4C2 - Designing with Composites

Cribs

1. (a) A laminate is made up of a stacked assembly of unidirectional plies, each having its fibre axis lying at a specified angle to a reference direction. They are used in preference to unidirectional plies because they are more isotropic.

A *balanced* laminate ($A_{16} = A_{26} = 0$) is one in which the laminate as a whole exhibits no tensile-shear interactions i.e. the tension-shear interaction terms contributed by the individual laminae all cancel out each other (a tensile stress induces no shear straining and a shear stress induces no normal strain).

A *symmetric* laminate is one possessing a mirror plane lying in the plane of the laminate i.e. the stacking sequence in the top half reflects that in the bottom half. A symmetric laminate does not exhibit bending-stretching coupling (the coupling stiffness $[B] = 0$), i.e. in-plane loading will not generate any out-of-plane distortion and vice versa.

(b)

$$E_1 = \left[fE_{\mathrm{f}} + (1-f)E_{\mathrm{m}} \right] = 0.5\times 76 + 0.5\times 3 = 39.5 \text{ GPa}$$

$$E_2 = \left[\frac{f}{E_{\mathrm{f}}} + \frac{(1-f)}{E_{\mathrm{m}}} \right]^{-1} = \left[\frac{0.5}{76} + \frac{0.5}{3} \right]^{-1} \approx 5.77 \text{ GPa}$$

$$\frac{\nu_{12}}{E_1} = \frac{\nu_{21}}{E_2} \Rightarrow \nu_{21} \approx 0.05$$

Calculate $[Q]$ in principal material axes (1, 2)

$$Q_{11} = \frac{E_1}{1-\nu_{12}\nu_{21}} = \frac{39.5}{1-0.3\times 0.05} = 40.10 \text{ GPa}$$

$$Q_{22} = \frac{E_2}{1-\nu_{12}\nu_{21}} = \frac{5.77}{1-0.3\times 0.05} = 5.86 \text{ GPa}$$

$$Q_{12} = \frac{\nu_{12}E_2}{1-\nu_{12}\nu_{21}} = \frac{0.3\times 5.77}{1-0.3\times 0.05} = 1.76 \text{ GPa}$$

$$Q_{66} = G_{12} = 2.4 \text{ GPa} \qquad Q_{16} = Q_{26} = 0$$

$$[Q] = \begin{bmatrix} 40.10 & 1.76 & 0 \\ 1.76 & 5.86 & 0 \\ 0 & 0 & 2.4 \end{bmatrix} \text{GPa}$$

Calculate the transformed stiffness matrix $[\overline{Q}]$ in the global x-y axes.

The transformed lamina stiffness matrix $[\overline{Q}]$ for the 0° plies is given by

$$[Q]_{0^\circ} = \begin{bmatrix} 40.10 & 1.76 & 0 \\ 1.76 & 5.86 & 0 \\ 0 & 0 & 2.4 \end{bmatrix} \text{GPa}$$

The transformed stiffness matrix for the +45° plies is given by

$$\left(\overline{Q}_{11}\right)_{45°} = Q_{11}c^4 + Q_{22}s^4 + 2\left(Q_{12} + 2Q_{66}\right)s^2c^2 = 14.77 \text{ GPa}$$

$$\left(\overline{Q}_{12}\right)_{45°} = \left(Q_{11} + Q_{22} - 4Q_{66}\right)s^2c^2 + Q_{12}\left(c^4 + s^4\right) = 9.97 \text{ GPa}$$

$$\left(\overline{Q}_{22}\right)_{45°} = Q_{11}s^4 + Q_{22}c^4 + 2\left(Q_{12} + 2Q_{66}\right)s^2c^2 = 14.77 \text{ GPa}$$

$$\left(\overline{Q}_{16}\right)_{45°} = \left(Q_{11} - Q_{12} - 2Q_{66}\right)c^3s - \left(Q_{22} - Q_{12} - 2Q_{66}\right)cs^3 = 8.56 \text{ GPa}$$

$$\left(\overline{Q}_{26}\right)_{45°} = \left(Q_{11} - Q_{12} - 2Q_{66}\right)cs^3 - \left(Q_{22} - Q_{12} - 2Q_{66}\right)c^3s = 8.56 \text{ GPa}$$

$$\left(\overline{Q}_{66}\right)_{45°} = \left(Q_{11} + Q_{22} - 2Q_{12} - 2Q_{66}\right)s^2c^2 + Q_{66}\left(s^4 + c^4\right) = 10.61 \text{ GPa}$$

where $c = \cos 45,\ s = \sin 45$

$$\left[\overline{Q}\right]_{45°} = \begin{bmatrix} 14.77 & 9.97 & 8.56 \\ 9.97 & 14.77 & 8.56 \\ 8.56 & 8.56 & 10.61 \end{bmatrix} \text{GPa}$$

The transformed lamina stiffness matrix $[\overline{Q}]$ for the $-45°$ plies is given by

$$\left[\overline{Q}\right]_{-45°} = \begin{bmatrix} 14.77 & 9.97 & -8.56 \\ 9.97 & 14.77 & -8.56 \\ -8.56 & -8.56 & 10.61 \end{bmatrix} \text{GPa}$$

The only difference between the stiffness matrices for the two plies is that the shear coupling terms (terms with subscripts 16 and 26) for the $-45°$ ply have the opposite sign from the corresponding terms for the +45° ply.

Set t (= 0.5×10^{-3} m) for lamina thickness

$$A_{11} = \left[\left(\overline{Q}_{11}\right)_{+45} + \left(\overline{Q}_{11}\right)_{-45} + 2\left(\overline{Q}_{11}\right)_0\right]\cdot 2t = 109.74 \text{ MN m}^{-1}$$

$$A_{12} = \left[\left(\overline{Q}_{12}\right)_{+45} + \left(\overline{Q}_{12}\right)_{-45} + 2\left(\overline{Q}_{12}\right)_0\right]\cdot 2t = 23.45 \text{ MN m}^{-1}$$

$$A_{22} = \left[\left(\overline{Q}_{22}\right)_{+45} + \left(\overline{Q}_{22}\right)_{-45} + 2\left(\overline{Q}_{22}\right)_0\right]\cdot 2t = 41.26 \text{ MN m}^{-1}$$

$$A_{16} = \left[\left(\overline{Q}_{16}\right)_{+45} + \left(\overline{Q}_{16}\right)_{-45} + 2\left(\overline{Q}_{26}\right)_0\right]\cdot 2t = 0$$

$$A_{26} = \left[\left(\overline{Q}_{26}\right)_{+45} + \left(\overline{Q}_{26}\right)_{-45} + 2\left(\overline{Q}_{26}\right)_0\right]\cdot 2t = 0$$

$$A_{66} = \left[\left(\overline{Q}_{66}\right)_{+45} + \left(\overline{Q}_{66}\right)_{-45} + 2\left(\overline{Q}_{66}\right)_0\right]\cdot 2t = 26.02 \text{ MN m}^{-1}$$

$$[A] = \begin{bmatrix} 109.74 & 23.45 & 0 \\ 23.45 & 41.26 & 0 \\ 0 & 0 & 26.02 \end{bmatrix} \text{MN m}^{-1}$$

Since A_{16}=A_{26}=0, the laminate is balanced.

(c) (i)

$$\begin{pmatrix} N_{\mathrm{x}} \\ N_{\mathrm{y}} \\ N_{\mathrm{xy}} \end{pmatrix} = [A] \begin{pmatrix} \varepsilon_{\mathrm{x}} \\ \varepsilon_{\mathrm{y}} \\ \gamma_{\mathrm{xy}} \end{pmatrix}$$

$$\begin{pmatrix} N_{\mathrm{x}} \\ N_{\mathrm{y}} \\ N_{\mathrm{xy}} \end{pmatrix} = \begin{bmatrix} A_{11} & A_{12} & 0 \\ A_{12} & A_{22} & 0 \\ 0 & 0 & A_{66} \end{bmatrix} \begin{pmatrix} \varepsilon_{\mathrm{x}} \\ \varepsilon_{\mathrm{y}} \\ \gamma_{\mathrm{xy}} \end{pmatrix}$$

$$N_y = 0 = A_{12}\varepsilon_{\mathrm{x}} + A_{22}\varepsilon_y \qquad \therefore \varepsilon_y = -\frac{A_{12}}{A_{22}}\varepsilon_{\mathrm{x}} = -0.57\varepsilon_{\mathrm{x}}$$

$$N_{xy} = 0 \qquad \therefore \gamma_{\mathrm{xy}} = 0$$

(ii) For the 0° ply: $\varepsilon_1 = \varepsilon_{\mathrm{x}}$ etc, so

$$\begin{pmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_{12} \end{pmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} \begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{pmatrix} = \begin{bmatrix} 40.10 & 1.76 & 0 \\ 1.76 & 5.86 & 0 \\ 0 & 0 & 2.4 \end{bmatrix} \begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{pmatrix}$$

Hence,

$$\sigma_1 = Q_{11}\varepsilon_1 + Q_{12}\varepsilon_2 = 40.10\ \varepsilon_x + (1.76 \times (-0.57\varepsilon_x) \simeq 39.10\varepsilon_x$$
$$\sigma_2 = Q_{12}\varepsilon_1 + Q_{22}\varepsilon_2 = 1.76\ \varepsilon_x + (5.86 \times (-0.57\varepsilon_x) \simeq -1.58\varepsilon_x$$
$$\sigma_{12} = 0 \text{ (since } \gamma_{12}\text{=0)}$$

For the +45° ply

$$\begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{pmatrix} = [T]^{-T} \begin{pmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{pmatrix} = \begin{pmatrix} c^2 & s^2 & sc \\ s^2 & c^2 & -sc \\ -2sc & 2sc & c^2 - s^2 \end{pmatrix} \begin{pmatrix} \varepsilon_x \\ \varepsilon_y \\ 0 \end{pmatrix}$$
$$= \begin{pmatrix} 0.5 & 0.5 & 0.5 \\ 0.5 & 0.5 & -0.5 \\ -1 & 1 & 0 \end{pmatrix} \begin{pmatrix} \varepsilon_x \\ -0.57\varepsilon_x \\ 0 \end{pmatrix} = \begin{pmatrix} 0.5\varepsilon_x - 0.285\varepsilon_x \\ 0.5\varepsilon_x - 0.285\varepsilon_x \\ -\varepsilon_x - 0.57\varepsilon_x \end{pmatrix} = \begin{pmatrix} 0.215\varepsilon_x \\ 0.215\varepsilon_x \\ -1.57\varepsilon_x \end{pmatrix}$$

where $c = \cos 45 = \sin 45 = \dfrac{\sqrt{2}}{2}$

$$\begin{pmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_{12} \end{pmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} \begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{pmatrix} = \begin{bmatrix} 40.10 & 1.76 & 0 \\ 1.76 & 5.86 & 0 \\ 0 & 0 & 2.4 \end{bmatrix} \begin{pmatrix} 0.215\varepsilon_x \\ 0.215\varepsilon_x \\ -1.57\varepsilon_x \end{pmatrix}$$

$$\sigma_1 = Q_{11}\varepsilon_1 + Q_{12}\varepsilon_2 = 40.10 \times 0.215\varepsilon_x + 1.76 \times 0.215\varepsilon_x \simeq 9\varepsilon_x$$
$$\sigma_2 = Q_{12}\ \varepsilon_1 + Q_{22}\varepsilon_2 = 1.76\ \times 0.215\varepsilon_x + 5.86 \times 0.215\varepsilon_x \simeq 1.64\varepsilon_x$$
$$\sigma_{12} = Q_{66}\gamma_{12} \simeq -3.77\varepsilon_x$$

For the $-45°$ ply

$$\begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{pmatrix} = [T]^{-T} \begin{pmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{pmatrix} = \begin{pmatrix} c^2 & s^2 & sc \\ s^2 & c^2 & -sc \\ -2sc & 2sc & c^2 - s^2 \end{pmatrix} \begin{pmatrix} \varepsilon_x \\ \varepsilon_y \\ 0 \end{pmatrix}$$
$$= \begin{pmatrix} 0.5 & 0.5 & -0.5 \\ 0.5 & 0.5 & 0.5 \\ 1 & -1 & 0 \end{pmatrix} \begin{pmatrix} \varepsilon_x \\ -0.57\varepsilon_x \\ 0 \end{pmatrix} = \begin{pmatrix} 0.5\varepsilon_x - 0.285\varepsilon_x \\ 0.5\varepsilon_x - 0.285\varepsilon_x \\ -\varepsilon_x - 0.57\varepsilon_x \end{pmatrix} = \begin{pmatrix} 0.215\varepsilon_x \\ 0.215\varepsilon_x \\ 1.57\varepsilon_x \end{pmatrix}$$

where $c = \cos(-45) = \dfrac{\sqrt{2}}{2}$ and $s = \sin(-45) = -\dfrac{\sqrt{2}}{2}$

$$\begin{pmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_{12} \end{pmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} \begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{pmatrix} = \begin{bmatrix} 40.10 & 1.76 & 0 \\ 1.76 & 5.86 & 0 \\ 0 & 0 & 2.4 \end{bmatrix} \begin{pmatrix} 0.215\varepsilon_x \\ 0.215\varepsilon_x \\ 1.57\varepsilon_x \end{pmatrix}$$

$$\sigma_1 \simeq 9\varepsilon_x$$
$$\sigma_2 \simeq 1.64\varepsilon_x$$
$$\sigma_{12} = Q_{66}\gamma_{12} \simeq 3.77\varepsilon_x$$

The only difference between the in-plane stresses for the ±45 plies is that σ_{12} for the $-45°$ ply have the opposite sign from the corresponding term for the +45° ply.

2 (a) Cost, manufacturing, material properties are key
Manufacturing
eg making high rate moulded parts by injection moulding
requires a short fibre material
- similarly hand-spray gives a low performance random structure

For medium performance moulding is common and now
vacuum infusion becomes relevant, with forming of dry fabrics
being important.

Hand lay-up or automated processes can deal with
more tailored lay-ups, including UD tows or filament
winding tows. This gives a wider range of lay-up options
Similarly pultrusion allows for different multiaxial and
UD fabrics to give different properties.

Mechanical properties can also dictate architecture, so UD gives
better in-plane properties along fibres but poor splitting resistance.
Where impact resistance is important woven or braided
fabrics are common

2 (b)(i) Tsai-Hill criterion

Need to find σ along and transverse to fibre direction each ply

$$[A] = \underset{\substack{\times 2 \\ GPa \\ mm}}{0.2} \begin{bmatrix} 139+45+45 & & \\ 2.7+31+31 & 9+45+45 & \\ 0 & 0 & 6.9+36+36 \end{bmatrix}$$

+45 and −45 symmetrical
Balanced symmetric => B = 0
$A_{16} = A_{26} = 0$

$$= \begin{bmatrix} 91.6 & 25.88 & 0 \\ 25.88 & 39.6 & 0 \\ 0 & 0 & 31.56 \end{bmatrix} \times 10^6 \, N m^{-1}$$

$$\begin{pmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{pmatrix} = [A]^{-1} \begin{pmatrix} N_x \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0.0134 & -0.0088 & 0 \\ -0.0088 & 0.0310 & 0 \\ 0 & 0 & 0.0317 \end{pmatrix} \begin{pmatrix} N_x \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0.0134 \\ -0.0088 \\ 0 \end{pmatrix} N_x \, 10^{-6} \, N^{-1} m$$

$$\left[\text{For } 0_0 \quad \begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{xy} \end{pmatrix} = \begin{pmatrix} 0.0134 \\ -0.0088 \\ 0 \end{pmatrix} N_x \times 10^{-6} \, N^{-1} m \right]$$

For +45°

$$\begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{pmatrix} = [T]^{-T} \begin{pmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{pmatrix} = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} \\ -1 & 1 & 0 \end{pmatrix} \begin{pmatrix} 0.0134 \\ -0.0088 \\ 0 \end{pmatrix} = \begin{pmatrix} 0.0023 \\ 0.0023 \\ -0.0221 \end{pmatrix} N_x \times 10^{-6} \, N^{-1} m$$

$$\begin{pmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_{12} \end{pmatrix} = Q \begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{pmatrix} = \begin{pmatrix} 139 & 2.7 & 0 \\ 2.7 & 9 & 0 \\ 0 & 0 & 6.9 \end{pmatrix} \begin{pmatrix} 0.0023 \\ 0.0023 \\ -0.0221 \end{pmatrix} \begin{matrix} \times 10^9 \, N/m^2 \\ \times 10^{-6} \, N^{-1} m \\ N_x \end{matrix}$$

$$= \begin{pmatrix} 0.329 \\ 0.027 \\ -0.153 \end{pmatrix} \times 10^3 \, N_x m^{-1} = \begin{pmatrix} 329 \\ 27 \\ -153 \end{pmatrix} N_x \, m^{-1}$$

2 (b)(i) Tsai Hill $S_L^+ = 1448$ MPa $S_T^- = 48.3$ MPa $S_{LT} = 62.1$ MPa

$$\left(\frac{329}{1448}\right)^2 - \left(\frac{329 \times 27}{1448^2}\right) + \left(\frac{27}{48.3}\right)^2 + \left(\frac{153}{62.1}\right)^2 = \frac{1}{N_x^2} \frac{(MPa)^2}{m^{-2}}$$

dominant

$N_x = 394\ kN\ m^{-1}$

Critical failure is shear failure of either +45 or −45 plies as the final term associated with shear is dominant.

(ii)

→

2(b) (ii) Need to consider the effect on the shear stress and strain in the ±45 plies

From Mohrs circle or T^{-1}, $\gamma_{12} = -\varepsilon_x + \varepsilon_y$

Changing the stiffness of just the 0° plies doesn't change ν_{xy}. Imagine imposing a ε_x, then the ±45 plies see the same ε_x as before, and the local ε_y of the 0° ply is also unchanged. So no change in the off axis ply loading and the same ν_{xy}

So increase the 0° ply stiffness decreases ε_x and ε_y, hence decreases γ_{12} in the off-axis plies and the failure load goes up.

The laminate stiffness increases by $\dfrac{139 \times 1.5 + 90}{139 + 90} = 1.30$

So the expected shear strain decreases by 30%

or the strength increases by 30%

[CCSM check - increase of 37%, the difference being due to the mixed σ failure criterion.]

3 (a) Handle - probably bending stiffness limited with torsion also being relevent

- may want a bit of flexibility
- can tailor the properties to suit.
- make hollow to reduce weight (which will be important)

At hitting face, need to ensure good strength and impact resistance - use a woven material probably with glass. Weight may not be so critical as this is needed to impart momentum to puck

Joining / shaping will be key considerations

Damage / abuse will require a tough exterior.

3 (b)

The idea is that the many tests at low levels underpin the overall design, helping also model complex shapes and different load cases. The more complex element and sub-component tests provide specific data. Just a few full tests validate the overall design.
In principle this provides a robust testing regime, but is very expensive & time consuming. Also it makes change relatively difficult. New materials or concepts need extensive re-working.
For the ice-hockey stick much less testing is needed, particularly with fewer coupon and element tests. Full scale tests will be important to demonstrate the less well-defined use cases, including impact and fatigue failure. With the failure less safety critical, overall more uncertainty can be tolerated.

3 (c)(i)

Failure - use ε allowable

- no need to worry about ν strain and neglect shear flow

$2\pi R^2 \times N_{xy} = Q \Rightarrow N_{xy} = \frac{Q}{2\pi R^2}$

$\frac{\sigma}{y} = \frac{M}{I} \Rightarrow \frac{N_x/t}{R} = \frac{M}{\pi R^3 t} \Rightarrow N_x = \frac{M}{\pi R^2}$

$\frac{N_x}{N_{xy}} = \frac{2M}{Q}$

$e^+ < e^-$ so critical

Failure $\quad \varepsilon_x = e^+ = N_x / E_x t$

$\gamma = e_{LT} = N_{xy} / G_{xy} t$

From Table 1 $\quad e^+ = 0.004, \; e_{LT} = 0.005$

So with failure by M and Q at the same time (minimises mass):

$\frac{E_x}{G_{xy}} = \frac{N_x / e^+}{N_{xy} / e_{LT}} = \frac{2 \times 5}{4} \times \frac{M}{Q} = 2.5 \frac{M}{Q}$

$\Rightarrow \frac{M}{Q} = 0.4 \times \frac{E_x}{G_{xy}}$

Limits $\quad M = 0 \Rightarrow 90\% \pm 45$

$Q = 0 \Rightarrow 90\% \; 0$

0% 0 : 90% ±45 $\Rightarrow E_x = 27, G_{xy} = 33$

$\Rightarrow M/Q = 0.33$

10% 0 : 80% ±45 $\Rightarrow E_x = 39, G_{xy} = 30$

$\Rightarrow M/Q = 0.52$

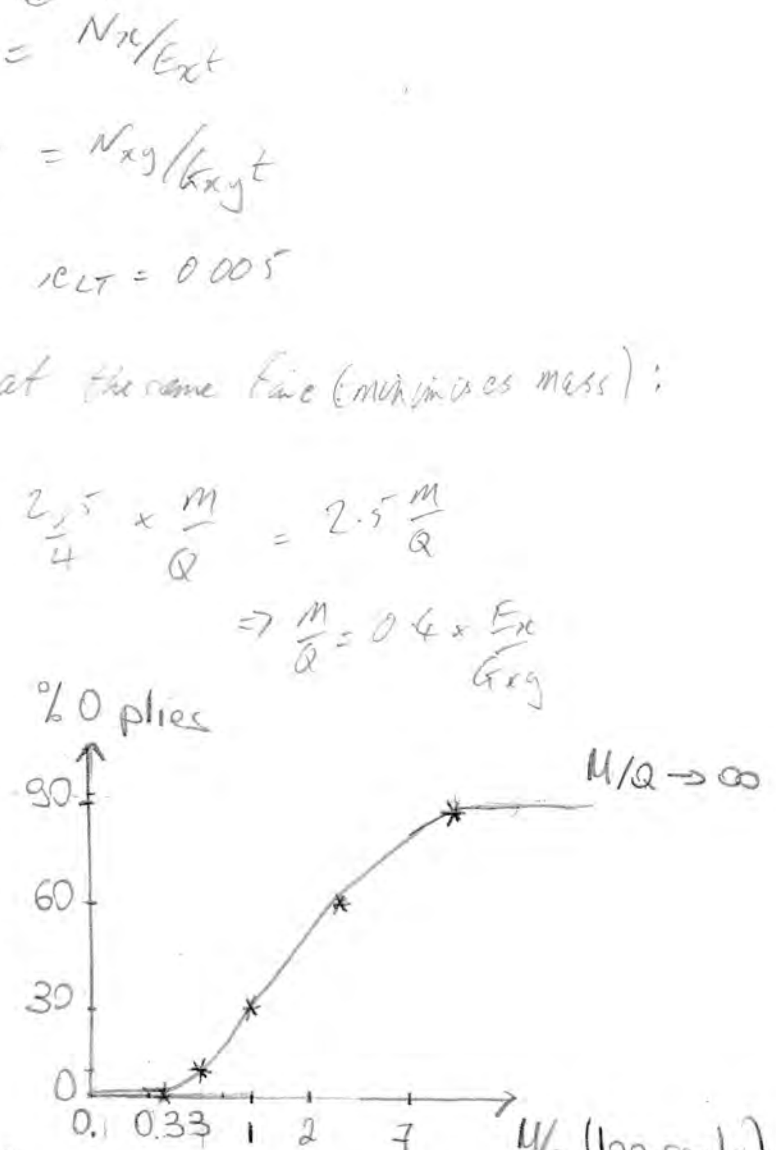
30% 0 : 60% ±45 $\Rightarrow E_x = 60, G_{xy} = 24$

$\Rightarrow M/Q = 1$

60% 0 : 30% ±45 $\Rightarrow E_x = 95, G_{xy} = 16$

$\Rightarrow M/Q = 2.38$

90% 0 : 0% ±45 $\Rightarrow E_x = 128, G_{xy} = 7 \Rightarrow M/Q = 7.31$



3 (c)(ii) For glass fibres in 45 plies

- this will affect N_{xy} loading where we expect failure in the ± 45 plies
- doesn't affect N_x loading so much

Since e_{LT} is the same for GFRP as CFRP

but G_{xy} is lower for GFRP compared with CFRP

$\Rightarrow$ composite tube is weaker in torsion

$\Rightarrow$ need to have higher proportion of ± 45 plies for given $\frac{M}{\epsilon_x}$, curve moves down.

4 (a) Transverse strength

debonding reduces strength

- debonding can be the weakest link.

- cracks join up

Tensile strength

- clustered / domino effect

when statistics of fibre strength and shear lag zone (affected by debonding) give failure at near neighbour fibre

or

- weaker bond ⇒
- longer shear lag zone ⇒
- more likely to get adjacent broken fibre

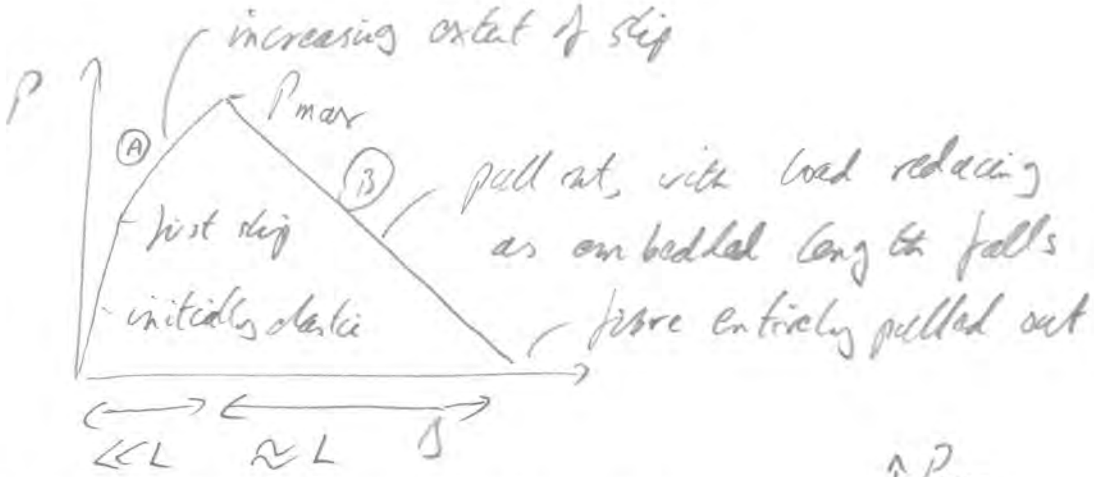
Toughness –

- work done due to pull out a key contributor to

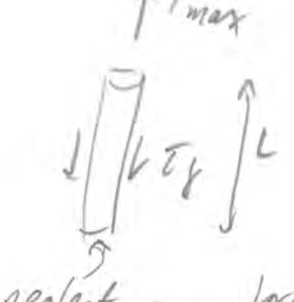
toughness where fibres bridge crack. weaker bond gives enhanced toughness.

In all cases sizing on the fibres enhances the bond strength.

4 (b)(i)



(ii) Maximum pullout force when shear stress reaches its peak value all along the embedded fibre $\Rightarrow P_{max} = \underbrace{\pi d L}_{\text{area}} \times \tau_f$

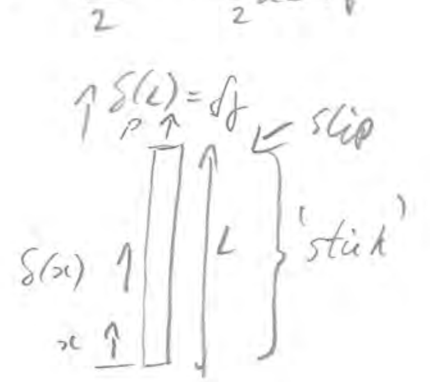


neglect force at this end, assume $L \gg d$

(iii) Assume that work done in (A) on curve (where there is limited slip) is much less than (B). This will be true assuming δ_f is small compared with L

Then work done is area under B section $= \frac{P_{max} L}{2} = \frac{\pi}{2} d L^2 \tau_f$

(iv) Slip first occurs at top of embedded section where there is maximum δ. Need to find δ as function of x



From equilibrium $\frac{\pi d^2}{4}\frac{d\sigma}{dx} = \tau \pi d \Rightarrow \frac{d\sigma}{dx} = 4\tau/d$

From strain equation $\frac{d\delta}{dx} = \varepsilon = \frac{\sigma}{E}$ ①

From elasticity $\varepsilon E = \sigma$

From slip law $\tau = \lambda \delta$ where $\lambda = \tau_f/\delta_f$

From ① $\frac{d^2\delta}{dx^2} = \frac{d\sigma}{dx}\frac{1}{E} = \frac{4\tau}{Ed} = \frac{4\lambda}{Ed}\delta = \mu^2\delta$ where $\mu^2 = \frac{4\tau_f}{E d \delta_f}$

Solution is sinh or cosh, here sinh with $\delta = 0$ at $x = 0$; also $x = L$, $\delta = \delta_f$

$\Rightarrow \delta = \delta_f \sinh \mu x / \sinh \mu L$, $P = \sigma_L \frac{\pi d^2}{4} = \frac{\pi d^2}{4} E \left.\frac{d\delta}{dx}\right|_{x=L} = \frac{\pi d^2}{4} E \mu \delta_f \coth(\mu L)$

Examiner's Comments

Question 1: Elastic Deformation

A popular question. Marks were lost mainly because of errors in estimating the laminate stiffness in part (b) and the stresses in part c(ii). A significant number of candidates erroneously took γ_{12} for ±45 plies as zero in part c(ii).

Question 2: Stress-based lamina failure

Part (a) wasn't answered very well due to lack of details. Several candidates focused on long and short fibre composites and didn't refer to the manufacturing methods or cost. Part b(i) was answered reasonably well but again several candidates took γ_{12} for ±45 plies as zero. Also, several candidates made numerical errors in estimating the stresses and ended up identifying a different failure mode. Part b(ii) was answered less well. However, the majority of candidates were able to deduce that the strength will increase.

Question 3: Laminate design (carpet plots)

Parts (a), (b) and c(ii) were answered reasonably well. In part c(i), several candidates made errors in the formulas for the shear and direct stress resultants, making it difficult to estimate M/Q. Despite errors, most candidates were able to deduce that, for a given M/Q, a higher proportion of ±45° plies was required for GFRP (part c(ii)).

Question 4: Micromechanics Strength)

This was a very unpopular question – only four candidates attempted this question. The candidates had to think a bit outside the box to answer this question. Part (a) was straightforward and was answered quite well. In part b(i), candidates sketched the applied load versus displacement with an initial elastic region but then assumed that applied load would remain constant during fibre pull-out. Parts b(ii-iv) were answered poorly as students seemed to run out of time.

Athina E. Markaki (Principal Assessor)