepsilon_L^+, \varepsilon_L^-, \varepsilon_T^+, \varepsilon_T^-\}$ considering both ply orientations

$< 0.5\% \, (\varepsilon_T^+)$

So the 45 ply is critical

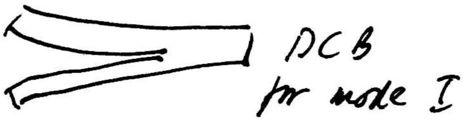
$$Q = \varepsilon_T^+ . 2\pi R^2 t . G = 35.8 \text{ kNm (twice (ci))}$$

(d) ~~Aeros~~ Aerospace so need ~~extent~~ extensive testing using pyramid, following compliance rules.



Coupon tests: tension and compression to establish modulus and strength

Maybe include cracking, and fatigue testing.



Shear: best to use tube to establish this property as most realistic. (answers didn't always include discussion)

Features: joints will be critical to test, including fatigue

Impact: may not be so critical assuming not an external part. maybe check for barely visible impact damage.

NDT: checking part quality eg with ultrasound will be important

Modelling: can be used to reduce testing.

Examiner's comments:

Q1 Laminate theory and elasticity
The standard laminate analysis part of this question was well answered. The rather challenging qualitative section discussing elastic was less well tackled.

Q2 Failure analysis
Again the open-ended question about manufacturing methods was the weak spot in this question, with some good answers to the failure analysis.

Q3 Qualitative design issues
This question contained a series of qualitative parts. Some of the material drew more or less strongly on recorded demo clips uploaded to the moodle site. These parts were less well addressed, though still there were some good answers.

Q4 Strain allowables and carpet plots
This analysis part of this question was quite simple. In part (c)(ii) a small minority of candidates launched into a more complicated laminate analysis rather than appreciating that the strain in the plies could be derived simply from the laminate strain. Again there were mixed answers to the qualitative sections.